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Chaos Synchronization of a New Hyperchaotic Finance System Via a Novel Chatter Free Sliding Mode Control Strategy

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Abstract: In this paper, a chatter free sliding mode control(SMC) strategy for chaos synchronization is proposed to the nonlinear uncertain chaotic systems in the presence of unknown bounded uncertainties and external disturbances. The sliding mode surface with differential operators is introduced to divert the discontinuous function swith term into the first derivative of the control input, and the control input is obtained for the chaotic systems with uncertainties. Based on the Lyapunov stability theory, stability analysis is performed and a theorem serving as designing a chatter free sliding mode control input is also introduced. Finally, numerical simulations results indicate the strategy is very effective to chaos synchronization.

Keywords: Sliding Mode Control(SMC); Chaos Synchroization; Hyperchatotic Finance System.

1 Introduction

In nonlinear areas, chaos study has increasingly become an important topic in recent years [1-2].A chaotic system is a nonlinear deterministic system that displays complex and unpredictable behavior and the sensitive dependence to initial condition and chaotic attractor. Since Pecora and Carrol have proposed a successful method to synchronize two identical chaotic systems with different initial conditions[3]. For many years, there are many control techniques that have been developed for control and synchronization hyperchaotic systems, such as the Ott-Grebogi-Yorke (OGY) method [4], adaptive control [5], feedback linearization method [6], backstepping design[7], etc. All these methods have been applied in experiments or practical problems, and have obtained many achievements.

Sliding monde control is a popular robust nonlinear control strategy in recent years. Because of the intrinsic nature of robustness of sliding mode, SMC has been effectively applied to control the systems with uncertainties [8]. For this superiority, some researchers have used this technique to cope with the control problem of chaotic system. However, the traditional SMC suffers from the problem of chattering, which is caused by the high-speed switching of the controller output in order to establish a sliding mode. It is essential to further investigate a novel SMC strategy to weaken the chatter in the control input, and synchronization between different structural chaotic systems with uncertainties can be achieved.

In this paper, we can propose this method which can via a novel chatter free sliding control strategy to the application of a new hyperchaotic finance system. In 2007, the U.S. subprime mortgage crisis triggered the global economic crisis once again shows that the existence of the butterfly effect and chaos in the finance system. The novel hyperchaotic finance system was constructed in this background [9].As this global economic crisis did not cause the great depression, this hyperchaotic finance system exactly reflects this finance phenomenon.

Sliding mode control (SMC) is a variable structure control utilizing a high speed switching control law to derive to a system state trajectory onto a specified and user chosen surface, the so-called sliding surface, and to maintain the system state trajectory on the sliding surface at subsequent times. The SMC is a well known method of robust trajectory control rather than controlling the states of the system directly [10]. In this paper, to solve this issue raised above, the reliability of the proposed strategy is proved by combing Lyapunov stability theory and SMC technique [11]. The chatter is weakened effectively, synchronization between different structural chaotic systems with uncertainties can be achieved.

The rest of this paper is organized as follows. In Section 2, the new financial hyperchaotic system is presented. In Section 3 and Section 4, Chatter free sliding mode controller design is proposed. In Section 5, chaos synchonizaiton of the financial hyperchaotic systems with uncertain parameters is investigated. Numerical examples are given in this section. Finally, the conclusions are drawn in Section 6.

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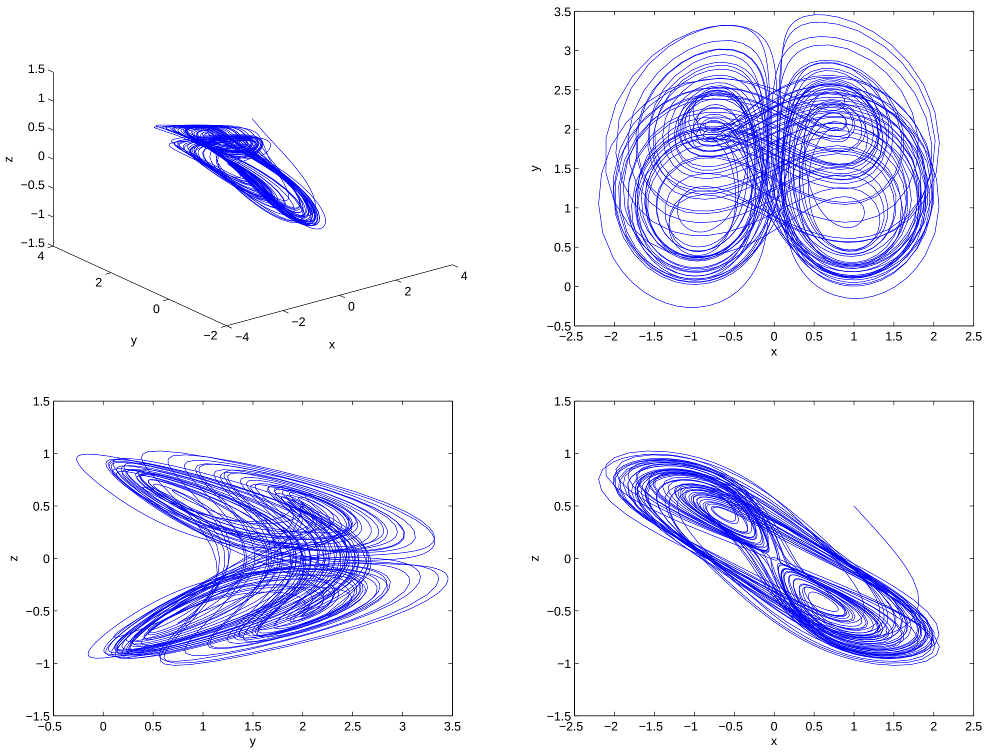

Figure 1: Phase portraits of hyperchaotic finance system (1)

2 Dynamic finance model

The novel financial hyperchaotic system [9] is described as

$$\begin{cases} \dot{x} = z + (y-a)x + w \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \\ \dot{w} = -dxy - kw \end{cases}, \tag{1}$$

where a,b,c,d and k are the parameter of the system, and they are all the positive constants. And when the parameters $a = 0.9, b = 0.2, c = 1.5, d = 0.2$ and $k = 0.17$, the Lyapunov exponents of the system are $L_1 = 0.34432, L_2 = 0.018041, L_3 = 0$ and $L_4 = -1.1499$. The hyperchaotic system has three equilibrium points:P_0 (0, 1/b, 0, 0), $P_{1,2}(\pm\theta, \frac{k+ack}{c(k-d)}, \mp\frac{\theta}{c}, \frac{d\theta(1+ac)}{cd-ck})$, where $\theta = \sqrt{\frac{kb+abck}{c(d-k)}} + 1$.

By calculation, equilibrium P_0 is an unstable saddle point in this nonlinear four-dimensional autonomous system and equilibrium P_1 and P_2 are unstable saddle points at where system (1) undergoes a Hopf bifurcation.

3 Problem formulation

Consider a class of controlled hyperchaotic systems described by the following equation:

$$\dot{x} = Ax + F(t,x) + d(t) + u(t), \tag{2}$$

where $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in R^n$ denotes the state vector of the system , $F(t,x) : R^+ \times R^n \to R^n$ indicates a continuous nonlinear vector field. $u(t) \in R^n$ is the control input vector, $d(t) \in R^n$ is the external disturbance of system. $A \in R^{n\times n}$ represents a coefficient matrix. The control goal considered in this paper is that for any given initial conditions, a chatter free sliding mode controller is designed, in spite of the uncertain terms, such that the asymptotic stability of the resulting state response of system can be achieved in the sense that $\lim_{t\longrightarrow\infty} \|x(t)\| = \lim_{t\longrightarrow\infty} \|(x_1, x_2, \cdots, x_n)^{\mathrm{T}}\| = 0,$

where $\| \bullet \|$ is the Euclidean norm of a vector. More importantly, it is also suitable for different structural synchronizations between two perturbed chaotic systems.

Remark 1 *As we investigate the hyperchaotic control of a nonlinear hyperchaotic system, $x \in R^n$ denotes the state vector, while it denotes the error vector as in the case of synchronization between hyperchaotic systems.*

Remark 2 *It is assumed that $d(t) \in R^n$ satisfies the conditions required to ensure that the system defined in Eq.(2) has a unique solution in the time interval $[t_0, +\infty), t_0 > 0$ for any given initial conditions.*

Remark 3 *In most of the earlier references concerning SMC, the proposed control input is not smooth and differentiable. This may lead chatter in controller, which could deteriorate system performance and also cause wear and tear in mechanical devices.*

4 Chatter free SMC design and stability analysis

For designing the SMC, there exist two basic steps. First, it needs to select an appropriate switching surface. Second, it needs to establish a control law which can guarantee stability of the sliding surface. First, the sliding mode surface with integral operator is defined as follows:

$$s_i(t) = \eta_i \int_0^t x_i(\tau)d\tau + x_i(t), \tag{3}$$

where η_i is a positive constant (i=1,2,,n). In order to obtain smooth and differentiable control inputs to further weaken the chatter, an idea, diverting the discontinuous sign function switch term into the first derivative of control input, is created. Using the sliding mode surfaces defined above, we put forward the following dynamical sliding mode surface as followings:

$$\sigma_i(t) = \dot{s}_i(t) + \lambda_i s_i(t), \tag{4}$$

where σ_i is a positive constant(i=1,2,,n).

It follows that
$\dot{\sigma}_i(t) = \ddot{s}_i(t) + \sigma_i \dot{s}_i(t)$
$= \ddot{x}_i(t) + (\lambda_i + \eta_i)\dot{x}_i(t) + \lambda_i \eta_i x_i(t)$
$= A_i\dot{x} + \sum_{j=1}^n \frac{\partial F_i(t,x)}{\partial x_j}\dot{x}_j + \frac{\partial F_i(t,x)}{\partial t} + \dot{d}_i(t) + \dot{u}_i + (\lambda_i + \eta_i)(A_i x + F_i(t,x) + d_i(t) + u_i(t)) + \lambda_i \eta_i x_i(t)$, where F_i denotes the ith row of matrix or vector F.

Having established the appropriate dynamical sliding mode surface, the next step is to design a sliding mode control scheme to drive the system trajectories onto the dynamical sliding mode surface $\sigma_i(t) = 0$ (i=1,2,,n). Before developing the controller design procedure, we introduce several as assumptions and the Barbalat lemma.

The uncertain term $d_i(t)$ and the derivate of the uncertain term $\dot{d}_i(t)$ is assumed to be bounded, i.e., there exists a positive bounded function $B_i(t)$ and $\bar{B}_i(t)$ making the following inequalities hold:
$|d_i(t)| < B_i(t)$,$|\dot{d}_i(t)| < \bar{B}_i(t)$ $\forall x \in R^n$(i=1,2,,n)

There exists a positive constant ξ_i satisfying the following inequalities hold:

$$\xi_i > \bar{B}_i(x) + (\lambda_i + \eta_i)B_i(t), \tag{5}$$

where the parameters λ_i and η_i are the same.(i=1,2,,n)

Lemma 4 *(Barbalat lemma)[12] If f(t) is nonnegative, integrable (has a finite integral) and uniformly continuous on the interval $[a, +\infty)$, then f(t) tends to 0 as t tends to $+\infty$.*

4.1 Main theorem.

Considering the nonlinear controlled hyperchaotic system (2), if the dynamics sliding mode control law is designed as follows:

$$\dot{u}_i = -A_i\dot{x} - \sum_{j=1}^n \frac{\partial F_i(t,x)}{\partial x_j}\dot{x}_j - \frac{\partial F_i(t,x)}{\partial t} - (\lambda_i + \eta_i)(A_i x + F_i(t,x) + u_i(t)) + \lambda_i \eta_i x_i(t) - \varepsilon_i sign(\sigma_i) \tag{6}$$

(i=1,2,,n), then the state vector $x(t)$ will converge asymptotically to zero.
Proof. Select the Lyapunov function of the system to be

$$V = \frac{1}{2}\sum_{i-1}^{n}\sigma_i^2. \tag{7}$$

Its first derivative with respect to time along the solution is given by
$\dot{V} = \sum_{i=1}^{n}\sigma_i\dot{\sigma}_i$
$= \sum_{i=1}^{n}(\sigma_i(\dot{d}_i(t) + (\lambda_i+\eta_i)\dot{d}_i(t) - \varepsilon_i sign(\sigma_i)))$
$= \sum_{i=1}^{n}(\sigma_i(\dot{d}_i(t) + (\lambda_i+\eta_i)\sigma_i\dot{d}_i(t) - \varepsilon_i|\sigma_i|)))$
$\leq \sum_{i=1}^{n}(|\sigma_i|(\bar{B}_i + (\lambda_i+\eta_i)B_i) - \varepsilon_i|\sigma_i|)$
$= -\sum_{i=1}^{n}(\varepsilon_i - (\bar{B}_i + (\lambda_i+\eta_i)B_i))|\sigma_i|$

From (6), we obtain According to $\dot{V} \leq 0$, we can find out that of σ_i is integrable and uniformly continuous on $[0, +\infty)$ with respect to time t, which indicates $\sigma_i \in L_2$. From the Barbalat lemma, we have $\sigma_i \longrightarrow 0$ as $t \longrightarrow 0$. Therefore, the dynamical sliding mode surface is globally asymptotically stable at its equilibrium point $\sigma_i = 0$ (i=1,2,,n). So the equivalent to $\dot{s}_i = -\lambda_i s_i$, which signifies that $s_i \longrightarrow 0$ as $t \longrightarrow 0$. Then, we have $x_i(t) = -k_i\int_o^t x_i(\tau)d\tau$ (i=1,2,,n) when s_i converges to zero. Differentiability of $x_i(t)$ draws $\dot{x}_i(t) = -k_i x_i(t)$(i=1,2,,n) Therefore, the state vector $x(t)$ will converge asymptotically to zero. Proof completes. ■

5 Synchronization of the two financial hyperchaotic systems with uncertain parameters

In this subsection, synchronization between two financial hyperchaotic systems is taken as an illustrative example to demonstrate and verify the performance of the proposed dynamical SMC strategy, where the drive system and the response system are denoted with x and y, respectively.

Now we also introduced the other new financial hyperchaotic system [13],

$$\begin{cases} \dot{x} = -a(x+y) + w \\ \dot{y} = -y - axz \\ \dot{z} = b + axy \\ \dot{w} = -dw - cxz \end{cases} \tag{8}$$

Consider unified financial hyperchaotic system as the drive system:

$$\dot{x} = Ax + F(t,x)$$

where

$$x^{\mathsf{T}} = (x_1, x_2, x_3, x_4),\ A = \begin{pmatrix} -a & -a & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d \end{pmatrix} + \begin{pmatrix} 0 \\ -ax_1x_2 \\ b + ax_1x_2 \\ -cx_1x_2 \end{pmatrix}.$$

Taking the perturbed financial hyperchaotic system as response system:

$$\dot{y} = By + G(t,y) + d(t) + u$$

where $y^{\mathsf{T}} = (y_1, y_2, y_3, y_4)$, $B = \begin{pmatrix} -a_1 & 0 & 1 & 1 \\ 0 & -b_1 & 0 & 0 \\ -1 & 0 & -c_1 & 0 \\ 0 & 0 & 0 & -k_1 \end{pmatrix}$,$G(t,y) = \begin{pmatrix} y_1y_2 \\ 1 - y_1^2 \\ 0 \\ -dy_1y_2 \end{pmatrix}$, $d(t) = \begin{pmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \\ d_4(t) \end{pmatrix}$.

Here the parameters are taken as a=3, b=15, c=0.2, d=0.12, a1=0.9, b1=0.2, c1=1.5, d1=0.2, and k1=0.17.

For the sake of simplicity, we select $d(t) = [\cos(t), \sin(t), \cos(t), \sin(t)]^{\mathsf{T}}$. According to theorem 1, the control $u(t) \in R^3$ can be expressed as following:
$\dot{u}_1 = a_1\dot{y}_1 - \dot{y}_3 - \dot{y}_4 - a\dot{x}_1 - a\dot{x}_2 + \dot{x}_4 - (\dot{y}_1y_2 + y_1\dot{y}_2) - (\lambda_1+\eta_1)(-a_1y_1 + y_2 + y_3 + ax_1 + ax_2 - x_4 + y_1y_2$
$+u_1) - \lambda_1\eta_1(y_1 - x_1) - \varepsilon_1 sign(\sigma_1)$
$\dot{u}_2 = b_1\dot{y}_2 - [-2y_1\dot{y}_1 + a(\dot{x}_1x_2 + x_1\dot{x}_2)] - (\lambda_2+\eta_2)(-b_1y_2 + x_2 + 1 - y_1^2 + ax_1x_2 + u_2) - \lambda_2\eta_2(y_2 - x_2) - \varepsilon_2 sign(\sigma_2)$

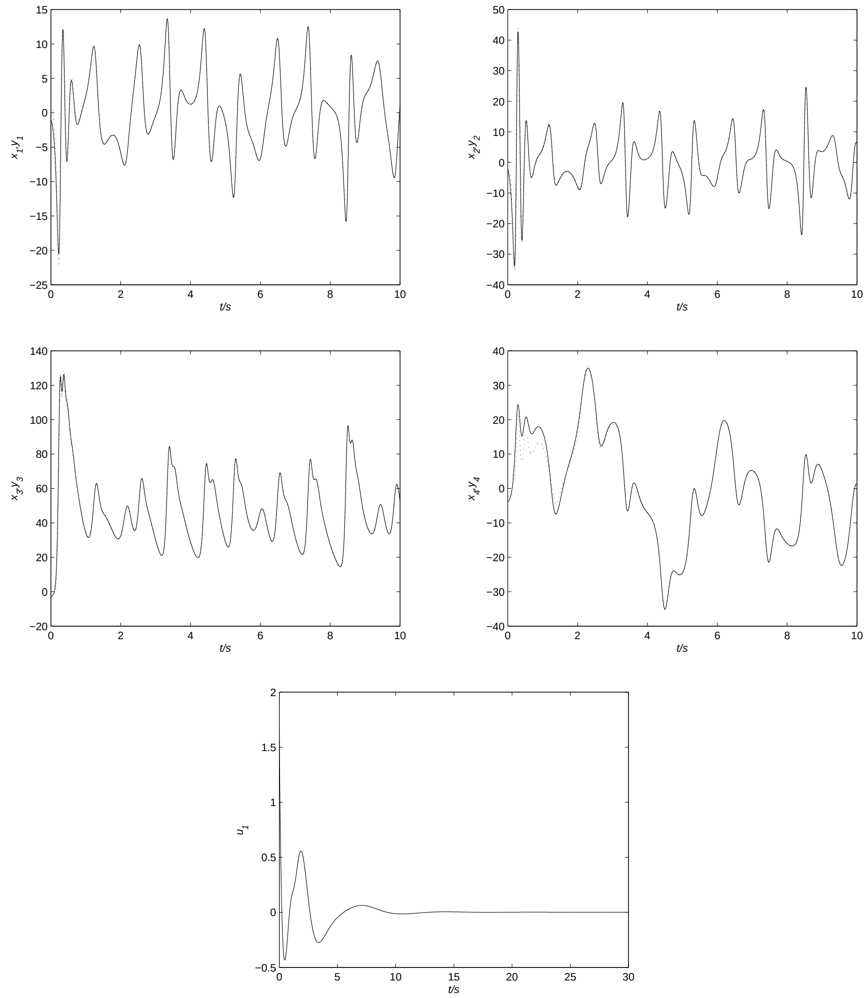

Figure 2: Simulation results of dynamic SMC for synchronization unified chaotic system

$\dot{u}_3 = \dot{y}_1 + c_1\dot{y}_3 + a(\dot{x}_1x_2 + x_1\dot{x}_2) - (\lambda_3 + \eta_3)(-y_1 - c_1y_3 - b - ax_1x_2 + u_3) - \lambda_3\eta_3(y_3 - x_3) - \varepsilon_3 sign(\sigma_3)$
$\dot{u}_4 = k_1\dot{y}_4 - d\dot{x}_4 - (-d_1\dot{y}_1y_2 - dy_1\dot{y}_2 + c\dot{x}_1x_3 + cx_1\dot{x}_3) - (\lambda_4 + \eta_4)(-k_1y_4 + dx_4 - d_1y_1y_2 + cx_1x_3 + u_4)$
$-\lambda_4\eta_4(y_4 - x_4) - \varepsilon_4 sign(\sigma_4)$

To verify the results, the numerical simulations are performed. The parameters of drive and response systems are chosen as . The initial values of the drive system and the response system are $(x1(0), x2(0), x3(0), x4(0)) = (-2, -3, -4, -5)$,and $(y1(0), y2(0), y3(0), y4(0)) = (2, 3, 4, 5)$ and select $\lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 2, \lambda_4 = 2$, and $k_1 = 2, k_2 = 2, k_3 = 2, k_4 = 2$ to result the sliding modes and chose $\varepsilon_1 = 15, \varepsilon_2 = 10, \varepsilon_3 = 10$, and $\varepsilon_4 = 10$. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with a time step size of 0.001.

In addition, it can be seen that the control input u_3 is chatter free even though the overall system is subjected to uncertainty and disturbance, because the discontinuous sign function switch term is transferred to the first derivative of the controller.

6 Conclusions

This paper has proposed a novel chatter free sliding mode control strategy for synchronization of hyperchaotic system. Based on the Lyapunov stability theory, a sliding-mode controller is designed for the regulation of the error state vector to a desired point in the state space. According to the simulations, the proposed method can be successfully applied to control problems of financial hyperchaotic system. The numerical simulations are also given to show the effectiveness of the proposed scheme.

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