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Quantitative Dynamics in Earthquake Sequences

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Abstract: The time dynamics of sequences of earthquakes occurred in three different seismic areas in Morocco (Al-Hoceima, Middle Atlas and West Rif) is investigated. All the series analyzed are characterized by $1/f$ temporal fluctuations, shown by the power-law behaviour of the Allan Factor, which allows to detect correlation properties in point processes.

Keywords: Earthquakes; Allan Factor

1 Introduction

Recently, increasing work has been carried out in the investigating the timescale properties of seismic sequences. The deep understanding of the correlation time struc-tures governing observational time series can provide information on the dynamical characterization of seismic processes and on the underlying geodynamical mechanisms. It is well known that several statistical features of earthquakes are scale invariant. Gutenberg and Richter [1] found that earthquake size follows a power-law distribution. Other scale-invariant features were determined in [2], [3]. Kagan [4] reviewed experimental evidences for earthquake scale-invariance. A theory to explain the presence of scale-invariance was proposed by Bak et al. [5] in a pioneer work; they introduced the idea of self-organized criticality (SOC) starting from a simple cellular automaton model, namely sandpile. In this model, sand is dropped slowly into random locations on a lattice, one grain at a time. For a critical slope, when a new grain is added many avalanches are triggered. Numerically they obtained that the distribution of avalanche sizes presents a power-law spectrum. It is widely assumed that the earthquakes are due to a stick-slip process involving sliding of the crust of the earth along faults ([6], [7]). When slip occurs at some location, the strain energy is released, and the stress propagates in the vicinity of that position. The SOC concept, then, is well suited for rationalizing observations on occurrences and magnitudes of earthquakes [8] that are an important part of the relaxation mechanism of the crust of the earth which is submitted to inhomogeneous increasing stresses accumulating at continental plate borders [9].The analysis of scaling laws concerning earthquakes has led to the development of a wide variety of physical models of seismogenesis, used to better characterize the seismicity patterns ([10], [11], [12], [13]). Several distributions have been used to model seismic activity. Among these, the Poisson distribution, which implies the independence of each event from the time elapsed since the previous event, is the most extensively used, since, in many cases, for large events a simple discrete Poisson distribution provides a close fit [14]. However, it has been shown that earthquake occurrence is possibly characterized by clustering properties with both short and long timescales with temporal correlation among the seismic events [4]. Like some random phenomena, such as noise and traffic in communication systems [15], biological ion-channel openings [16], trapping times in amorphous semiconductors [17], seismic events occur at some random locations in time. A stochastic point process is a mathematical description wich represents these events as random points on the time axis [18]. Such a process may be called fractal if some relevant statistics display scaling,

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characterized by a power-law behaviour, with related scaling coefficients, that indicate that the represented phenomenon contains clusters of points over a relatively large set of timescales [19]. Recently, some authors ([20], [21], [22], [23], [24]) focused their attention to observational evidences of time clustering properties in earthquake sequences of some different seismic areas, showing the existence of a range of timescales with scaling behaviour. The method they used, the Cantor dust method [25], consists in dividing the time interval T, over which N earthquake occur, into a series of n smaller intervals of length $t = T/n$ with $n = 2, 3, 4, ...$ and computing the number R of intervals of length t which contain at least one event. If the distribution of events has a fractal structure [26] then $R \sim t^{1-D}$, where D is the fractal dimension, which has subunitary values: the clustering is higher as D approaches 0, while a value of 1 corresponds to a uniform distribution (events equally spaced in time). But the parameter R does not give information about the temporal fluctuations, because it is not directly correlated to the power spectral density $S(f)$ of the process itself. In this paper any earthquake sequence is assumed to be a realization of a point process, with events occurring at some random locations in time, and it is completely defined by the set of event times, or equivalently by the set of interevent intervals. If the point process is Poissonian, the occurrence times are uncorrelated; for this memoryless process the interevent-interval probability density function $p(t)$ behaves as a decreasing exponential function $p(t) = e^{-\lambda t}$, for $t \geq 0$, with the mean rate of the process. If the point process is characterized by scaling behaviour, the interevent-interval probability density function $p(t)$ generally decreases as a power-law function of the interevent time, $p(t) = kt^{-(1+\alpha)}$, with α the so called fractal exponent [27]. The exponent measures the strength of the clustering and represents the scaling coefficient of the decreasing power-law spectral density of the process $S(f) \approx f^{-\alpha}$ ([28]). The power spectral density furnishes the information about how the power of the process is concentrated at various frequency bands [29], and it gives information about the nature of the temporal fluctuations of the process. If $\alpha \approx 0$, then the process is Poissonian, while if $\alpha \neq 0$, the process present correlation structures and is characterized by memory effects. In this paper, we analyze the seismicity of three different zones in Morocco in order to detect the presence of $1/f^{\alpha}$ temporal fluctuations, by means of the Allan Factor that allows to estimate the fractal exponent.

2 The method

A sequence of earthquakes can be mathematically expressed by a finite sum of Dirac's delta functions centered on the occurrence times t_i with amplitude A_i proportional to the magnitude M_i of the i-th event:

$$y(t) = \sum_{i=1}^{n} A_i \delta(t - t_i). \tag{1}$$

A representation of the process is given by the series of interevent time intervals. A common encountered first-order statistic for a stochastic point process is the interevent interval probability density function $p(t)$, that highlights the behaviour of the times between adjacent events, but reveals none of the information contained in the relationships among these times, such as correlation between adjacent time intervals. For a Poissonian process the interevent-interval density function decreases exponentially with constant rate λ. For a fractal process, that displays clustering properties, $p(t)$ generally behaves as a power-law function of the interevent time t with exponent $(1 + \alpha)$, where α, called fractal exponent, characterizes the clustering of the process. If we represent the point process by the set of the interevent times, some inference on the nature, clustered or not, of the time-occurrence sequence of the point process can be drawn on the basis of the coefficient of variation C_V, defined as $C_V = \sigma_t / < t >$, where $< t >$ is the mean interevent time and σ_t is its standard deviation [30]: a Poissonian process (completely random) has a $C_V = 1$ whereas a clustered process is characterised by a $C_V > 1$. Another useful representation of a point process is given by dividing the time axis into equally spaced contiguous counting windows of duration τ, and producing a sequence of counts $\{N_k(\tau)\}$, with $N_k(\tau)$ denoting the number of earthquakes in the k-th window:

$$N_k(\tau) = \int_{t_{k-1}}^{t_k} \sum_{j=1}^{n} A_i \delta(t - t_j) dt \tag{2}$$

This sequence is a discrete-random process of non-negative integers. An important feature of this representation is that it preserves the correspondence between the discrete time axis of the counting process N_k and the "real" time axis of the underlying point process, and the correlation in the process N_k refers to correlation in the underlying point process [27]. With this representation the Allan Factor (AF) statistics can be used to reveal the presence of fractality in the process. The Allan Factor (AF) is in relation with the variability of successive counts ([31], [32]), and is defined as the variance of successive counts for a specified counting time τ divided by twice the mean number of events in that counting time:

$$AF(\tau) = \frac{\langle (N_{k+1}(\tau) - N_k(\tau))^2 \rangle}{2\langle N_k(\tau) \rangle}. \tag{3}$$

The AF of a fractal point process varies with the counting time τ with a power-law form:

$$AF(\tau) = 1 + (\frac{\tau}{\tau_1})^{\alpha} \tag{4}$$

Over a relatively large range of counting times τ [30]; τ_1 is the fractal onset time for the AF. For Poissonian processes AF assumes values near unity and the fractal exponent $\alpha = 0$. The exponent α conveys the information about the temporal fluctuations of the process, because it is the exponent of the power spectrum, that for fractal processes decays as a power-law function of the frequency f, $S(f) \propto f^{-\alpha}$, with α measuring the strength of the clusterization.

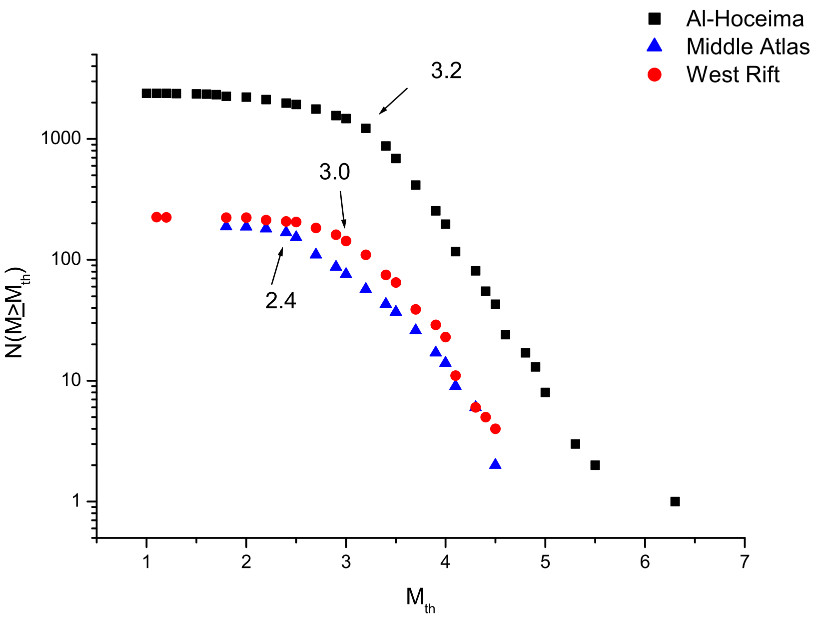

Figure 1: Gutenberg-Richter analysis for the three areas in Morocco. The completeness magnitude is 3.2, 3.0 and 2.4 for Al-Hoceima, West Rif and Middle Atlas respectively

3 Data analysis

Fig. 1 shows the cumulative distribution of the magnitudes for the three investigated areas. This analysis, known as Gutenberg-Richter analysis, is performed in order to estimate the completeness magnitude, which is the lowest magnitude for which the catalogue could be considered complete, as indicated by the linear behaviour of the distribution plotted in lin-log scales. In particular the three seismic catalogues are complete for magnitudes higher or equal to 3.2 for Al-Hoceima, 3.0 for West Rif and 2.4 for Middle Atlas. The Allan Factor method was applied to the three seismic areas. The linear range is visible in all the plots, but the fractl onset times are different: Al-Hoceima is characterized by a fractal onset time $\tau_1 = 3 \cdot 10^3 s$, the West Rif by $\tau_2 = 5 \cdot 10^4 s$, and Middle Atlas by $\tau_3 = 1.9 \cdot 10^5 s$. The AF curves appear quite rough due to the AF definition involving the difference between successive counts, but the scaling behaviour is quite clear for timescales larger than the fractal onset time. The values of the scaling exponent, estimated as the slope of the line fitting the linear part of the AF curves by the least square method, are 0.65, 013 and 0.17 for Al-Hoceima, West Rif and Middle Atlas respectively. The values of the scaling exponent indicates that Al-Hoceima is more clusterized than the other two regions, whose time behaviour can be considered quasi-Poissonian. Maybe the higher clustering degree of Al-Hoceima seismicity could be due to the many aftershocks following the February 24, 2004 earthquake.

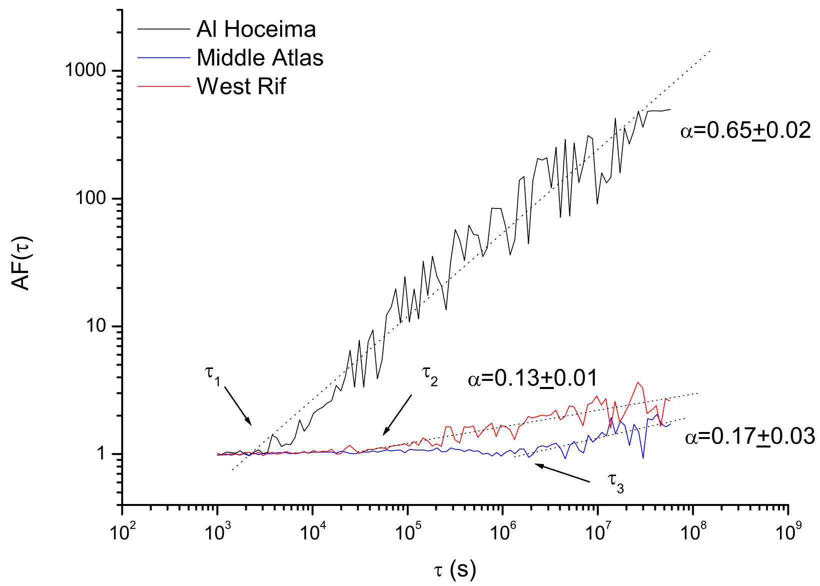

Figure 2: Allan Factor for the three areas in Morocco. The scaling exponent is 0.65, 0.13 and 0.17 for Al-Hoceima, West Rif and Middle Atlas respectively

4 Conclusions

We have investigated the $1/f$ temporal fluctuations of seismicity of three different zones in Morocco. We estimated the fractal exponent α using the Allan Factor statistics. The presence of time-correlation structures is revealed by the power-law form, that this measure assumes; the scaling exponent, calculated by means of a least square method, estimates the power-law index of the power spectral density. Our findings are: i) the sequences of Moroccan earthquakes show a clear clustering effect; ii) the West Rif and the Middle Atlas seismicities are characterized by a background activity with low value of the fractal exponent; iii) the Al-Hoceima seismicity is characterized by a higher clustering degree, maybe due to the large number of aftershocks, which followed the strong event occurred on February 24, 2004.

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