Optimal Extraction of Exhaustible Resources under Resource Tax Reform in China

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(Received 14 September 2010, accepted 18 November 2010)

Abstract: This paper discusses the impacts of resource tax reform on the optimal resources extraction in China when the time of resource tax reform and the ways of taxes levy are uncertain. Based on the assumption that the process for accepting resource tax reform follows Poisson process, we first divide resource extraction into two processes by using the idea of dynamic planning. We then apply the optimal control theory to build dynamic optimal extraction model and give the optimal extraction path by solving the model. The results show that: (1) when the extent of uncertain is fixed, the initial resources extraction and extraction rate increase by increasing the taxes rate; (2) when taxes rate is fixed, extraction rate increase by increasing uncertainty of the reform, but initial resources extraction is related with taxes rate.

Keywords: exhaustible resources; resources taxes reform; ad valorem taxation; dynamic optimization

1 Introduction

Exhaustible resources have limited reserves and cannot be regenerated. In China, exhaustible resources were considered to be free of charge for a long time. As a result, the valuable resources were wasted tremendously and exhausted earlier in order to seek short-term profits. So the China government has carried out resource tax reform in recent years to protect these resources. However, the resource tax rate taken by the country was too low to distort resources’ prices, which prevents exploiting and using resources effectively. Also it is hard to keep economic sustainable development. In 2007, the resource tax rates for some mineral resources in China were adjusted. In 2008, an officer of State Administration of Taxation pointed out that the project of resources taxes reform should change levy ways from ”specific taxation” to ”ad valorem taxation”. But the concrete reform program isn’t yet offered. Also we can find that the details of the taxes reform have been subject to quite frequent adjustments. Reform type is often understood and anticipated in advanced, but the time of reform and taxes parameter is uncertain. In this paper, we analyze optimal extraction of exhaustible resources under changes of levy ways and noise about timing.

Since Hotelling (1931), optimal extraction problems of exhaustible resources have been studied by many researchers. They described the patterns of resources depletion under different assumptions about resources demand and availability, market structure, the possibility of backstop resources, and so forth, e.g., Dasgupta and Heal (1974,1979), Stiglitz (1976), Gilbert (1979), Pindyck (1980). Besides, the resources taxation is a subject by itself in exhaustible resources extraction. In this area, the studies can be divided into the impact of resources taxes on the resources exploitation and optimal taxation of exhaustible resources. The former not only includes the impact of some resources taxes on the resources extraction but also compares the effectiveness of different types of tax affecting resources allocation. Conrad and Hool(1984) investigated the impact of three common types of taxes on intertemporal extraction patterns and recovery form mineral deposits. Gamponia and Mendelsohn (1985) compared the effectiveness of different types of tax. Slade (1984,1986) discussed the influence of various types of tax policy from the theoretical and practical aspects and via combining with resources
production at all stages. Rowse (1997) studied the impact of ad valorem taxation on production of natural gas by using the numerical simulation. Optimal taxation is designed to ensure resources allocation or exploitation effectively. Khaltabari (1977) investigated the optimal extraction path of resources under imperfect market. He found that excess exploitation is not because of imperfect competition but of unclear definition of property rights. Then he gave optimal taxation of the public. Dore(1992) studied the taxation design of exhausted resources under monopolistic competition. Jeong-Bin Im (2002) discussed the ineffectiveness of the allocation of resources under monopoly, and discussed design of various taxes to correct the inefficient resources allocation under market failure. However, these literatures didn’t analyze resources policy uncertainty. Taxes policy uncertainty is often discussed in investment area. For example, AlvarezKanniainen and S?dersten(1998) studied enterprises investment under taxes policy uncertainty by using the method of optimal control and they found impact of uncertainty is great.

The purpose of the paper is to find optimal extraction of exhaustible under resources taxes reform in China, and analyze the impact of reform uncertainty on resources exploitation. The organization of the paper is as follows. Section 2 introduces our model and section 3 states the optimal extraction behavior when the uncertainty concerns the taxes levy ways and timing. Some simplifications of functions are useful in the light of the complexity of the mechanisms. Section 4 discusses the impact of reform uncertainty on resources extraction using a numerical example. Finally, section 5 concludes.

2 The model

Now we present the model in detail. We begin with a brief description of symbol and then consider some assumptions. Then, we describe the mathematical formulation that we use to model the production process of exhaustible resources. Finally, we set up the model that we use to formulate the optimization problem as an optimal control problem.

2.1 Symbol description

For year $t$ the choice variables are: $X_t$, resource extraction rate from proved reserves; $S_t$, the resources stock; $Q_t$, resources demanded; $P_t$, resources price; $Q_t(P_t)$, the demand function; $C_t(X_t)$, the cost function; $T$, the exhaustible date of resources; $A$, the fixed resources stock at the initial period; $B$, the final sock at the exhaustible period (for simplicity, fix $B = 0$); $r$, fixed discount rate; $a$, the positive parameter of demand function (for simplicity, fix $a = 1$); $b$, constant parameter of cost function (for simplicity, fix $c = 0$).

2.2 Assumptions

We adopt a continuous-time model to represent the problem. We shall also assume that the exploration stages have been completed. On the basis of existing literatures, the restricted conditions about exhaustible resources are available as follows:

\begin{align}
S_t &= -X_t \\
S_0 &= A, S_T = B, S_t \geq 0.
\end{align}

Besides, the following assumptions are made to describe our problem:

A1: The firm operates in a market characterized by complete competition.

A2: All the resources exploited are inputted into the market. And when the market is cleared, resources supply by firms is equal to that demanded, i.e.:

$$Q_t = X_t.$$  

A3: Demand function is weak concave, it satisfied: $dQ_t/dP_t < 0$, $d^2Q_t/dP_t^2 \geq 0$.Simply, we regard it as linear function, noting its form as following:

$$Q_t = a(\bar{P} - P_t) = \bar{P} - P_t.$$  

Wherein, the chock price ($\bar{P}$) and the lowest price ($P_t$) of resources are determined by consumers and producers. When $P_t > \bar{P}$, resources consumption of consumers is reduced to zero. So resources exploitation and consumption is stopped, and it is dropped out of market. Then these kinds of resources may be replaced by substitution. When $P_t < \bar{P}$, the producers will have no profit and the output will be decreased to zero.

A4: Cost function is weak convex and it depends on time $t$ and extraction rate $X_t$. So $\frac{\partial C_t}{\partial X_t} > 0$, $\frac{\partial^2 C_t}{\partial X_t^2} \geq 0$. According to these characters, we select the following cost function:

$$C_t(X_t) = \frac{bX_t^2}{2} + c = \frac{bX_t^2}{2}.$$  

2.3 The resources taxes reform

The taxes reform we consider is expected to be of the change in taxes levy ways. In the initial period of taxes reform the way of taxes levy is "specific taxation". In this paper, a specific taxation is expressed as dollars per unit of the extracted resources, and suppose the government is levying a per unit tax $\gamma_0$ on the extracted resources. After reform, the way will shift to "ad valorem taxation". Suppose the government is levying an ad valorem taxation $\beta$ on the price of the resources.

In the paper, we take the date of taxes reform ($t^*$) as random variable. Let $\lambda_t$ denote the probability rate of reform occurring at $t^*$ conditional on it not having occurred earlier. When $t \geq 0$, $\lambda_t$ is continuous. From $t^* = 0$, let $\Pi_t$ be the probability density of taxes reform being made at $t^*$. And $\Omega_t$ is the probability of taxes reform. Then, we can get following relations:

$$\Omega_t = \int_0^t \Pi_t dt,$$  \hspace{0.5cm} (5)

$$\lambda_t = \frac{\Pi_t}{1 - \Omega_t}.$$  \hspace{0.5cm} (6)

For simplicity, we have the following assumptions:

A5: the process of taxes reform follows the Poisson process, i.e.:

$$\lambda_t = \lambda > 0, \quad \Pi_t = \lambda e^{-\lambda t}.$$  

2.4 Objective function

The aim of the firm is to maximize its' profit. In the paper, we regard it as producer surplus, which is equal to the income of selling goods subtracts cost and tax. Then the forms of firm’s aims before and after resources taxes reform are as follows:

$$\text{Max}: V_1 = P_t Q_t - C_t(X_t) - \gamma_0 X_t,$$ \hspace{0.5cm} (7)

$$\text{Max}: V_2 = (1 - \beta)P_t Q_t - C_t(X_t).$$ \hspace{0.5cm} (8)

The time ($t^*$) divides the process of resources extraction into two stages. According to A5, we can study the whole extraction process by dynamic programming. The problem to solve is how to solve the following optimal problem at time $\tau (\tau \geq t^*)$:

$$\text{Max}: V(S_t) = \int_{t^*}^\tau V_2 e^{-r(t^*-\tau)} dt, \text{ s.t.}: \left\{ \begin{array}{l} \int_{t^*}^\tau X_t d\tau \leq S_t^*, \\ X_t \geq 0, S_T = 0. \end{array} \right.$$ \hspace{0.5cm} (9)

So the objective function of whole extraction process is as follows:

$$\text{Max}: \int_0^\tau [V_1(1 - \Omega_t) + \Pi_t V(S_t)]e^{-rt} dt.$$ \hspace{0.5cm} (10)

2.5 The optimal extraction model

Uniting equations (1) and (10), we build the optimal extraction model:

$$W_t = \text{Max} \int_0^\tau [V_1(1 - \Omega_t) + \Pi_t V(S_t)]e^{-rt} dt, \text{ s.t.}: \left\{ \begin{array}{l} \dot{S}_t = -X_t, \\ S_0 = A, S_T = 0, S_t \geq 0. \end{array} \right.$$ \hspace{0.5cm} (11)

3 The optimal extraction path

3.1 The solution of optimal problem (9)

According to the assumptions, we know that this optimization satisfies convexity and ensures that the optimal solution exists. Then putting equations (2),(3),(4) and (8) into problem (9), we get the Hamilton-Jacobi-Bellman equation of the problem (Liutang Gong, 2002). as follows:

$$r V(S_t) = \max[(1 - \beta)(P - X_t)X_t - \frac{b X_t^2}{2} - \frac{\partial V(S_t)}{\partial S_t} X_t].$$ \hspace{0.5cm} (12)

In order to satisfy equation (12)we need to differentiate it about $X_t$, and make it equal to zero. Then we get

$$\frac{\partial V(S_t)}{\partial S_t} = (1 - \beta)\dot{P} - (2 - 2\beta + b)X_t.$$ \hspace{0.5cm} (13)
3.2 The system optimal solution

According to A5 and equation (13) we can solve the optimal problem (11) by using optimal control theory. The current value Hamiltonian of the problem (11) is (Chiang, 1999):

\[ H_c = \{ [\bar{P} - X_t - \gamma_0]X_t - \frac{bX_t^2}{2}e^{-\lambda t} + \lambda e^{-\lambda t}V(S_t) \} - m_tX_t, \]  

where \( m_t \) is the dynamic shadow price of the resource stock at time \( t \). And \( m_t = m_0e^{rt}, m_0 \) is the shadow price of the initial resources. The first term (in brackets) is the expected net earnings of resources in year \( t \). The second term is a value in which the resources reserves \( S_t \) is gradually decreased in year \( t \). So shadow price is also referred to as the user cost of the resources. Then the necessary and transversal conditions for optimal resources extraction of the problem (11) are

\[ \dot{S}_t = \frac{\partial H_c}{\partial m_t} = -X_t, \]

\[ \dot{m}_t = \frac{\partial H_c}{\partial S_t} + rm_t = -\lambda e^{-\lambda t} \frac{\partial V(S_t)}{\partial S_t} + rm_t, \]

\[ \frac{\partial H_c}{\partial X_t} = [\bar{P} - \gamma_0 - (2 + b)X_t]e^{-\lambda t} - m_t = 0, \]

\[ [H_c]_{t=T} = 0, \]

where equations (15) and (16) are the equations of motion of the resources stock and the resources shadow price, respectively. Equation (17) is used to determine the resources extraction path. And equation (18) is transversal condition when the final time is free.

From equation (17), we have

\[ m_t = [\bar{P} - \gamma_0 - (2 + b)X_t]e^{-\lambda t}. \]  

First we differentiate equation (19) about \( t \), then put equations (13) and (19) into equation (16). We have From equation (17), we have

\[ (2 + b)\ddot{S}_t - [r(2 + b) + 2\lambda \beta]S_t = (r + \lambda)(\bar{P} - \gamma_0) - \lambda(1 - \beta)\bar{P}. \]  

Solving equation (20), and uniting initial condition, we have

\[ X_t = \frac{A - NT}{e^{r't} - 1} r' e^{r't} + N, \]

where \( r' = \frac{2r + rb + 2\lambda \beta}{2 + b}, N = \frac{(r + \lambda \beta)P - (r + \lambda)\gamma_0}{2(r + rb + 2\lambda \beta)}, T \) is determined by equations (18) and (21).

4 An Example and Results

Equation (21) is an analytical solution of our model. From this equation, we can find that if \( \lambda = 0 \), the optimal problem is degenerated to the situation of no change in taxation levy ways. Also we find that parameter \( \lambda \) and \( \beta \) in the equation (21) not only appear in denominator but also appear in the index. It is hard to differentiate \( X_t \) about \( \lambda \) and \( \beta \) to discuss the impact of taxes reform on resources extraction. So it is explained by numerical solutions in this paper. The numerical results are often specific and understood only in the neighborhood of values explored. As a result, the general and elegant conclusions aren’t found. When using numerical approaches, it is therefore necessary to test a wide range of parameter values to find the trend of the impact.

In order to explain the impact of taxes reform on the resources extraction path, we can only discuss changes in the resources extraction rate caused by changing parameter values of \( \lambda \) and \( \beta \) respectively. First of all, we begin by setting special values of some variants, such as \( r = 0.1, A = 10^4, b = 5, \bar{P} = 500, \gamma_0 = 10 \). Then putting these data into equation (21), we can get

\[ X_t = \frac{10^4 - TN'}{e^{r''t} - 1} r'' e^{r''t} + N', \]  

where \( r'' = \frac{0.7 + 2\lambda \beta}{2 + b}, N' = \frac{49 + 500\lambda \beta - 10\lambda}{2(0.7 + 2\lambda \beta)} \).

According to (22), we can analyze impact of \( \lambda \) and \( \beta \) respectively on resources optimal extraction. When discussing one parameter, we explore low, medium and high value of the other. Suppose values of these tree values are 0.2, 0.5 and
Then we can get the resources optimal extraction. And the results of impact analysis are presented in Table 1 and Table 2.

<table>
<thead>
<tr>
<th>values of λ</th>
<th>low taxes rate (β = 0.2)</th>
<th>medium taxes rate (β = 0.5)</th>
<th>high taxes rate (β = 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low taxes rate</td>
<td>medium taxes rate</td>
<td>high taxes rate</td>
</tr>
<tr>
<td>r^T</td>
<td>N^T</td>
<td>T</td>
<td>r^T</td>
</tr>
<tr>
<td>0.1</td>
<td>0.106 78.38 136 0.114</td>
<td>91.25 118 0.123 102.33</td>
<td>105</td>
</tr>
<tr>
<td>0.2</td>
<td>0.111 85.9 125 0.129</td>
<td>107.78 100 0.146 124.51</td>
<td>87</td>
</tr>
<tr>
<td>0.3</td>
<td>0.117 92.68 116 0.143</td>
<td>121 89 0.169 140.68</td>
<td>76</td>
</tr>
<tr>
<td>0.4</td>
<td>0.123 98.84 109 0.157</td>
<td>131.82 82 0.191 152.99</td>
<td>70</td>
</tr>
<tr>
<td>0.5</td>
<td>0.129 104.44 103 0.171</td>
<td>140.83 76 0.214 162.67</td>
<td>66</td>
</tr>
<tr>
<td>0.6</td>
<td>0.134 109.57 98 0.186</td>
<td>148.46 72 0.237 170.48</td>
<td>62</td>
</tr>
<tr>
<td>0.7</td>
<td>0.14 114.29 94 0.2</td>
<td>155 69 0.26 176.92</td>
<td>60</td>
</tr>
<tr>
<td>0.8</td>
<td>0.146 118.63 91 0.214</td>
<td>160.67 66 0.283 182.32</td>
<td>58</td>
</tr>
<tr>
<td>0.9</td>
<td>0.151 122.64 88 0.229</td>
<td>165.63 64 0.306 186.92</td>
<td>56</td>
</tr>
</tbody>
</table>

As shown in the Table 1, change rate of resources extraction rate are rising by increasing the probability of taxes reform or taxes rate. If the probability rate of reform is selected, the exhaustible time of resources is shorten by increasing taxes rate. When the taxes rate is selected, the time is also shorten by increasing the probability.

As is evident from viewing Table 2, the initial extraction rate and change rate of resources extraction rate are rising by increasing the tax rate. If the probability of taxes reform is given, the higher the tax rate is, the higher are initial extraction rate and change rate of extraction. It leads to exhaust fast.

### 5 Summary and conclusions

This paper has analyzed the optimal extraction path of exhaustible resources under taxes reform uncertainty. We explored a simple model characterized by some special functions. There are two uncertainty of taxes reform, which are crucial to the results of the paper: taxation levy ways and time uncertainty.

The analyses have shown that change rate of resources extraction rate rises by increasing the tax rate or probability of taxes reform. It is interesting to point out that higher probability of taxes reform does not always yield more initial extraction rate, it also depends on the tax rate. So in practice, we must decide a suitable tax rate. Meanwhile, the resources...
extraction behavior can be conducted by government, and the government should expose its reform project to correct the inefficient resources extraction to protect resources. Because of numerical solution in the paper, the conclusions are not general, so we must study further.

6 Acknowledgements

The authors gratefully acknowledge the financial support from the Key Project of the National Social Science Foundation of China (NSSFC) under the grants Nos. 08ZD046, the National Natural Science Foundation of China (NSFC) under the grants Nos. 70873058 and 70702015, the Humanities and Social Sciences Foundation of Ministry of Education (HSSF) (grants Nos.10YJC790062), the Higher School Philosophy and Social Sciences Foundation of Jiangsu Province Education Department under grants Nos. 2010SJB790014, and the Nanjing University of Finance and Economics Foundation of China (C0905).

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