Risk Control of Bank Loans under the Marketization of Interest Rate

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Abstract: On the basis of marketization of interest rate, a real option model is used to get the feasible range of credit rationing and the borrowing probability of corporate’s investment decision. Some discussion are made on the fine for the bank, then some conclusions are obtained.

Keywords: credit rationing; marketization of interest rate; real options

Over the past 10 years, financial option valuation techniques are widely used in corporate strategic decision-making, because options is the subject of physical assets, so it is may called real options. The characteristics of this method lies in facing uncertainty about the future, giving companies the rights such as waiting, increase, conversion, abandonment, to use the financial options pricing formula or numerical simulation techniques to make more accurately quantitative analysis on the flexibility of enterprise management, and make proposed strategy for different market environment, then give real-time control. Compared with traditional methods the method attaches great importance to the value of flexibility in decision-making, namely, management flexibility, which means when managers are facing the future market situation of uncertainty, they have the rights to adjust the investment program at any time to change the original operation of the established strategy to make the maximum profits.

1 Changes of credit rationing situation

Credit rationing situation has always been an important feature of the loan market and is also involved in the study of theory and practice of the hot issues. Over the past 30 years economists explain it from adverse selection and moral hazard point. The classical credit rationing model was set up on the basis of Stights and Weiss (1981). Whose model includes a commercial banking and various forms of risk-neutral business who make applications for loan. Enterprise initial wealth $W$, the volume of investment $I$, therefore enterprises need to borrow $L = I - W$. Assuming the project expected profit is equal(recorded as $A$), the difference between enterprises are yields and success probability. Suppose $P_0$ be the first phase of the profit and $R_i$ the yield for success (failure hypothesis value of 0), $q_i$ the success probability, then the net present value for No.i enterprises is

$$NPV_i = -I + L + P_0 - rL + (R_i - rL)q_i \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} = -W + L(r + q_i) + A.$$ (1)

If NPV is zero, we get the interest rate called Reservation rates $r_i = (A - W)/L - q_i$. If higher interest rate is obtained, business will give up the loan investment.

Stiglitz-Weiss model shows a low-risk enterprises (who has higher probability of success$q_i$) have lower reservation interest rates. If banks raise lending rates, it will reduce the quality of loan applicants, because the loan interest rates is above the reservation interest rate of high quality borrowers(in this case enterprises decide not to invest). So Stiglitz and Weiss suggested that all banks should not raise the lending rates but by reducing the supply of loans to reduce risk.

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According to theory of credit rationing, raising lending interest rates will lead low-risk businesses to leave the credit market or to switch to high-risk projects, so the ratio of bank loan repayment decreases. However, Zhang Yuan (2005) found an interesting phenomenon inconsistent with the classical theory: Some high-risk groups did not reduce the repayment rate after they obtain loans, the article identified a number of examples of risk items, such as Grameen Bank, China Mount Everest region pilot, etc., which shows that high-risk enterprises increase the repayment rate after they are given higher interest rate.

Lensink and Sterken (2002) use real options method to question the theory of credit rationing. Considering the enterprises have the right to wait, assuming that the first phase of return is less than the initial investment, they use a simple option and get an opposite conclusion: high interest rates will enable the high-risk businesses to leave the credit market. Enterprises have flexibility in investment when \( t = 0 \), enterprises can choose to invest or wait, if the enterprise invest currently, the net present value is recorded as \( NPV_{i,t=0} \) (the value is equal to formula(1)), if they choose to wait until \( t = 1 \), the net present value investment will be

\[
NPV_{i,t=1} = -q_i W/(1 + r) + q_i R_i/ r - q_i L.
\]

The difference between the two is the value of option to defer investment, after the collation

\[
V_i = (1 - q_i/(1 + r))W - P_0 + r L.
\]

Lensink and Sterken made the definition of reservation interest rate be the rate which makes the real option value, and thus

\[
\frac{dr}{d q_i} = \frac{W}{I - P_0 + 2 r L}.
\]

Suppose \( P_0 < I \), it is easy to obtain that \( q_i \) increases when \( r \) increases, which means that banks raising lending rates will make bad business leave the credit markets.

2 The loan investment decision-making model based on the IRR

Enterprises or individuals obtain loans from the bank for investment, the uncertain cost is mainly the interest payments, the net value of loan interest rates are random fluctuated comparing with exchange rate, prices, policy changes, and heterogeneity of the business or individual will also cause differences in rates, then businesses or individuals will face significant cost uncertainties. Here Cox-Ingersoll-Ross model is used to describe the uncertainty of interest rates:

\[
dr = \eta(\bar{r} - r) dt + \sigma \sqrt{r} dz
\]

where \( dz \) is the standard Wiener process, \( \eta \) indicates the reversion speed of average interest rate, \( \bar{r} \) is long-term average interest rate, \( \sigma \) is interest rate risk factor.

Stiglitz-Weiss model and Lensink-Sterken model consider the discount rate as lending rates, so that at constant rate they get the NPV value and real option value, however, the loan interest rates is a dynamic process comparing with the relatively risk-free interest rate, which makes the value of real option showed by exponential integration of two random variables, and it is very difficult to solve. In this regard, we introduce a real options model where IRR is a criteria for judging.

If we do not take into account other costs, when the IRR is higher than the lending rate \( r \), the enterprise will consider borrowing \( L \) from banks and invest it. Enterprises are able to observe current IRR, assuming that it also could anticipate future IRR and IRR is a constant, then the loan decision-making process can be seen as a kind of real option held by enterprises. It has the right to implement the option at any time when \( r \) does not exceed IRR-loan and invest, otherwise, not to implement the option – does not loan, wait-and-see future market change and then make decision. IRR of enterprises may be obtained through public data, such as the financial statements. Similarly, for personal loans, we also introduced the concept. When we put IRR of personal loans as the rate return, it is related to the personal income.

This is an American put option based on the underlying asset \( r \), exercise price IRR, its real option value is

\[
F(r) = E(\max(IRR - r, 0))
\]

IINS homepage: http://www.nonlinearscience.org.uk/
where IRR and $r$ is the net value relative to inflation, and risk-free interest rate, $E$ is the mathematical expectation operator.

According to the no-arbitrage principle, then

$$E(dF) = 0.$$  

Boundary conditions are

$$F(r^*) = IRR - r^*,
F'(r^*) = -1,
F(0) = IRR,$$

(which shows $r^*$ has nothing to do with the IRR, and it is completely determined by the dynamic changes parameters of interest rate $\eta, \sigma, \bar{r}$ where $r^*$ is the best lending rates, which means at this time the business or individual’s net income rate is $IRR - r^*$. Letting $a = 2\eta/\sigma^2$, then the value of the real option based on the IRR is

$$F(r) = IRR + \gamma[\Gamma(1 - a\bar{r}, -ar) - \Gamma(1 - a\bar{r})]$$

where $\Gamma$ is gamma function, coefficient

$$\gamma = \frac{-r^*}{\Gamma(1 - a\bar{r}, -ar) - \Gamma(1 - a\bar{r})}.$$  

Trigger point for loan $r^*$ is the solution for the equation

$$(-ar^*)^{1-a\bar{r}}e^{ar^*} = \Gamma(1 - a\bar{r}, -ar^*) - \Gamma(1 - a\bar{r})$$

Investment projects often have a limited lifespan, letting it be $T$, after which businesses or individuals will give up the investment projects. Then we estimate the probability of corporate loans before $T$, that is the probability of investment before $T$. This probability is a function of $t$ and $r$ (the lending rate of $t$), letting it be $P(r, t)$, which is the probability for the first time through $r^*$ before $T$, then

$$P(r, t) \equiv P(\tau(r^*) < T),$$

where $\tau(r^*)$ is the time through $r^*$. $P(r, t)$ is the solution of differential equations as the following:

$$P_t + P_r\eta(\bar{r} - r) + \frac{1}{2}P_{rr}\sigma^2 r = 0.$$  

Boundary conditions are

$$P(r^*, t) = 1, \ \forall \ t < T
P(r, T) = \max\left(\frac{IRR - r}{IRR - r^*}, 0\right) , \ \forall \ r > r^*$$

(Here we change the boundary conditions $P(r, T) = e^{-(r-r^*)}, \ \forall \ r > r^*$ in Zhang Xiao-qian (2007), because the interest rate of corporate loans is limited, assuming the business loan ceiling is $IRR$.)

$$P(r, T) = 1, \ \forall \ r \leq r^*$$

Rational bank should set up lending rates at intervals $[r^*, IRR]$, the specific value may be determined by their bargaining power, the more to the right shows that the stronger the bargaining power of banks, the more the left shows that the stronger the bargaining power of enterprises. Defining the interval as the extent practicable of the lending rate. Then for two enterprises $i, j$ with different IRR, assuming $IRR_i < IRR_j$, there are two possible trigger point of loans $r^*_i \leq r^*_j$ (Figure 1) or $r^*_i > r^*_j$ (Figure 2).

Case 1. $r^*_i \leq r^*_j$. Enterprises with better qualification are more likely to get loans, which is shown in Figure 1, where the IRR of enterprise $j$ is higher than that of enterprise $i$. At the same loan interest rates of loans, enterprise $j$ of high-quality has a larger possibility to get loans. Bank can control the loans by interest rates.
Case 2. $r_i^* > r_j^*$. At the same level of interest rates, the enterprises with high-quality have smaller probability to obtain loans, which is shown in the shadow region of Figure 2, the loan probability of high-quality enterprise $j$ is lower than that of enterprise $i$, where $r_E$ is the loan rate when the two enterprises have the same possibility of lending. Among $[r_j^*, r_E]$ (that is, the shadow area), banking regulation and control on companies through interest rates fails, enterprise of poor quality is more likely to obtain loans. If supply is insufficient, enterprises with high quality (that is, high IRR) are unable to obtain loans and leave credit market.

Since 1996 the process of market-oriented interest rate has been officially launched in our country, the floating range of lending rates of financial institutions has been expanded many times during more than a decade. Market-oriented interest rate optimizes the allocation of financial resources, from a certain extent, and resolve the finance plight of a number of corporate.

Considering whether there is credit rationing under the market-oriented interest rate. Enterprises face dynamic interest rates before they obtain loan, after the loan is obtained, interest rate was fixed, its NPV can still be expressed as formula (1), lower-risk enterprises (with higher probability of success $q_i$) has a lower NPV, so its IRR is high, which is corresponding to enterprise $j$ in figure. In this paper, under a dynamic interest rates, corporate lending probability is used as a measure of approach, the results show that credit rationing only occur in the shadow region of Figure 2, Lensink-Sterken conclusions are correct in the remaining cases.

In fact, if the banks do not provide different interest rates, lending rates of all business are the same parameters, which will leads to the same trigger point $r^*$, that is shown in Figure 1 (a), and interest rate regulation is not obvious at this time, the lending probability of high-quality corporate is lower than that in Figure 1 (b). If banks provide different interest rates, Figure 1 (b) and Figure 2 may appear, at this time setting the interest rate parameters reasonably to prevent the emergence of credit rationing has become a key of interest rate regulation.
3 Fine setting

There are two kinds of borrowers: the honest and the dishonest. The honest are intended to pay for loans at the beginning. Once they have possibility they will pay for their loans. The dishonest will make their payment decisions after they comparing the results of the lowest departure cost and the repayment amount. Let \( p \) be the borrowers’ probability of pay off. Banks can not get the exact values of probability \( p \). This is because of the prior information asymmetry of capacity of enterprise managing, enterprise projects or enterprise quality, as well as the post information asymmetry of enterprise managers’ selecting investment projects, and their efforts.

We will consider the effects of adjusting the interest rates on the bank profits.

Considering the problem of the bank profits at time \( t \) under the marketization of interest rate. We have

\[
R (r, t) = \left( e^{rt} - 1 \right) L (r) p (r) + \left( f (r, t) L (r) - L (r) \right) (1 - p (r)),
\]

where \( R (r, t) \) presents the bank profits at time \( t \), \( L (r) \) is the volume of loans when the interest rate is \( r \), \( p (r) \) is the probability of borrower’s repayment when the interest rate is \( r \), and \( f (r, t) \) presents the fine per amount on borrower from the banks when the borrower defaults at time \( t \), including the value of collateral, the fine to staff after loan defaults etc, which is the controllable variable of bank Increasing with the interest rate.

Because \( R (r, t) \) is related not only to \( f (r, t) \) and \( p (r) \), but also related to \( L (r) \), in practice we will obtain its comparative significance from the relative volume, that is, we consider

\[
S (r) = \left( \frac{R (r, t)}{L (r)} \right)^{1/t}
\]

whose economic meaning is the income per loans at unit time, then

\[
S (r) = \left( \frac{R (r, t)}{L (r)} \right)^{1/t} = \left( \frac{1 + IR (r, t)}{e^{rt}} \right)^{1/t} = \left[ e^{rt} p (r) + f (r, t) (1 - p (r)) \right]^{1/t},
\]

where \( IR (r, t) \) is the yield rate per unit at time \( t \).

We get it easily

\[
f (r, t) = \frac{[S (r)]^t - e^{rt}}{(1 - p (r))} + e^{rt}.
\]

4 Discussion:

Case 1. When \( S (r) = e^r \), then

\[
f (r, t) = e^{rt},
\]

which is independent with \( p (r) \).

At this time the fine is the same as the loan which should be paid.

Case 2. When \( S (r) > e^r \), we have \( f (r, t) > e^{rt} \). Then \( f (r, t) \) increases as \( p (r) \) increases.

As shown before, for a rational bank, the lending rates should be set on interval \( [r^*, IRR] \). In Fig.1, business with better quality are more likely to get loans from banks. When lending rates \( r \) increases, \( p (r) \) will increase and \( f (r, t) \) will increase.

Explanation: At this point, although the repayment rate will increase as the interest rate \( r \) increases, because of its high-income settings, the fine is also high.

In Fig. 2, when lending rate \( r \leq r_E \) (under credit rationing situation), given the same level of interest rates, the probability to obtain loans for a high-quality enterprise is smaller. So that \( p (r) \) reduces as the lending rate in this interval increase, therefore \( f (r, t) \) decrease (or, when the lending rate \( r \) decreases, \( p (r) \) will increases, so that \( f (r, t) \) will increase too).

Explanation: At this point when the interest rate \( r \) decreases, the banks think that the probability to obtain loans for low-grade corporate increases, although the repayment rate increases, but the risk increases too. Therefore fine is high.
When the lending rate $r > r_E$, given the same level of interest rates, the probability to obtain loans for the high-quality enterprises becomes larger. So when the lending rate $r$ increases, $p(r)$ will also increases, thus $f(r,t)$ will increase.

**Explanation:** At this point when the interest rate $r$ increases, the repayment rate will increase. But because of its high-income settings, so fines is also high.

**Case 3 When** $S(r) < e^r$, **we have** $f(r,t) < e^{rt}$.

$f(r,t)$ decreases as $p(r)$ increases.

As shown before, for a rational bank, the lending rates should be set on interval $[r^*, IRR]$. In Fig.1, business with better quality are more likely to get loans from banks.

When lending rates $r$ increases, $p(r)$ will increase and $f(r,t)$ will decrease.

**Explanation:** At this point, although the repayment rate will increase as the interest rate $r$ increases, because of its low-income settings, the fine is also low.

In Fig. 2, when lending rate $r \leq r_E$ (under credit rationing situation), given the same level of interest rates, the probability to obtain loans for a high-quality enterprise is smaller. So that $p(r)$ reduces as the lending rate in this interval increases, therefore $f(r,t)$ increases.

**Explanation:** At this point, when the interest rate $r$ increases, the banks think that the probability to obtain loans for low-grade corporate increases, so does the risk. Therefore fine is high.

When the lending rate $r > r_E$, given the same level of interest rates, the probability to obtain loans for the high-quality enterprises becomes larger. So when the lending rate $r$ increases, $p(r)$ will also increases, thus $f(r,t)$ will decrease.

**Explanation:** At this point when the interest rate $r$ increases, the repayment rate will increase. But because of its low-income settings, so fine is also low.

This shows the impact of interest rate risk on the fine is not robust.

### 5 Analysis and application

A real option approach can not change the theory of credit rationing fully, in certain interest rate range (the shaded area in Figure 2), there still exists credit rationing, market-oriented interest rates may not exist. In Figure 1, banks can use interest rate as an effective instrument in distinguishing qualification and the regulation of credit quality. This requires banks to use the same level of loan rates for the enterprises and individuals with similar IRR, and eliminate institutional discrimination. Such as in the real estate investment, loan rates should be based on loan duration and IRR levels, rather than watching his qualifications, history, scale and so on. For personal loans, loan rates should be strictly in accordance with its proceeds, rather than as is now the mortgage market on the basis of their availability of a job, and the remaining amount of the size of a credit loan to determine whether the mortgage interest rate should be given discount. Otherwise, the credit rationing may exist, high-quality customer may loss, the banks have more risk. In the process of providing loans, reasonable collateral requirements setting, strengthen surveillance by regulating fines, are the effective protections to raise bank profits and eventually achieve market-oriented interest rate.

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