Adaptive Control and Slow Manifold Analysis of a New Chaotic System

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Abstract: The problem of Slow Manifold Analysis and adaptive control for a new system——Liu system is studied in this paper. Firstly, the Liu system is considered as slow-fast autonomous dynamical system in order to obtain the equations of the slow manifold from three different ways as well as geometric characterization of the attractor. And a global qualitative description of its dynamics is given. Then a brief comparison is presented among these methods. Secondly, a single new adaptive controller is designed. The analytical expression of controller and adaptive law of unknown parameters are given based on Lyapunov stability theory. With this method, parameters identification and control can be achieved simultaneously with only one controller. Numerical simulations verify the effectiveness and feasibility of the method.

Keywords: Liu system; slow manifold; equation; adaptive control

1 Introduction

In recent years, the study of the nonlinear chaotic dynamics is a popular problem in the field of the nonlinear science and great progress has been made in the research of nonlinear chaotic dynamics. In 1963, Lorenz found the first classical chaotic attractor [1] in three-dimension autonomous system. In 1999, Chen and Ueta found another similar but not topological equivalent chaotic attractor to Lorenz’s [2]. In 2002, Lü found the critical chaotic attractor [3] between the Lorenz and Chen attractor. It is noticed that these systems can be classified into three types by the definition of Vaněcěand Cělikovsky [4]: the Lorenz system satisfies the condition $a_{12}a_{21} > 0$, the Chen system satisfies $a_{12}a_{21} < 0$, and the Lü system satisfies $a_{12}a_{21} = 0$, where $a_{12}$ and $a_{21}$ are the corresponding elements in the linear part matrix $A = (a_{ij})_{3 \times 3}$ of the system.

Recently, Liu etc.discovered another chaotic system by using physical electrical circuits and named it the Liu system [5]. It provides a new domain for the study of chaotic system. The autonomy differential equations that describe the system as:

$$\begin{cases}
\dot{x} = a (y - x) \\
\dot{y} = bx - kxz \\
\dot{z} = -cz + hx^2
\end{cases}$$

where $a = 10$, $b = 40$, $c = 2.5$, $h = 4$, $k = 1$. If we select the initial values of the system as $(2.2, 2.4, 3.8)$, it has a chaotic attractor. According to the critical term $a_{12}a_{21}$which was proposed by Vaněcěand Cělikovsky: The Liu system satisfies the condition $a_{12}a_{21} > 0$. The chaotic attractor obtained from this system is also the butterfly-shaped attractor. This attractor is similar but not equivalent to the Lorenz chaotic attractor. The third differential equation has one quadratic item that can produce folding trajectories. The strange attractors are shown in Figure 1.

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It is known that the slow-fast autonomous dynamical systems (S-FADS) show a dichotomy of motion which is alternatively slow and fast. This is confirmed theoretically by some recent studies [6] in which brought to light that S-FADS have slow manifolds. It is confirmed that the Lorenz system and the Chua system had slow manifolds in [7]. Slow manifold of Lorenz-Haken System is discussed in [8]. In this paper, we will draw the Liu system’s slow manifold.

![Figure 1: The strange attractor of the Liu system](image)

Recently, some results have been reported about the Liu system. Chen et al. discussed the nonlinear feedback synchronization control of Liu chaotic system in [9], synchronization of Liu chaotic system with Lorenz system and Chen system was studied in [10], but they are based on the parameters known before. However, for some uncertain systems, the before mentioned methods [9-16] will fail. In this paper, the observer is designed to identify the unknown parameters and control Liu system to bounded points simultaneously.

The rest of the paper is organized as follows. Three different methods are derived to get the equations of slow manifold in section 2. In section 3, the qualitative behavior and the orbits of the Liu system are analyzed. Adaptive method for the uncertain Liu system is discussed in Section 4. Section 5 contains the conclusion of this paper.

## 2 Methods of construction of the slow manifold equation

### 2.1 Some results

In the study of nonlinear chaotic dynamics, one of methods is to consider some systems with chaotic behaviors as slow-fast systems in order to make qualitative analysis. Slow-fast systems have the extensive value of practical application. Geometric singular perturbation theory is a good method for qualitative analysis of slow-fast autonomous dynamical system. However, searching the expression of slow manifold is an important job in geometric singular perturbation theory. If we can obtain the expression of slow manifold, it will allow us to restore a part of the deterministic property of the system that was lost because of the sensitivity to initial conditions. In [6], it has shown that slow-fast autonomous dynamical systems (S-FADS) (i.e. systems that are modeled by differential equations having a small parameter \( \varepsilon \) multiplying one of their velocity components) have slow manifolds. In the Liu system, the number of \( a \) only multiplies the derivative of \( x \). So in this paper, we can consider this system as a slow-fast autonomous dynamic system. In S-FADS, variables are separated into two groups: one is fast variable, others are slow variables. The behaviors of the whole system are determined by its slow variables. Three methods are put forward to obtain the slow manifold equation of the Liu system. Firstly, the method is to build the slow manifold equation by considering that the slow manifold is locally defined by a plan orthogonal to the tangent system’s left fast eigenvector. Secondly, the method is to get the equation of slow manifold by using the tangent system’s slow
eigenvectors. Thirdly, from the geometric singular perturbation point of view, we discuss the slow manifold equation of the Liu system and obtain the first order expression and the second order expression of slow manifold of the Liu system.

2.2 With the first method to get the equation of slow manifold

For the parameters \( a = 10, b = 40, c = 2.5, h = 4, k = 1 \), the Liu system has a strange attractor, so we have the following system:

\[
\begin{align*}
\dot{x} &= 10(y - x) \\
\dot{y} &= 40x - xz \\
\dot{z} &= -2.5z + 4x^2
\end{align*}
\]  

(2.1)

For these parameter values, it has a real, negative and dominant eigenvalue (i.e. fast eigenvalue) for Jacobian matrix in a large part of the attractor’s phase space domain.

Now, write the Jacobian matrix at point \( X = (x, y, z)^T \)

\[
J(X) = \begin{pmatrix}
-10 & 10 & 0 \\
40 - z & 0 & -x \\
8x & 0 & -2.5
\end{pmatrix}
\]  

(2.2)

Let \( \lambda_1(X) = \lambda_1(x, y, z) \) be the fast eigenvalue and \( \lambda_2(X), \lambda_3(X) \) be the two slow ones. Then eigenvector of \( J^t(X) \) (the superscript “t” denotes “transpose”) is given by

\[
Z_{\lambda_1}^T(x, y, z) = \begin{pmatrix}
1 \\
\frac{10}{\lambda_1(x, y, z)} - \frac{10x}{(\lambda_1(x, y, z) + 2.5)}
\end{pmatrix}
\]  

(2.3)

On the attractive parts of the phase space (i.e. where \( J(X) \) have a fast eigenvalue), the equation of the slow manifold is given by \( Z_{\lambda_1}^T(x, y, z) \)

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix} = 0
\]

We can use the last equation to define the equation of the slow manifold. If we replace \( Z_{\lambda_1}(x, y, z) \) with its expression given by (2.3) and \( \begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix} \) by (2.1), we can write the equation of the slow manifold:

\[
10(y - x) \lambda_1(x, y, z) (\lambda_1(x, y, z) + 2.5) + 10(40x - xz) (\lambda_1(x, y, z) + 2.5) - 10x (-2.5z + 4x^2) = 0
\]  

(2.4)

where \( \lambda_1(x, y, z) \) is the fast eigenvalue of \( J(X) \). Because \( \lambda_1(x, y, z) \) is uncertain eigenvalue, it is not easy to use this implicit equation to draw a slow manifold representation in the \( 3 - D \) phase space.

2.3 To get the slow manifold equation with two slow eigenvectors

The slow manifold of nonlinear chaotic system can be regarded as surface generated by the two slow eigenvectors associated with the two eigenvalues \( \lambda_2(X) \) and \( \lambda_3(X) \) of \( J(X) \). We know that the local manifold in the neighborhood of \( X \) is generated by those two vectors. With this method, we can derive a new equation of the slow manifold. It is easy to show that for \( k \in \{2, 3\} \), it is possible to write

\[
Z_{\lambda_k}(x, y, z) = \begin{pmatrix}
1 \\
u_k(x, y, z) \\
v_k(x, y, z)
\end{pmatrix}
\]  

(2.5)

where

\[
u_k(x, y, z) = 1 + \frac{\lambda_k(x, y, z)}{10}
\]  

(2.6)
The equation of the slow manifold can be derived from

\[
\begin{bmatrix}
\dot{x} & \dot{y} & \dot{z} \\
1 & u_2(x, y, z) & v_2(x, y, z) \\
1 & u_3(x, y, z) & v_3(x, y, z)
\end{bmatrix} = 0
\]

which gives

\[
\begin{bmatrix}
10(y - x) & 4x - xz & -2.5z + 4x^2 \\
1 & u_2(x, y, z) & v_2(x, y, z) \\
1 & u_3(x, y, z) & v_3(x, y, z)
\end{bmatrix} = 0
\]

At this point, we have to know that for some points \( X = (x, y, z)^t \), the two slow eigenvalues \( \lambda_2(X) \) and \( \lambda_3(X) \) are complex conjugate numbers and so are \( Z_{\lambda_2}(x, y, z) \) and \( Z_{\lambda_3}(x, y, z) \) (in fact their second and third components).

For \( k \in \{2, 3\} \), we have \( u_3(x, y, z) = [u_2(x, y, z)]^* \) and \( v_3(x, y, z) = [v_2(x, y, z)]^* \), where "*" denotes complex conjugate operation.

Now, the slow manifold equation must be real. So we have to take any linear combination of the slow eigenvectors that leads to a real determinant.

\[
\begin{align*}
\tilde{Z}_{\lambda_2}(x, y, z) &= \frac{1}{2} \left[ Z_{\lambda_2}(x, y, z) + Z_{\lambda_3}(x, y, z) \right] \\
\tilde{Z}_{\lambda_3}(x, y, z) &= \frac{1}{2} \left[ (1 + i) Z_{\lambda_2}(x, y, z) + (1 - i) Z_{\lambda_3}(x, y, z) \right]
\end{align*}
\]

So we have

\[
\begin{align*}
Z_{\lambda_2}(x, y, z) &= \begin{pmatrix} 1 \\ Re[u_2(x, y, z)] \\ Re[v_2(x, y, z)] \end{pmatrix} \\
Z_{\lambda_3}(x, y, z) &= \begin{pmatrix} 1 \\ Re[u_2(x, y, z)] - Im[u_2(x, y, z)] \\ Re[v_2(x, y, z)] - Im[v_2(x, y, z)] \end{pmatrix}
\end{align*}
\]

The new real expression of the determinant is

\[
\begin{bmatrix}
10(y - x) & 40x - xz & -25z + 4x^2 \\
1 & Re[u_2(x, y, z)] & Re[v_2(x, y, z)] \\
1 & Re[u_2(x, y, z)] - Im[u_2(x, y, z)] & Re[v_2(x, y, z)] - Im[v_2(x, y, z)]
\end{bmatrix} = 0
\]

The general case [equations (2.9) and (2.14)] for both real and complex eigenvalues can be combined into a union equation of the slow manifold as follows

\[
\begin{bmatrix}
10(y - x) & 40x - xz & -25z + 4x^2 \\
1 & Re[u_2(x, y, z)] & Re[v_2(x, y, z)] \\
1 & Re[u_3(x, y, z)] - Im[u_2(x, y, z)] & Re[v_3(x, y, z)] - Im[v_2(x, y, z)]
\end{bmatrix} = 0
\]

Let

\[
h_1(x, y, z) = Re[u_2(x, y, z)]
\]

\[
h_2(x, y, z) = Re[v_2(x, y, z)]
\]
\[ h_5(x, y, z) = \text{Re} [u_3(x, y, z)] - \text{Im} [u_2(x, y, z)] \]  
\[ h_4(x, y, z) = \text{Re} [v_3(x, y, z)] - \text{Im} [v_2(x, y, z)] \]  
\[ F_1(x, y, z) x + F_2(x, y, z) y + F_3(x, y, z) z + F_4(x, y, z) x^2 + F_5(x, y, z) xz = 0 \]

If we expand Eq (2.15) in terms of these notations, we would obtain the equation

\[ F_1(x, y, z) = 40 [h_2(x, y, z) - h_4(x, y, z)] + 10 [h_2(x, y, z) h_3(x, y, z) - h_1(x, y, z) h_4(x, y, z)] \]  
\[ F_2(x, y, z) = 10 [h_1(x, y, z) h_4(x, y, z) - h_2(x, y, z) h_3(x, y, z)] \]  
\[ F_3(x, y, z) = 2.5 [h_1(x, y, z) - h_3(x, y, z)] \]  
\[ F_4(x, y, z) = 4 [h_3(x, y, z) - h_1(x, y, z)] \]  
\[ F_5(x, y, z) = h_4(x, y, z) - h_2(x, y, z) \]

2.4 With geometric singular theory to get the equation of slow manifold

Now, we give the third method to derive the slow manifold equation of the Liu system. Taking \( \varepsilon = 1/10 \), then we can treat the system (2.1) as a slow-fast autonomous system. Therefore we can precede the qualitative analysis to it by using geometric singular perturbation method. The slow system is

\[ \begin{align*}
\varepsilon \dot{x} &= y - x \\
\dot{y} &= 40 - xz \\
\dot{z} &= -2.5z + 4x^2
\end{align*} \]  

(2.26)

where \( x \) is the fast variable, \( y \) and \( z \) are slow variables. The dualistic system of (2.26), namely fast system is

\[ \begin{align*}
x' &= y - x \\
y' &= \varepsilon (40x - xz) \\
z' &= \varepsilon (-25z + 40x^2)
\end{align*} \]  

(2.27)

where \( \tau = t/\varepsilon \) is called the fast-time scale, while \( t \) is named the slow-time scale. As long as \( \varepsilon \neq 0 \), the system (2.26) is equivalent to the system (2.27).

In fast system, letting \( \varepsilon \to 0 \), we obtain a zero order approximate slow manifold \( M_0: x = y \). Obviously the dimension of \( M_0 \) is 2. On \( M_0 \), the Jacobin matrix of the system (2.27) is

\[ J = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

which just contains two eigenvalues with zero real part (in fact, they are zero real roots), therefore \( M_0 \) is normal hyperbolic. Fenichel’s invariable manifold theorem guarantees that a local invariable manifold \( M_e \) exists, which is \( o(\varepsilon) \) close to \( M_0 \). \( M_e \) is local invariable. If \( M_0 \) is \( C^r \) \((0 < r < \infty) \) and described as the graph of a smooth function, so is \( M_e \).

In general, one cannot directly calculate the expression of \( M_e \), but from the invariable manifold theory by Fenichel, we can obtain it, because \( M_e \) is \( o(\varepsilon) \) close to \( M_0 \). The relation of variables \( x \) and \( y \) in \( M_e \) is

\[ x = y + o(\varepsilon) \]

We can expand it in \( \varepsilon \) series:

\[ x = y + \varepsilon H_1(y, z) + \varepsilon^2 H_2(y, z) + o(\varepsilon^2) \]  

(2.28)

Because \( M_e \) is local invariable manifold of (2.26), derivate (2.28) to get
\[ \dot{x} = \dot{y} + \varepsilon \left( \frac{\partial H_1}{\partial y} \dot{y} + \frac{\partial H_1}{\partial z} \dot{z} \right) + \varepsilon^2 \left( \frac{\partial H_2}{\partial y} \dot{y} + \frac{\partial H_2}{\partial z} \dot{z} \right) + o(\varepsilon^3) \]  

(2.29)

And

\[ \varepsilon \dot{x} = \varepsilon \dot{y} + \varepsilon^2 \left( \frac{\partial H_1}{\partial y} \dot{y} + \frac{\partial H_1}{\partial z} \dot{z} \right) + o(\varepsilon^3) \]

\[ = \varepsilon (40x - xz) + \varepsilon^2 \left( \frac{\partial H_1}{\partial y} \dot{y} + \frac{\partial H_1}{\partial z} \dot{z} \right) + o(\varepsilon^3) \]

\[ = \varepsilon (40 - z) \left[ y + \varepsilon H_1(y, z) + \varepsilon^2 H_2(y, z) + o(\varepsilon^3) \right] + \varepsilon^2 \left( \frac{\partial H_1(y,z)}{\partial y} \dot{y} + \frac{\partial H_1(y,z)}{\partial z} \dot{z} \right) + o(\varepsilon^3) \]

\[ = (40y - yz) \varepsilon + \left[ (40 - z) H_1(y, z) + \left( \frac{\partial H_1(y,z)}{\partial y} \dot{y} + \frac{\partial H_1(y,z)}{\partial z} \dot{z} \right) \right] \varepsilon^2 + o(\varepsilon^3) \]  

(2.30)

Taking expression (2.28) into the first term of (2.26), we can obtain

\[ \varepsilon \dot{x} = y - x = y - \left[ y + \varepsilon H_1(y, z) + \varepsilon^2 H_2(y, z) + o(\varepsilon^3) \right] \]  

(2.31)

Comparing the coefficients of the same power of \( \varepsilon \) in (2.30) and (2.31), one gets

\[ H_1(y, z) = yz - 40y \]

\[ H_2(y, z) = 2y (z - 40)^2 + y (-2.5z + 4y^2) \]

Thus the second order expression of the slow manifold \( M_\varepsilon^2 \) of the Liu system is

\[ x = y + \varepsilon (yz - 40y) + \varepsilon^2 \left[ 2y (z - 40)^2 + y (-2.5z + 4y^2) \right] \]  

(2.32)

If we expand it in \( \varepsilon \) series:

\[ x = y + \varepsilon H(y, z) + o(\varepsilon^2) \]

Using the same method given above, we can get the first order expression of the slow manifold \( M_\varepsilon^1 \) of the Liu system

\[ x = y + \varepsilon (yz - 40y) \]  

(2.33)

Its graphs are given in Figure 2(a) and (b).

From the discussion above, we can get the expression of slow manifold clearly and easily by geometric singular perturbation analysis. The acquired equations of the slow manifold (2.28) and (2.29) are concrete and brief, so it is more convenient for the equation (2.29) to be used for qualitative analysis and numerical simulation.
3 The geometric characterization of the attractor and a global qualitative description of its dynamics

It is easy to prove that three equilibria of the Liu system, \( S_0 = (0, 0, 0), S_1 = (5, 5, 40), S_2 = (-5, -5, 40) \) all satisfy equations (2.32) and (2.33), which means that they are all in the slow manifold \( M^2 \) and \( M^3 \). Therefore we can use geometric singular perturbation theory to analyze the qualitative behavior and orbits of the Liu system.

Given initial value \((x(0), y(0), z(0))\), because the velocities of \( x, y, z \) are different, \( x \) is a fast variable, \( y \) and \( z \) slow variables. The fast movement takes place first. \( x \) changes fast, while \( y \) and \( z \) remain almost unchanged, so \( x \) attains the half stability condition. In the slow manifold, \( x, y \) and \( z \) all change slowly and the movement attain a certain equilibrium point (one of the \( S_0, S_1, S_2 \)), but they cannot stay in the equilibrium point forever. Because \( x \) just attains half stability condition on slow manifold, it must lose stability and begin fast movement again. The fast movement and the slow movement change alternatively. The behavior of the system is like this: it goes out from the point \( S_0 \) and comes into the point \( S_1 \). Then it turns out from \( S_1 \) into \( S_0 \), and leaves \( S_0 \) to reach \( S_0 \). Next it turns out from \( S_2 \) again and comes into \( S_0 \). A period begins right after the ending of last period. The back and forth is continuous. The round of the points \( S_1 \) and \( S_2 \) is on the plan approximately, however, its type and numbers of rotation are irregular. As a result, it becomes the strange attractor like the wings of butterfly, and results in chaos.

Now a simple comparison is made among these methods. In general, the first two methods are difficult to get clear forms for slow manifold of the Liu system. Because it is quite difficult to calculate eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \), particularly when the elements of the Jacobian matrix are the functions of state variables. But the thought is clear. The third method is relatively easy, because we only need to calculate the eigenvalues of degenerate fast subsystem and make power expand. The degenerate system is brief, so the expressions of the fast variables change slowly, while \( y, z \) remain almost unchanged, so \( x \) attains the half stability condition. In the slow manifold, \( x, y \) and \( z \) all change slowly and the movement attain a certain equilibrium point (one of the \( S_0, S_1, S_2 \)), but they cannot stay in the equilibrium point forever. Because \( x \) just attains half stability condition on slow manifold, it must lose stability and begin fast movement again. The fast movement and the slow movement change alternatively. The behavior of the system is like this: it goes out from the point \( S_0 \) and comes into the point \( S_1 \). Then it turns out from \( S_1 \) into \( S_0 \), and leaves \( S_0 \) to reach \( S_0 \). Next it turns out from \( S_2 \) again and comes into \( S_0 \). A period begins right after the ending of last period. The back and forth is continuous. The round of the points \( S_1 \) and \( S_2 \) is on the plan approximately, however, its type and numbers of rotation are irregular. As a result, it becomes the strange attractor like the wings of butterfly, and results in chaos.

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4 Adaptive backstepping control with only one controller

In the following part, we will control uncertain Liu system with only one controller and identify all unknown parameters, where \( k = 1 \). The controlled system of system (1.1) is described as follows:

\[
\begin{align*}
\dot{x} &= a(y - x) \\
\dot{y} &= bx - xz \\
\dot{z} &= -cz + hx^2 + u
\end{align*}
\]

(4.1)

where \( a, b, c, h \) are unknown parameters to be identified, and \( u \) is the controller to be designed.

Next we will identify the five unknown parameters and control uncertain Liu system to a bounded point simultaneously. Here we suppose \( a \) is positive. Backstepping method is used to design the controller. At each step \( i \), an intermediate virtual control function \( \alpha_i \) will be developed using an appropriate Lyapunov function \( V_i \).

**Step 1.** Starting from the first equation, a stabilizing function \( \alpha_1(x) \) has to be designed for the virtual control \( y \) in order to make the derivative of \( V_1(x) = x^2/2 \), i.e., \( \dot{V}_1(x) = x \dot{x} \) be negative definite. Assume that \( \alpha_1(x) = px \) \((0 < p < 1)\) and define an error variable \( \tilde{y} = y - \alpha_1(x) \), then

\[
\dot{x} = a\tilde{y} - a(1 - p)x
\]

\[
\dot{V}_1(x) = ax\tilde{y} - a(1 - p)x^2
\]

**Step 2.** In this step, we will deal with the singularity problem caused by \(-xz\) in the second equation of (1.1). The derivative of \( \tilde{y} \) is

\[
\begin{align*}
\tilde{y} &= \tilde{y} - \dot{\alpha}_1 = \tilde{y} - px = -xz + bx - ap\tilde{y} + ap(1 - p)x \\
&= -x(z + \alpha_2) + bx - ap\tilde{y} + ap(1 - p)x - (b - \tilde{a})x - (\tilde{a} - a)p(1 - p)x
\end{align*}
\]
where $\bar{z} = z - \alpha_2$, $\alpha_2$ is the virtual control to be defined.

Let $\hat{a}, \hat{b}, \hat{c}, \hat{h}$ be estimates of $a, b, c, h$, and introduce the parameter errors:

$$\hat{a} = \bar{a} - a, \hat{b} = \bar{b} - b, \hat{c} = \bar{c} - c, \hat{h} = \bar{h} - h.$$ 

then we obtain the $(x, \bar{y})$-subsystem:

$$\begin{cases}
\dot{x} = a\bar{y} - a (1 - p) x \\
\dot{\bar{y}} = -x ( \bar{z} + \alpha_2 ) + \bar{b} x - a p \bar{y} + \bar{a} p (1 - p) x + - (\bar{b} - b) x - (\bar{a} - a) p (1 - p) x 
\end{cases}$$

(4.2)

We choose the following Lyapunov function candidate:

$$V_2 (x, \bar{y}) = V_1 (x) + \frac{1}{2} \bar{y}^2 + \frac{1}{2} \alpha \dot{\bar{a}} + \frac{1}{2} \bar{b}^2$$

Calculating its time derivative along system (4.1), we have

$$\dot{V}_2 = -a (1 - p) x^2 + a x \bar{y} + \bar{y} (\bar{y} - \dot{\bar{a}}) + \alpha \ddot{\bar{a}} + \ddot{\bar{b}}$$

$$= -a (1 - p) x^2 - a p \bar{y}^2 - x \bar{y} \bar{z} + x \bar{y} (-\alpha_2 + (\bar{b} + b) + (\bar{a} + a) (1 + p - p^2))$$

$$+ a (\bar{a} - x \bar{y} (1 + p - p^2)) + \bar{b} (\dot{\bar{b}} - x \bar{y})$$

Choosing

$$\hat{a} = (1 + p - p^2) x \bar{y} - \bar{a}$$

$$\dot{\bar{b}} = x \bar{y} - \bar{b}$$

$$\alpha_2 = (\bar{a} + a) (1 + p - p^2) + (\bar{b} + b)$$

then

$$\dot{\bar{V}}_2 = -a (1 - p) x^2 - a p \bar{y}^2 - a^2 - \bar{b}^2 - x \bar{y} \bar{z}$$

The term $-x \bar{y} \bar{z}$ will be canceled in the final step.

**Step 3.** The derivative of $\bar{z}$ is

$$\dot{\bar{z}} = \bar{z} - \alpha_2 = -c z + h x^2 + u - \frac{\partial \alpha_2}{\partial \bar{a}} \dot{\bar{a}} - \frac{\partial \alpha_2}{\partial \bar{b}} \dot{\bar{b}}$$

Then we get the following system in the $(x, \bar{y}, \bar{z})$-coordinates:

$$\begin{cases}
\dot{x} = a\bar{y} - a (1 - p) x \\
\dot{\bar{y}} = -x ( \bar{z} + \alpha_2 ) + \bar{b} x - a p \bar{y} + \bar{a} p (1 - p) x + - (\bar{b} - b) x - (\bar{a} - a) p (1 - p) x \\
\dot{\bar{z}} = -c z + h x^2 - \frac{\partial \alpha_2}{\partial \bar{a}} \dot{\bar{a}} - \frac{\partial \alpha_2}{\partial \bar{b}} \dot{\bar{b}} + u
\end{cases}$$

(4.3)

We choose the following Lyapunov function candidate:

$$V_3 (x, \bar{y}, \bar{z}) = V_2 (x, \bar{y}) + \frac{1}{2} \bar{z}^2 + \frac{1}{2} \hat{c}^2 + \frac{1}{2} \hat{h}^2$$

then

$$\dot{\bar{V}}_3 = \dot{\bar{V}}_2 + \bar{z} \ddot{\bar{z}} + \dot{\bar{c}} \dot{\bar{c}} + \dot{\bar{h}} \dot{\bar{h}} = -a (1 - p) x^2 - a p \bar{y}^2 - a \bar{y} \bar{z} - \bar{b}^2 - \bar{z} \bar{z} - \bar{c} \bar{c} - \hat{h}^2$$

$$= -a (1 - p) x^2 - a p \bar{y}^2 + \bar{z} \left( u - x y + (p + \hat{h}) x^2 - \dot{\bar{c}} - c - \frac{\partial \alpha_2}{\partial \bar{a}} \dot{\bar{a}} - \frac{\partial \alpha_2}{\partial \bar{b}} \dot{\bar{b}} \right) + \bar{c} (\bar{c} + \bar{z}) + \hat{h} \left( \hat{h} - x^2 \bar{z} \right)$$

In order to make $\dot{\bar{V}}_3$ be negative definite, we choose

$$\dot{\bar{c}} = -\bar{z} \bar{c} - \bar{c}$$

IINS homepage: http://www.nonliearscience.org.uk/
\[ \dot{h} = x^2 z - h \]

\[ u = -q z + xy - \left( p + \dot{h} \right) x^2 + \dot{c} z + \frac{\partial \alpha_2}{\partial \dot{a}} \dot{a} + \frac{\partial \alpha_2}{\partial \dot{b}} \dot{b} \]

where \( q \) is a positive constant, which yields

\[ \dot{V}_3 = -a (1 - p) x^2 - ap y^2 - q z^2 - \alpha_2^2 - \dot{b}^2 - c^2 - \dot{h}^2 \]

Since \( \dot{V}_3 \) is negative definite, we prove that \( (x, y, z, \alpha, \beta, \dot{c}, \dot{h}) \) system:

\[
\begin{align*}
\dot{x} &= a \dot{y} - a (1 - p) x \\
\dot{y} &= x (\dot{z} + \alpha_2) + bx - ap \dot{y} + \ddot{a} p (1 - p) x - (\dot{b} + b) x - (\alpha - a) p (1 - p) x \\
\dot{z} &= -cz + h x^2 - \frac{\partial \alpha_2}{\partial \dot{a}} \dot{a} - \frac{\partial \alpha_2}{\partial \dot{b}} \dot{b} + u \\
\dot{\alpha} &= (1 + p - p^2) x \dot{y} - \ddot{a} \\
\dot{\beta} &= x \dot{y} - \ddot{b} \\
\dot{c} &= -z \dot{z} - \ddot{c} \\
\dot{h} &= x^2 \dot{z} - \ddot{h}
\end{align*}
\]  

is globally asymptotically stabilized at the origin point. In view of \( \ddot{y} = y - \alpha_1 (x), \alpha_1 (x) = px, x \to 0 \), it implies that \( \ddot{y} \to 0 \).

From \( \ddot{z} = z - \alpha_2 \) and \( \alpha_2 = (\ddot{a} + a) (1 + p - p^2) + (\ddot{b} + b), \) we can conclude that \( z \) is bounded. From \( \ddot{a} = \ddot{a} - a, \ddot{b} = \ddot{b} - b, \ddot{c} = \ddot{c} - c, \ddot{h} = \ddot{h} - h \), we have that \( \ddot{a} \to a, \ddot{b} \to b, \ddot{c} \to c, \ddot{h} \to h \). Using \( u = -q \ddot{z} + xy - (p + \dot{h}) x^2 + \dot{c} \ddot{z} + \frac{\partial \alpha_2}{\partial \dot{a}} \dot{a} + \frac{\partial \alpha_2}{\partial \dot{b}} \dot{b} \), one concludes that the controller \( u \) is also bounded.

The adaptive backstepping controlling method presents a systematic procedure for selecting a proper controller in chaos control. It needs only one controller, so it is easy to implement.

Numerical simulations show the effectiveness of the above methods. In the simulations, we assume that \( a = 10, b = 40, c = 2.5, h = 4, \) initial conditions \( (x_0, y_0, z_0) = (2.2, 2.4, 38) \), \( (\ddot{a}_0, \ddot{b}_0, \ddot{c}_0, \ddot{h}_0) = (10, 10, 10, 10) \), \( p = 0.5, q = 30 \).

Figure 3-4 show the effectiveness of the adaptive backstepping control techniques. Figure 3 (a) displays the states of the controlled Liu system. Figure 3 (b) displays the system parameters identification results. Figure 4 is the controller action \( u \). As can be seen from the figure, \( u \) is bounded as \( t \to \infty \).

![Figure 3: (a)System states: x(-), y(-), z(-)  (b)Identification of unknown parameters: \( \ddot{a}(-), \ddot{b}(-), \ddot{c}(-), \ddot{h}(-) \)](image-url)
5 Conclusion

In this paper, we discuss a new Liu chaotic system and regard it as slow-fast autonomous dynamical systems. Three different methods are derived to get the equation of slow manifold. By considering the slow manifold, we give a deterministic description of the global slow motion. Furthermore, the trajectories and slow manifold of the Liu system are presented. Then adaptive control method is used to stabilize Liu system to bounded points with one controller and identify the unknown parameters simultaneously. Numerical simulations show the effectiveness and feasibility of this technique.

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References


