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Boundary Control of Viscous Dullin-Gottwald-Holm Equation

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Abstract: his paper investigates the boundary control of viscous DGH equation on [0,1], results the existence of solution of viscous DGH equation in a short time interval with the boundary condition given in the paper and proves the global exponential stablility of DGH equation in L^2, H^1, H^2, H^3 with the given boundary condition.

Key words: Dullin-Gottwald-Holm equation;Boundary control;Viscous

1 Introduction

In [1], D.D.Holm and M.F.Staley introduced the b family PDEs that describe the balance between convection and stretching for small viscosity in the dynamics of 1D nonlinear waves in fluids:

$$m_t + \underbrace{um_x}_{convection} + \underbrace{bu_x m}_{stretching} = \underbrace{\varepsilon m}_{vis\cos ity}, u = g * m, \tag{1.1}$$

Here $u = g*m$ denotes $u(x) = \int_{-\infty}^{\infty} g(x-y)m(y)dy$. The convolution relates velocity u to momentum density m by integration against the kernel $g(x)$. [1] studied the effects of the balance parameter b and kernel $g(x)$ on the solitary wave structures and investigated their interactions analytically for $\varepsilon = 0$ and numerically for small viscosity, $\varepsilon \neq 0$.

When (1.1) restrict to the peakon case $g(x) = e^{-|x|/\alpha}$ with length scale α, $m = u - \alpha^2 u_{xx}$, the equation (1.1) may be expressed solely in terms of the velocity $u(x,t)$ as

$$u_t + (b+1)uu_x - \varepsilon u_{xx} = \alpha^2 (u_{xxt} + uu_{xxx} + bu_x u_{xx} - \varepsilon u_{xxxx}) \tag{1.2}$$

For $b = 2$, (1.2) is the one-dimensional version of the three dimensional Navier-Stokes-alpha model for turbulence [2, 3] as $\varepsilon \neq 0$. For $b = 2$, $\varepsilon = 0$, it becomes Camassa-Holm equation [11]. It has a bi-Hamiltionian structure and is completely integrable [12-16]. In [4] Danping Ding, Lixin Tian researched the solution of 1D Navier-Stokes-alpha model or viscosity Camassa-Holm equation and got the existence of the global attractor of viscosity Camassa-Holm equation.For $b = 3$, $\varepsilon = 0$, (1.2) is Degasperis-Procesi equation.

$$u_t - u_{txx} + 4uu_x = 3u_x u_{xx} + uu_{xxx}, t > 0\ ,\ x \in R \tag{1.3}$$

It is completely integrable.

Dullin,Gottwald,Holm discussed the following 1+1 quadratically nonlinear equation in this class for a unidirectional water wave with fluid velocity $u(x,t)$,

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$$m_t + c_0 u_x + u m_x + 2 m u_x = -\gamma u_{xxx} \tag{1.4}$$

where $m = u - \alpha^2 u_{xx}$ is a momentum variable, the constants α^2 and γ/c_0 are squares of length scales, and $c_0 = \sqrt{gh}$ is the linear wave speed for undisturbed water at rest at spatial infinity. (1.4)is derived by using asymptotic expansions directly in the Hamiltonian for Euler's equations in the shallow water regime and thereby shown to be bi-Hamiltonian.It also has a Lax pair formulation. Equation(1.4) combines the linear dispersion of the KdV equation with the nonlinear/nonlocal dispersion of CH equation, yet still preserves integrability via the inverse scattering transform (IST)method.This IST-integrable class of equations contains both the KdV and CH equation as limit cases.

Many researches have been researched on DGH equation.In[9],Lixin Tian,Guilong Gui Yue Liu studied some properties of the solutions of the Cauchy problem and the scattering problem associated to (1.4).The issue of passing to the limit as the dispersive parameter tends to zero for the solution of the DGH equation is investigated, and the convergence of solutions to the DGH equation as $\alpha^2 \to 0$ is studied, and the scattering data of the scattering problem for the equation can be explicty expressed;the new exact peaked solitary wave solutions are obtained in the DGH equation.Besides,Tian and Fan [10] also study the attacttor of DP equation which also enlighten the author.

In the recent 20 years,boundary control has been constantly investigated.More people have paid attentions to boundary control of Kdv,KdVB, MKdV and K-S equation.Byrnes et. al studied local stability of Burgers equationVan ly et. al. consummated this result furtherbut still local.Miroslav Kristic studied global stability of Burgers equation[5].Biler Rassel Zhang Bingyu studied KdVB equation under periodical boundary condition[17]-[19].Liu and Kristic studied the stability of KdVB equation in a limited area[6].Wei-jiu Liu,Miroslav Krstic studied global stability of K-S equation [7], Haixia Chao also did some studies on boundary control of K-S equation[8]. Boundary control of DGH equation has not been studied. As above mentioned,the added viscosity is reasonable.In this paper, we study the following viscous DGH equation

$$\begin{cases} u_t - \alpha^2 u_{xxt} - \varepsilon\,(u - u_{xx})_{xx} + 2\omega u_x + \gamma u_{xxx} + 3 u u_x = \alpha^2 (2 u_x u_{xx} + u u_{xxx}) \\ u_0 = u(x,0) \\ u(0,t) = u(1,t) \\ u_x(1,t) = u_x(0,t) \\ u_{xx}(0,t) = u_{xx}(1,t) \end{cases} \tag{1.5}$$

where $t > 0, x \in \Omega, \Omega = [0,1]$. Let $v = u - \alpha^2 u_{xx}$,(1.5) becomes

$$\begin{cases} (1-\alpha^2) v_t - \varepsilon v_{xx} + 2\omega u_x + u v_x + 2 u_x v = 0 \\ u_0 = u(x,0) \\ u(0,t) = u(1,t) \\ u_x(1,t) = u_x(0,t) \\ u_{xx}(0,t) = u_{xx}(1,t) \end{cases} \tag{1.6}$$

where $t > 0, x \in \Omega, \Omega = [0,1]$.

In this paper,we discuss the global well-posedness of (1.5). Let $T > 0$,$u_0 \in H^3$, (1.6) admits a unique solution$u \in ((0,T), H^3(0,1)) \bigcap C'((0,T), H^2(0,1))$ .The proof is standard;the local well-posedness is established first by using Galerkin Procedure and then the global well-posedness is obtained by finding the needed global a priori estimate.

The outline of this paper is as follows.In section 1, we introduce the differences and relatives among C-H and DGH equation and some relative studies are given out. In section 2, we introduce the main definitions and theorems. In section 3, we establish the proof of the main theorems.

2 Main Theorem

The following notation is used in this paper: $(,)$ is the Ł^2\$$innerproductand\|.\|$the correspongding L^2norm, $\|u\|_{H^m(\Omega)} = \|D^m u\|_{L^2(\Omega)}$.The inner product here is equivalent to the nature inner product in $H^m\left(\Omega\right)$ when $mes\left(\Omega\right) < +\infty$;

$$\|u\|_{L^2(\Omega)} \triangleq |u|\,, \|Du\|_{L^2(\Omega)} \triangleq \|u\|\,, \|D^m u\|_{L^2(\Omega)} \triangleq |D^m u|\,;$$

$B(u,v) = u\nabla v$,where ∇ is $Hamilton$ operator.

Here we only study one dimentional viscous DGH equation, then $\nabla u = u_x$.

Denote $b(u,v,w) = (B(u,v), w) = \int_\Omega (u\nabla v) w dx$

With following boundary condition, we have

$$(B(u,v), w) = -(B(u,w), v) - (B(w,u), v)$$

$$(B(v,u), w) = -(B(w,v), u) - (B(v,w), u)$$

Then $(B(u,v), u) = -2(B(u,u), v)\,, (B(u,v), u) = -2(B(v,u), u)$

$(B(u,u), v) = (B(v,u), u)\,, (B(u,u), u) = 0$

Denote $A = -\Delta$, Δ is Laplace operator, $v = u + \alpha^2 Au$

Eq. (1.6) follows that

$$\begin{cases} \frac{dv}{dt} + \varepsilon Av + 2\omega\nabla u + \gamma\nabla Au + B(u,v) + 2B(v,u) = 0 \\ u_0 = u(x,0) \\ u(0,t) = u(1,t) \\ u_x(1,t) = u_x(0,t) \\ u_{xx}(0,t) = u_{xx}(1,t) \end{cases} \tag{2.1}$$

where A is a self-adjoint positive operator with compact inverse. Hence the space H has an orthonormal basis $\{w_j\}_{j=1}^{\infty}$ of eigenfunctions of$A$, i.e. $Aw_j = \lambda_j w_j$, with $0 < \lambda_1 \le \lambda_2 \le \cdots, \lambda_j \to \infty$ when $j \to \infty$

Theorem: With $u_0 \in H^3$, Eq.(2.1)has a global solution the solution satisfis the following L^2, H^1, H^2, H^3 stability:

(1)$|u_m|^2 + \alpha^2 \|u_m\|^2 \le \left(|u_{m_0}|^2 + \alpha^2 \|u_{m_0}\|^2\right) \exp(-2\varepsilon\lambda_1 t$

(2)$\|u\|_{H^2}^2 \le \left(\|u_0\|_{H^1}^2 + \|u_0\|_{H^2}^2\right) \exp(3C_1 + C_2 + C_3 - \varepsilon\lambda_1$, where$\varepsilon \in \left(\frac{3C_1+C_2+C_3}{\lambda_1}, +\infty\right)$

(3)$\|u\|_{H^3}^2 \le \left(\|u_0\|_{H^2}^2 + \|u_0\|_{H^3}^2\right) \exp\left(\|u_0\|_{H^1}\left(\|u_0\|_{H^1}^2 + \|u_0\|_{H^2}^2\right) - \varepsilon\lambda_1\right) t$,

where $\varepsilon \in \left(\frac{\|u_0\|_{H^1}\left(\|u_0\|_{H^1}^2+\|u_0\|_{H^2}^2\right)}{\lambda_1}, +\infty\right)$

3 The proof of the theorem

The proof of theorem 1, Galerkin Procedure is used to prove global existence.

Let $\{\phi_j\}_{j=1}^{\infty}$ be an orthonormal basis of H consisting of eigenfunctions of the operator A.

The Galerkin Procedure for Eq.2.1is the ordinary differential system,

$$\begin{cases} \frac{dv_m}{dt} + \varepsilon Av_m + 2\omega\nabla u_m + \gamma\nabla Au_m + p_m B(u_m, v_m) + p_m 2B(v_m, u_m) = 0 & (3.1a) \\ u_m(0) = p_m u(0) & (3.1b) \end{cases}$$

where $v_m = u_m + \alpha^2 Au_m$. Since the nonlinear term is quadratic in u_m, the system (3.1) has a unique solution for a short interval of time $(0, T_m)$ by the classical theory of ordinary differential equationsour purpose is to show the existence of the solution under given boundary control.

We take the inner product of (3.1a) with u_m in Ω,

$$\frac{1}{2}\frac{d}{dt}\left(|u_m|^2 + \alpha^2\|u_m\|^2\right) + \varepsilon\left(\|u_m\|^2 + \alpha^2|Au_m|^2\right) = 0 \tag{3.2}$$

By poincare inequation,

$$\frac{d}{dt}\left(|u_m|^2 + \alpha^2\|u_m\|^2\right) + 2\varepsilon\lambda_1\left(|u_m|^2 + \alpha^2\|u_m\|^2\right) \le 0$$

By Gronwall inequality,

$$|u_m|^2 + \alpha^2 \|u_m\|^2 \leq \left(|u_{m_0}|^2 + \alpha^2 \|u_{m_0}\|^2\right) \exp\left(-2\varepsilon\lambda_1 t\right) \leq |u_{m_0}|^2 + \alpha^2 \|u_{m_0}\|^2 \triangleq r_1 \tag{3.3}$$

Integrating (3.3) over the interval $[t, t+r]$

$$\int_t^{t+r} \left(\|u_m(s)\|^2 + \alpha^2 |Au_m(s)|^2\right) ds \leq r_1$$

Take the inner product of $(3.1a)$ with Au_m in $(0,1)$,

$$\begin{aligned}&\tfrac{1}{2}\tfrac{d}{dt}\left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) + \varepsilon\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right) + p_m\left(B\left(u_m, v_m\right), Au_m\right) + \\ &2p_m\left(B\left(v_m, u_m\right), Au_m\right) = 0\end{aligned} \tag{3.4}$$

Then, we have

$$|p_m\left(B\left(u_m, v_m\right), Au_m\right) + 2p_m\left(B\left(v_m, u_m\right), Au_m\right)|$$

$$\leq \left(B\left(u_m, u_m\right), Au_m\right) + \left(B\left(Au_m, u_m\right), Au_m\right) + \left(B\left(u_m, Au_m\right), Au_m\right) \tag{3.5}$$

By $Agmon$ inequality when $n = 1$:

$$\|\varphi\|_{L^\infty} \leq c\|\varphi\|_{L^2}^{\frac{1}{2}} \|\varphi\|_{H^1}^{\frac{1}{2}}$$

$$|p_m\left(B\left(u_m, Au_m\right), Au_m\right)| \leq \frac{1}{2}\|\nabla u_m\|_{L^\infty} |Au_m|^2 \leq \frac{c_4}{2}\|u_m\|^{\frac{1}{2}} |Au_m|^{\frac{5}{2}}$$

$$|p_m\left(B\left(Au_m, u_m\right), Au_m\right)| \leq \|\nabla u_m\|_{L^\infty} |Au_m|^2 \leq c_5 \|u_m\|^{\frac{1}{2}} |Au_m|^{\frac{5}{2}}$$

$$|p_m\left(B\left(u_m, u_m\right), Au_m\right)| \leq \frac{1}{2}\|\nabla u_m\|_{L^\infty} \|\nabla u_m\|^2 \leq \frac{c_1}{2}\|u_m\|^{\frac{5}{2}} |Au_m|^{\frac{1}{2}}$$

From above inequalities, we get

$$|p_m\left(B\left(u_m, v_m\right), Au_m\right) + 2p_m\left(B\left(v_m, u_m\right), Au_m\right)|$$

$$\leq \frac{3c_1}{2}\|u_m\|^{\frac{5}{2}} |Au_m|^{\frac{1}{2}} + \left(\frac{2c_5 + c_4}{2}\right)\|u_m\|^{\frac{1}{2}} |Au_m|^{\frac{5}{2}}$$

$$\leq \frac{3}{2}c_1 \|u_m\|^{\frac{5}{2}} |Au_m|^{\frac{1}{2}} + \left(\frac{2c_5 + c_4}{2}\right)\|u_m\|^{\frac{1}{2}} |Au_m|^{\frac{5}{2}}$$

$$\leq c_6 \|u_m\|^{\frac{1}{2}} |Au_m|^{\frac{1}{2}} \left(\|u_m\|^2 + |Au_m|^2\right)$$

$$\leq \frac{1}{2}\varepsilon\lambda_1 \left(\|u_m\|^2 + |Au_m|^2\right) + c_7 \|u_m\| |Au_m| \left(\|u_m\|^2 + |Au_m|^2\right) \tag{3.6}$$

$$c_6 = \max\left(3c_1, 2c_5 + c_4\right), c_7 = \frac{c_6^2}{8\varepsilon\lambda_1}$$

By $Young$ inequality, it follows that

$$\frac{1}{2}\frac{d}{dt}\left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) + \varepsilon\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right)$$

$$\leq \varepsilon\lambda_1 \left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) + c_7 \|u_m\| |Au_m| \left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) \tag{3.7}$$

By $Poincare$ inequality $|Au_m|^2 > \lambda_1 \|u_m\|^2, |\nabla Au_m|^2 > \lambda_1 |Au_m|^2$ and(3.4)
We get

$$\frac{d}{dt}\left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) \le 2c_7 \|u_m\| |Au_m| \left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right)$$

By $Young$ inequality and above inequalities,

$$\frac{d}{dt}\left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) \le c_7 \left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right)^2 \tag{3.8}$$

Denote $y = \|u_m(s)\|^2 + \alpha^2 |Au_m(s)|^2, g = c_7 \left(\|u_m(s)\|^2 + \alpha^2 |Au_m(s)|^2\right)^2$
By inequality (3.6), we have

$$\int_t^{t+r} y(s)ds \le \frac{r_1}{\varepsilon}, \int_t^{t+r} g(s)ds \le \frac{r_1 c_7}{\varepsilon}$$

By $Gronwall's$ inequality we have

$$\left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right) \le \frac{r_1}{r\varepsilon} \exp\left(\frac{r_1 c_7}{\varepsilon}\right) \triangleq r_3, t > t_0 + r \tag{3.9}$$

where $r, r_1, c_7$are nonnegative constants, Integrating3.7over the interval $[t, t+r]$

$$\varepsilon \int_t^{t+r} \left(|Au_m(s)|^2 + \alpha^2 |\nabla Au_m(s)|^2\right) ds$$

$$\le \int_t^{t+r} \left(\varepsilon\lambda_1 \left(\|u_m(s)\|^2 + \alpha^2 |Au_m(s)|^2\right) + \frac{c_7}{2}\left(\|u_m(s)\|^2 + \alpha^2 |Au_m(s)|^2\right)^2\right) ds$$

$$+ \left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right)$$

$$\le \left(\varepsilon\lambda_1 r_2 + \frac{c_7 r_3^2}{2}\right) r + r_3 \triangleq r_4 \tag{3.10}$$

Take the inner product of $(3.1a)$ with $A^2 u_m$ in Ω

$\frac{1}{2}\frac{d}{dt}\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right) + \varepsilon\left(|\nabla Au_m|^2 + \alpha^2 \left|A^2 u_m\right|^2\right) + p_m\left(B(u_m, v_m), A^2 u_m\right) + 2p_m\left(B(v_m, u_m), A^2 u_m\right) = 0$

By $Young$ inequality, we have

$$\begin{aligned}&\tfrac{1}{2}\tfrac{d}{dt}\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right) + \varepsilon\left(|\nabla Au_m|^2 + \alpha^2 \left|A^2 u_m\right|^2\right)\\ &\le \varepsilon\lambda_1\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right) + c_8 \|u_m\| |Au_m| \left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right)\end{aligned} \tag{3.11}$$

By $Poincare$ inequality and $Young$ inequality, we have

$$\begin{aligned}&\tfrac{d}{dt}\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right) \le c_8 \|u_m\| |Au_m| \left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right)\\ &\le \tfrac{c_8}{2}\left(\|u_m\|^2 + \alpha^2 |Au_m|^2\right)\left(|Au_m|^2 + \alpha^2 |\nabla Au_m|^2\right)\end{aligned}$$

From(3.6)and(3.10),we get

$$\frac{c_8}{2}\int_t^{t+r} \left(\|u_m(s)\|^2 + \alpha^2 |Au_m(s)|^2\right) ds \le \frac{c_8 r_1}{2\varepsilon}$$

$$\int_t^{t+r} \left(|Au_m(s)|^2 + \alpha^2 |\nabla Au_m(s)|^2\right) ds \le \frac{r_3}{\varepsilon}$$

By $Gronwall's$ inequality we have

$$|Au_m|^2 + \alpha^2 |\nabla Au_m|^2 \le \frac{r_4}{r\varepsilon} \exp \frac{c_8 r_1}{2\varepsilon} \triangleq r_5 \quad t > t_0 \tag{3.12}$$

Integrating(3.12)over the interval $[t, t+r]$, we get

$$\varepsilon \int_t^{t+r} \left(|\nabla Au_m(s)|^2 + \alpha^2 \left| A^2 u_m(s) \right|^2 \right) ds \le \left(\varepsilon \lambda_1 r_5 + \frac{c_8 r_2 r_5}{2} \right) r + r_5$$

Take the inner product of $(3.1a)$ with $A^3 u_m$ in Ω,

$$\frac{1}{2} \frac{d}{dt} \left(|\nabla Au_m|^2 + \alpha^2 \left| A^2 u_m \right|^2 \right)$$

$$\le c_9 \|u_m\|^{\frac{1}{2}} |Au_m|^{\frac{1}{2}} \left(|\nabla Au_m|^2 + \alpha^2 \left| A^2 u_m \right|^2 \right) + c_{10} |Au_m|^8 |\nabla Au_m|^2$$

By $Gronwall's$ inequality we have

$$|\nabla Au_m|^2 + \alpha^2 \left| A^2 u_m \right|^2 \le \left(\frac{r_5}{\varepsilon r} + 2c_{10} r_2^4 r_5 r \right) \exp \left(c_9 r_2^{\frac{1}{2}} r \right) \triangleq r_6$$

Now we get: $|u_m|, \|u_m\|, |Au_m|, |\nabla Au_m|$ and $\left|A^2 u_m\right|$are finite, then $|v_m|, \|v_m\|$ and $|Av_m|$ are finite, then we get $\left|\frac{du_m}{dt}\right|$ and $\left|\frac{dv_m}{dt}\right|$ are also finite. By $Aubin's$ compactness Theorem., we conclude that there is a subsequence u_m', such that $u_m' \to u$, or equivalently $v_m' \to v$. Relable u_m' and v_m' by u_m and v_m. Now we prove u, v satisfy equation (2.1).

Let $w \in D(A)$we know $|w|$ is finite from the above discussion, from ordinary differential equations $(3.1a)$ we get,

$$(v_m(t), w) + \varepsilon \int_{t_0}^t (v_m(s), p_m Aw) ds + 3 \int_{t_0}^t (B(v_m(s), u_m(s)), p_m w) ds +$$

$$\int_{t_0}^t (B(u_m(s), v_m(s)), p_m w) ds = (v_m(t_0), w)$$

Now, it is clear that

$$\lim_{m \to \infty} \int_{t_0}^t (v_m(s), Aw) ds = \int_{t_0}^t (v(s), Aw) ds$$

thus

$$(v_m(t), w) \to (v(t), w), \int_{t_0}^t (v_m(s), p_m Aw) ds \to \int_{t_0}^t (v(s), Aw) ds \quad m \to \infty$$

$$\left| \int_{t_0}^t (\nabla v_m(s), p_m w) ds - \int_{t_0}^t (\nabla v(s), w) ds \right|$$

$$\le \left| \int_{t_0}^t (\nabla v_m(s), p_m w - w) ds \right| + \left| \int_{t_0}^t (\nabla v_m(s) - \nabla v(s), w) ds \right|$$

$$\le \|v_m\| |p_m w - w| + \|v_m - v\| |w| \to 0$$

$$\left| \int_{t_0}^t (B(v_m(s), u_m(s)), p_m w) ds - \int_{t_0}^t (B(v(s), u(s)), w) ds \right|$$

$$\le I_m^1 + I_m^2 + I_m^3$$

where

$$I_m^1 = \left| \int_{t_0}^t (B(v_m(s), u_m(s)), p_m w - w) ds \right| \le \int_{t_0}^t |v_m(s)| \|u_m(s)\| |p_m w - w| ds \to 0$$

$$I_m^2 = \left| \int_{t_0}^{t} \left(B\left(v_m\left(s \right) - v\left(s \right), u_m\left(s \right) \right), w \right) ds \right| \leq \int_{t_0}^{t} \left| v_m\left(s \right) - v\left(s \right) \right| \left\| u_m\left(s \right) \right\| \left| w \right| ds \to 0$$

$$I_m^3 = \left| \int_{t_0}^{t} \left(B\left(v\left(s \right), u_m\left(s \right) - u\left(s \right) \right), w \right) ds \right| \leq \int_{t_0}^{t} \left| v\left(s \right) \right| \left\| u_m\left(s \right) - u\left(s \right) \right\| \left| w \right| ds \to 0$$

From the above discussionand by $Lebesgue$ theormwe get,

$$\lim_{m\to\infty} \int_{t_0}^{t} \left(B\left(v_m\left(s \right), u_m\left(s \right) \right), p_m w \right) ds = \int_{t_0}^{t} \left(B\left(v\left(s \right), u\left(s \right) \right), w \right) ds$$

Similarly, we have

$$\lim_{m\to\infty} \int_{t_0}^{t} \left(B\left(u_m\left(s \right), v_m\left(s \right) \right), p_m w \right) ds = \int_{t_0}^{t} \left(B\left(u\left(s \right), v\left(s \right) \right), w \right) ds$$

Above all$w \in D\left(A \right)$we get

$$\begin{aligned} &\left(v\left(t \right), w \right) + \varepsilon \int_{t_0}^{t} \left(v\left(s \right), Aw \right) ds + 3\int_{t_0}^{t} \left(B\left(v\left(s \right), u\left(s \right) \right), w \right) ds + \\ &\int_{t_0}^{t} \left(B\left(u\left(s \right), v\left(s \right) \right), w \right) ds = \left(v\left(t_0 \right), w \right) \end{aligned}$$

let u,v be the same two solutions of (2.1),and $\omega = u - v$,we get

$$\omega_t - \omega_{xxt} - \varepsilon \left(\omega - \omega_{xx} \right)_{xx} + 3\left(u\omega_x + u_x\omega - \omega\omega_x \right)$$

$$= 2\left(u_x\omega_{xx} + u_{xx}\omega_x - \omega_x\omega_{xx} \right) + \left(u\omega_{xxx} + u_{xxx}\omega - \omega\omega_{xxx} \right) \tag{3.13}$$

Take the inner product of (3.13) in [0,1],

$$\frac{1}{2}\frac{d}{dt}\left(\left| \omega \right|^2 + \left\| \omega \right\|^2 \right) + \varepsilon \left(\left\| \omega \right\|^2 + \left| A\omega \right|^2 \right) = 2\left(u_x, \omega\omega_x \right) + \left(u_x, \left(\omega - \omega_x \right)\omega_{xx} \right)$$

by(3.9),we get$\left\| u \right\| \leq r_1^{\frac{1}{2}} e^{-\varepsilon\lambda_1}$,$\left| Au \right|^2 < r_2$
from(3.5),we get

$$\frac{d}{dt}\left(\left| \omega \right|^2 + \left\| \omega \right\|^2 \right) + M\left(\left| \omega \right|^2 + \left\| \omega \right\|^2 \right) \leq 0$$

where M is constant then

$$\left| \omega \right|^2 + \left\| \omega \right\|^2 \leq \left(\left| \omega_0 \right|^2 + \left\| \omega_0 \right\|^2 \right) e^{-Mt} = 0$$

so $\left\| \omega \right\| = 0$ i.e. $\omega = 0, u = v$
By (3.3),we have

$$\left| u_m \right|^2 + \alpha^2 \left\| u_m \right\|^2 \leq \left(\left| u_{m_0} \right|^2 + \alpha^2 \left\| u_{m_0} \right\|^2 \right) \exp\left(-2\varepsilon\lambda_1 t \right)$$

Now, take the inner product of (2.1) with Au in $(0,1)$,

$$\left(\left| Au \right|^2 + \alpha^2 \left| \nabla Au \right|^2 \right) + \left(B\left(u, v \right), Au \right) + 2\left(B\left(v, u \right), Au \right) = 0$$

By computing, we have

$$\left| \left(B\left(u, v \right), Au \right) + 2\left(B\left(v, u \right), Au \right) \right| = 3\left(B\left(u, u \right), Au \right) + \left(B\left(Au, u \right), Au \right) + \left(B\left(u, Au \right), Au \right)$$

$$\left| \left(B\left(u, u \right), Au \right) \right| \leq \left(\left\| u \right\|^2 + \left| Au \right|^2 \right)$$

$$|(B\left(Au,u\right),Au)|\leq\|u\|\left|Au\right|^{2}\leq c_{2}\left(\|u\|^{2}+|Au|^{2}\right)$$

$$|(B\left(u,Au\right),Au)|\leq c_{3}\left|Au\right|^{2}\leq c_{3}\left(\|u\|^{2}+|Au|^{2}\right)$$

$$\frac{1}{2}\frac{d}{dt}\left(\|u\|^{2}+|Au|^{2}\right)\leq\left(3C_{1}+C_{2}+C_{3}-\varepsilon\lambda_{1}\right)\left(\|u\|^{2}+|Au|^{2}\right)$$

$$\|u\|^{2}+|Au|^{2}\leq\left(\|u_{0}\|^{2}+|Au_{0}|^{2}\right)\exp\left(3C_{1}+C_{2}+C_{3}-\varepsilon\lambda_{1}\right)t$$

So we get Th.(2)
Take the inner product of (2.1) with A^2u in (0,1),

$$\tfrac{1}{2}\tfrac{d}{dt}\left(|Au|^{2}+|\nabla Au|^{2}\right)+\varepsilon\left(|\nabla Au|^{2}+\left|A^{2}u\right|^{2}\right)+\left(B\left(u,v\right),A^{2}u\right)+2\left(B\left(v,u\right),A^{2}u\right)=0$$

By $Young$ inequality, we have

$$\begin{aligned}&\tfrac{1}{2}\tfrac{d}{dt}\left(|Au|^{2}+|\nabla Au|^{2}\right)+\varepsilon\left(|\nabla Au|^{2}+\left|A^{2}u\right|^{2}\right)\\&\leq\left(|Au|^{2}+|\nabla Au|^{2}\right)+\|u\|\left|Au\right|\left(|Au|^{2}+|\nabla Au|^{2}\right)\\&\leq\left(|Au|^{2}+|\nabla Au|^{2}\right)+\|u_{0}\|\left(\|u_{0}\|^{2}+|Au_{0}|^{2}\right)\left(|Au|^{2}+|\nabla Au|^{2}\right)\end{aligned}$$

By $Poincare$ inequality and $Young$ inequality, we have

$$\frac{d}{dt}\left(|Au|^{2}+|\nabla Au|^{2}\right)\leq\left(\|u_{0}\|\left(\|u_{0}\|^{2}+|Au_{0}|^{2}\right)-\varepsilon\lambda_{1}\right)\left(|Au|^{2}+|\nabla Au|^{2}\right)$$

$$|Au|^{2}+|\nabla Au|^{2}\leq\left(|Au_{0}|^{2}+|\nabla Au_{0}|^{2}\right)\exp\left(\|u_{0}\|\left(\|u_{0}\|^{2}+|Au_{0}|^{2}\right)-\varepsilon\lambda_{1}\right)t$$

We get Th.(3).The proof is completed.

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