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Homotopy Perturbation Method for the Fisher's Equation and Its Generalized

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Abstract: More recently, Wazwaz [An analytic study of Fisher's equation by using Adomian decomposition method, Appl. Math. Comput. 154 (2004) 609–620] employed the Adomian decomposition method (ADM) to obtain exact solutions to Fisher's equation and to a nonlinear diffusion equation of the Fisher type. In this paper, He's homotopy perturbation method is employed for these equations to overcome the difficulty arising in calculating Adomian polynomials.

Keywords: Homotopy perturbation method; Fisher's equation; generalized Fisher's equation; nonlinear diffusion

AMS Subject Classification: 34L30, 34L16.

1 Introduction

In this paper, following [11], we consider the Fisher's equation

$$u_t = u_{xx} + \alpha\, u(1-u), \tag{1}$$

and the generalized Fisher's equation

$$u_t = u_{xx} + \alpha\, u(1-u^{\beta}), \tag{2}$$

and a nonlinear diffusion equation of the Fisher type

$$u_t = u_{xx} + u(1-u)(u-a), \qquad 0 < a < 1, \tag{3}$$

where $u_t = \frac{\partial u}{\partial t}, u_{xx} = \frac{\partial^2 u}{\partial x^2}$.

Fisher proposed Eq. (1) as a model for the propagation of a mutantgene, with u denoting the density of an advantageous. This equation is encountered in chemical kinetics [7] and population dynamics which includes problems such as nonlinear evolution of a population in a one-dimensional habitat, neutron population in a nuclear reaction. Moreover, the same equation occurs in logistic population growth models [1], flame propagation, neurophysiology, autocatalytic chemical reactions, and branching Brownian motion processes.

The exact solution of Eq. (2) for $\beta = 1$ obtained by Wang [10] was given by

$$u(x,t) = \left\{\frac{1}{2}\,\tanh[-\frac{\alpha}{2\sqrt{2\alpha+4}}(x - \frac{\alpha+4}{\sqrt{2\alpha+4}}t) + \frac{b}{2}] + \frac{1}{2}\right\}^{\frac{2}{\alpha}}, \tag{4}$$

where b is a constant.

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In order to solve above equations, many researchers have used various method. Authors studied variational iteration method for Fisher's equation [8], and also, employed a modified of variational iteration method for generalized Fisher's equation [9], and Wazwaz [11] studied Adomian decomposition method for Fisher's equation. We know that Adomian decomposition method requires the use of Adomian polynomials for nonlinear terms, and this need more work.

In this paper, we solve the Fisher's equation (1), the generalized Fisher's equation (2) and nonlinear diffusion equation of the Fisher type (3) via homotopy perturbation method (in short HPM) to overcome the difficulty arising in calculating Adomian polynomials. HPM introduced by He [3–6] has been used by many mathematicians and engineers to solve various functional equations. In this method the solution is considered as the sum of an infinite series which converges rapidly to the accurate solutions. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$ which is considered as a *small parameter*.

2 Homotopy perturbation method

To illustrate the basic ideas of this method, we consider the following equation:

$$A(u) - f(r) = 0, \qquad r \in \Omega, \tag{5}$$

with the boundary condition

$$B(u, \frac{\partial u}{\partial n}) = 0, \qquad t \in \Gamma, \tag{6}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Eq. (5) can therefore be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \qquad r \in \Omega. \tag{7}$$

By the homotopy technique, we construct a homotopy $V(r,p) : \Omega \times [0,1] \longrightarrow \mathbb{R}$,
which satisfies:

$$H(V,p) = (1-p)[L(V) - L(u_0)] + p\,[A(V) - f(r)] = 0, \quad p \in [0,1],\ r \in \Omega, \tag{8}$$

or

$$H(V,p) = L(V) - L(u_0) + p\,L(u_0) + p\,[N(V) - f(r)] = 0, \tag{9}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is an initial approximation of Eq. (5), which satisfies the boundary conditions. Obviously, from Eqs. (8) or (9) we will have

$$H(V,0) = L(V) - L(u_0) = 0, \tag{10}$$

$$H(V,1) = A(V) - f(r) = 0. \tag{11}$$

The changing process of p form zero to unity is just that of $V(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called *homotopy*. According to the HPM, we can first use the embedding parameter p as a *small parameter*, and assume that the solution of Eqs. (8) or (9) can be written as a power series in p :

$$V = V_0 + p\,V_1 + p^2V_2 + p^3V_3 + \cdots, \tag{12}$$

and the exact solution is obtained as follows:

$$u = \lim_{p\to 1} V = \lim_{p\to 1}(V_0 + p\,V_1 + p^2V_2 + p^3V_3 + \cdots) = \sum_{j=0}^{\infty} V_j\,. \tag{13}$$

The series (13) is convergent for most cases, and the rate of convergence depends on $L(u)$ [2].

3 HPM for the Fisher's equation

In this section, we consider the Fisher's equation

$$u_t = u_{xx} + \alpha\, u(1-u), \tag{14}$$

subject to the initial condition $u(x,0) = f(x)$.

For solving Eq. (14), by homotopy perturbation method we construct a homotopy as follows:

$$(1-p)\,\{\frac{\partial V}{\partial t} - \frac{\partial u_0}{\partial t}\} + p\,\{\frac{\partial V}{\partial t} - \frac{\partial^2 V}{\partial x^2} - \alpha\, V + \alpha\, V^2\} = 0, \tag{15}$$

or

$$\frac{\partial V}{\partial t} - \frac{\partial u_0}{\partial t} + p\,\frac{\partial u_0}{\partial t} + p\,\{-\frac{\partial^2 V}{\partial x^2} - \alpha\, V + \alpha\, V^2\} = 0. \tag{16}$$

Therefore, we have

$$\frac{\partial V}{\partial t} = \frac{\partial u_0}{\partial t} + p\,\{\frac{\partial^2 V}{\partial x^2} + \alpha\, V - \alpha\, V^2 - \frac{\partial u_0}{\partial t}\}. \tag{17}$$

Suppose the solution of Eq. (17) has the form

$$V(x,t) = V_0(x,t) + p\, V_1(x,t) + p^2\, V_2(x,t) + \cdots = \sum_{i=0}^{\infty} p^i\, V_i(x,t), \tag{18}$$

where $V_i(x,t)$ are functions yet to be determined. Substituting (18) into (17), and equating the terms with identical powers of p, we have

$$p^0\ :\ \frac{\partial V_0}{\partial t} = \frac{\partial u_0}{\partial t}, \qquad V_0(x,0) = f(x), \tag{19}$$

$$p^1\ :\ \frac{\partial V_1}{\partial t} = \frac{\partial^2 V_0}{\partial x^2} + \alpha\, V_1 - \alpha\, V_0^2 - \frac{\partial u_0}{\partial t}, \qquad V_1(x,0) = 0, \tag{20}$$

$$p^{k+1}\ :\ \frac{\partial V_{k+1}}{\partial t} = \frac{\partial^2 V_k}{\partial x^2} + \alpha\, V_k - \alpha \sum_{j=0}^{k} V_j V_{k-j}, \qquad V_{k+1}(x,0) = 0,\ \ k \geqslant 1, \tag{21}$$

Considering $u_0(x,t) = u(x,0) = f(x)$, we have

$$V_0(x,t) = u_0(x,t) = f(x), \tag{22}$$

$$V_{k+1}(x,t) = \int_0^t \left\{ \frac{\partial^2 V_k}{\partial x^2}(x,s) + \alpha\, V_k(x,s) - \alpha \sum_{j=0}^{k} V_j(x,s) V_{k-j}(x,s) \right\} ds. \tag{23}$$

where $k \geqslant 0$. Not that in this case $\frac{\partial u_0}{\partial t} = 0$.

Therefore, the exact solution of (14) can be obtained by setting $p = 1$, i.e.

$$u(x,t) = \lim_{p\to 1} V(x,t) = \sum_{k=0}^{\infty} V_k(x,t).$$

Following [11], two important cases of nonlinear diffusion, which correspond to some real physical processes, will be investigated to show the reliability of the proposed scheme.

case I: In Eq. (14), we set $\alpha = 1$ and $u(x,0) = f(x) = \lambda$, where λ is a constant. By Eqs. (22) and (23) we have

$$V_0(x,t) = \lambda, \qquad V_1(x,t) = \lambda(1-\lambda)t, \qquad V_2(x,t) = \lambda(1-\lambda)(1-2\lambda)\frac{t^2}{2!},$$

$$V_3(x,t) = \lambda(1-\lambda)(1-6\lambda+6\lambda^2)\frac{t^3}{3!}, \qquad V_4(x,t) = \lambda(1-\lambda)(1-2\lambda)(1-12\lambda+12\lambda^2)\frac{t^4}{4!},$$

$$V_5(x,t) = \lambda(1-\lambda)(1-30\lambda+150\lambda^2-240\lambda^3+120\lambda^4)\tfrac{t^5}{5!},$$

and so on for other components. The solution in a closed form is given by

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} V_k \\ &= \lambda + \lambda(1-\lambda)t + \lambda(1-\lambda)(1-2\lambda)\frac{t^2}{2!} + \lambda(1-\lambda)(1-6\lambda+6\lambda^2)\frac{t^3}{3!} + \cdots \\ &= \frac{\lambda e^t}{1-\lambda+\lambda e^t}, \end{aligned}$$

which is exactly the same as obtained by Adomain decomposition method [11]. Therefore, the HPM avoids the need for calculating the Adomian polynomials which can be difficult.

case II: In Eq. (14), we set $\alpha = 6$ and $u(x,0) = f(x) = \frac{1}{(1+e^x)^2}$. By Eqs. (22) and (23) we have

$$V_0(x,t) = \frac{1}{(1+e^x)^2}, \qquad V_1(x,t) = 10\frac{e^x}{(1+e^x)^3}t, \qquad V_2(x,t) = 50\frac{e^x(2e^x-1)}{(1+e^x)^4}\frac{t^2}{2!},$$

$$V_3(x,t) = -250\frac{e^x(-4e^{2x}+7e^x-1)}{(1+e^x)^5}\frac{t^3}{3!}, \quad V_4(x,t) = 1250\frac{e^x(8e^{3x}-33e^{2x}+18e^x-1)}{(1+e^x)^6}\frac{t^4}{4!},$$

$$V_5(x,t) = \frac{78125}{12}\frac{e^x(16e^{4x}-131e^{3x}+171e^{2x}-41e^x+1)}{(1+e^x)^7}\frac{t^5}{5!},$$

and so on. The solution in a closed form is given by

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} V_k \\ &= \frac{1}{(1+e^x)^2} + 10\frac{e^x}{(1+e^x)^3}t + 50\frac{e^x(2e^x-1)}{(1+e^x)^4}\frac{t^2}{2!} - 250\frac{e^x(-4e^{2x}+7e^x-1)}{(1+e^x)^5}\frac{t^3}{3!} + \cdots \\ &= \frac{1}{(1+e^{x-5t})}, \end{aligned}$$

which is exactly the same as obtained by Adomain decomposition method [11].

4 HPM for the generalized Fisher's equation

In this section, we consider the generalized Fisher's equation

$$u_t = u_{xx} + \alpha\, u(1-u^\beta), \tag{24}$$

subject to the initial condition $u(x,0) = f(x)$.

For solving Eq. (24), by homotopy perturbation method we construct a homotopy as follows:

$$(1-p)\,\{\frac{\partial V}{\partial t} - \frac{\partial u_0}{\partial t}\} + p\,\{\frac{\partial V}{\partial t} - \frac{\partial^2 V}{\partial x^2} - \alpha\, V + \alpha\, V^{\beta+1}\} = 0, \tag{25}$$

or

$$\frac{\partial V}{\partial t} - \frac{\partial u_0}{\partial t} + p\,\frac{\partial u_0}{\partial t} + p\,\{-\frac{\partial^2 V}{\partial x^2} - \alpha\, V + \alpha\, V^{\beta+1}\} = 0. \tag{26}$$

Therefore, we have

$$\frac{\partial V}{\partial t} = \frac{\partial u_0}{\partial t} + p\,\{\frac{\partial^2 V}{\partial x^2} + \alpha\, V - \alpha\, V^{\beta+1} - \frac{\partial u_0}{\partial t}\}. \tag{27}$$

Suppose the solution of Eq. (27) has the form

$$V(x,t) = V_0(x,t) + p\,V_1(x,t) + p^2\,V_2(x,t) + \cdots = \sum_{i=0}^{\infty} p^i\,V_i(x,t), \tag{28}$$

where $V_i(x,t)$ are functions yet to be determined. Substituting (28) into (27), and equating the terms with identical powers of p, we have

$$p^0\ :\ \frac{\partial V_0}{\partial t} = \frac{\partial u_0}{\partial t}, \qquad V_0(x,0) = f(x), \tag{29}$$

$$p^1\ :\ \frac{\partial V_1}{\partial t} = \frac{\partial^2 V_0}{\partial x^2} + \alpha\,V_0 - \alpha\,V_0^{\beta+1} - \frac{\partial u_0}{\partial t}, \qquad V_1(x,0) = 0, \tag{30}$$

$$p^{k+1}\ :\ \frac{\partial V_{k+1}}{\partial t} = \frac{\partial^2 V_k}{\partial x^2} + \alpha\,V_k - \alpha\sum_{j_1=0}^{k}\sum_{j_2=0}^{j_1}\cdots\sum_{j_\beta=0}^{j_{\beta-1}}(V_{j_\beta}V_{j_{\beta-1}-j_\beta}\cdots V_{k-j_1}), \tag{31}$$

where $V_{k+1}(x,0) = 0,\ \ k \geqslant 1$.

Considering $u_0(x,t) = u(x,0) = f(x)$, we have

$$V_0(x,t) = u_0(x,t) = f(x), \tag{32}$$

$$V_{k+1} = \int_0^t \left\{ \frac{\partial^2 V_k}{\partial x^2} + \alpha\,V_k - \alpha\sum_{j_1=0}^{k}\sum_{j_2=0}^{j_1}\cdots\sum_{j_\beta=0}^{j_{\beta-1}}(V_{j_\beta}V_{j_{\beta-1}-j_\beta}\cdots V_{k-j_1}) \right\} ds, \tag{33}$$

where $k \geqslant 0$. Therefore, the exact solution of (24) can be obtained by setting $p = 1$, i.e.

$$u(x,t) = \lim_{p\to 1} V(x,t) = \sum_{k=0}^{\infty} V_k(x,t).$$

In Eq. (24), following [11], we set $\alpha = 1$, $\beta = 6$ and $u(x,0) = f(x) = \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}}$. By Eqs. (32) and (33) we have

$$V_0(x,t) = \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}}, \tag{34}$$

$$V_{k+1} = \int_0^t \left\{ \frac{\partial^2 V_k}{\partial x^2} + V_k - \sum_{i=0}^{k}\sum_{j=0}^{i}\sum_{m=0}^{j}\sum_{n=0}^{m}\sum_{r=0}^{n}\sum_{s=0}^{r}(V_s V_{r-s} V_{n-r} V_{m-n} V_{j-m} V_{i-j} V_{k-i}) \right\} ds, \tag{35}$$

where $k \geqslant 0$. Consequently, we obtain

$$V_0(x,t) = \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}}, \qquad V_1(x,t) = \frac{5\,e^{\frac{3}{2}x}}{4(1+e^{\frac{3}{2}x})^{\frac{2}{3}}}t, \qquad V_2(x,t) = \frac{25\,e^{\frac{3}{2}x}(e^{\frac{3}{2}x}-3)}{16(1+e^{\frac{3}{2}x})^{\frac{5}{3}}}\frac{t^2}{2!},$$

$$V_3(x,t) = \frac{125\,e^{\frac{3}{2}x}(18e^{\frac{3}{2}x}-e^{3x}-9)}{64(1+e^{\frac{3}{2}x})^{\frac{8}{3}}}\frac{t^3}{3!}, \qquad V_4(x,t) = \frac{625\,e^{\frac{3}{2}x}(e^{6x}-80e^{\frac{9}{2}x}+90e^{3x}+144e^{\frac{3}{2}x}-27)}{6(1+e^{\frac{3}{2}x})^{\frac{16}{3}}}\frac{t^4}{4!},$$

$$V_5(x,t) = -\frac{78125\,e^{\frac{3}{2}x}(-e^{9x}+334e^{\frac{15}{2}x}-1255e^{6x}-220e^{\frac{9}{2}x}+585e^{3x}+1134e^{\frac{3}{2}x}-81)}{24576(1+e^{\frac{3}{2}x})^{\frac{22}{3}}}\frac{t^5}{5!},$$

and so on. The solution in a closed form is given by

$$\begin{aligned}
u(x,t) &= \sum_{k=0}^{\infty} V_k \\
&= \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}} + \frac{5e^{\frac{3}{2}x}}{4(1+e^{\frac{3}{2}x})^{\frac{2}{3}}}t + \frac{25e^{\frac{3}{2}x}(e^{\frac{3}{2}x}-3)}{16(1+e^{\frac{3}{2}x})^{\frac{5}{3}}}\frac{t^2}{2!} + \frac{125e^{\frac{3}{2}x}(18e^{\frac{3}{2}x}-e^{3x}-9)}{64(1+e^{\frac{3}{2}x})^{\frac{8}{3}}}\frac{t^3}{3!} + \cdots \\
&= \left\{\frac{1}{2}\tanh[-\frac{3}{4}(x-\frac{5}{2}t)] + \frac{1}{2}\right\}^{\frac{1}{3}},
\end{aligned}$$

which is exactly the same as obtained by Adomain decomposition method [11]. Therefore, the HPM avoids the need for calculating the Adomian polynomials which can be difficult.

5 HPM for diffusion equation of the Fisher type

In this section, following [11], we examine the nonlinear diffusion equation of the Fisher type

$$u_t = u_{xx} + u(1-u)(u-a), \qquad 0 < a < 1, \tag{36}$$

subject to the initial condition

$$u(x,0) = f(x). \tag{37}$$

In [11] an exact solution of (36) which describes the coalescence of two traveling fronts of the same sense into a front connecting two stable constant states is found.

For solving Eq. (36), by homotopy perturbation method we construct a homotopy as follows:

$$(1-p)\,\{\frac{\partial V}{\partial t} - \frac{\partial u_0}{\partial t}\} + p\,\{\frac{\partial V}{\partial t} - \frac{\partial^2 V}{\partial x^2} + a\,V - (1+a)\,V^2 + V^3\} = 0, \tag{38}$$

or

$$\frac{\partial V}{\partial t} - \frac{\partial u_0}{\partial t} + p\,\frac{\partial u_0}{\partial t} + p\,\{-\frac{\partial^2 V}{\partial x^2} + a\,V - (1+a)\,V^2 + V^3\} = 0. \tag{39}$$

Therefore, we have

$$\frac{\partial V}{\partial t} = \frac{\partial u_0}{\partial t} + p\,\{\frac{\partial^2 V}{\partial x^2} - a\,V + (1+a)\,V^2 - V^3 - \frac{\partial u_0}{\partial t}\}. \tag{40}$$

Suppose the solution of Eq. (40) has the form

$$V(x,t) = V_0(x,t) + p\,V_1(x,t) + p^2\,V_2(x,t) + \cdots = \sum_{i=0}^{\infty} p^i\,V_i(x,t), \tag{41}$$

where $V_i(x,t)$ are functions yet to be determined. Substituting (41) into (40), and equating the terms with identical powers of p, we have

$$p^0 \,:\, \frac{\partial V_0}{\partial t} = \frac{\partial u_0}{\partial t}, \qquad V_0(x,0) = f(x), \tag{42}$$

$$p^1 \,:\, \frac{\partial V_1}{\partial t} = \frac{\partial^2 V_0}{\partial x^2} - a\,V_0 + (1+a)\,V_0^2 - V_0^3 - \frac{\partial u_0}{\partial t}, \qquad V_1(x,0) = 0, \tag{43}$$

$$p^{k+1} \,:\, \frac{\partial V_{k+1}}{\partial t} = \frac{\partial^2 V_k}{\partial x^2} - a\,V_k + (1+a)\sum_{i=0}^{k}(V_i V_{k-i}) - \sum_{i=0}^{k}\sum_{j=0}^{i}(V_j V_{i-j} V_{k-i}), \tag{44}$$

where $V_{k+1}(x,0) = 0$ and $k \geqslant 1$.

Considering $u_0(x,t) = u(x,0) = f(x)$, we have

$$V_0(x,t) = u_0(x,t) = f(x), \tag{45}$$

$$V_{k+1} = \int_0^t \left\{ \frac{\partial^2 V_k}{\partial x^2} - a\, V_k + (1+a)\sum_{i=0}^{k}(V_i V_{k-i}) - \sum_{i=0}^{k}\sum_{j=0}^{i}(V_j V_{i-j} V_{k-i}) \right\} ds, \tag{46}$$

where $k \geqslant 0$. Note that in this case $\frac{\partial u_0}{\partial t} = 0$.

Therefore, the exact solution of (36) can be obtained by setting $p = 1$, i.e.

$$u(x,t) = \lim_{p \to 1} V(x,t) = \sum_{k=0}^{\infty} V_k(x,t).$$

In Eq. (36), following [11], we set $u(x,0) = f(x) = \frac{1}{1+e^{-\frac{x}{\sqrt{2}}}}$. By Eqs. (45) and (46) we have

$$V_0(x,t) = \frac{1}{1+A}, \qquad V_1(x,t) = \frac{A(1-2a)}{2(1+A)^2}t, \qquad V_2(x,t) = \frac{A(A-1)(1-2a)^2}{4(1+A)^3}\frac{t^2}{2!},$$

$$V_3(x,t) = \frac{A(A^2-4A-1)(1-2a)^3}{8(1+A)^4}\frac{t^3}{3!}, \qquad V_4(x,t) = \frac{A(A-1)(A^2-10A+1)(1-2a)^4}{16(1+A)^5}\frac{t^4}{4!},$$

$$V_5(x,t) = \frac{25\, A(A^4-26A^3+66A^2-26A+1)(1-2a)^5}{768(1+A)^6}\frac{t^5}{5!},$$

and so on, where $A = e^{-\frac{x}{\sqrt{2}}}$. The solution in a closed form is given by

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} V_k \\ &= \frac{1}{1+e^{-\frac{x}{\sqrt{2}}}} + \frac{e^{-\frac{x}{\sqrt{2}}}(1-2a)}{2(1+e^{-\frac{x}{\sqrt{2}}})^2}t + \frac{e^{-\frac{x}{\sqrt{2}}}(e^{-\frac{x}{\sqrt{2}}}-1)(1-2a)^2}{4(1+e^{-\frac{x}{\sqrt{2}}})^3}\frac{t^2}{2!} \\ &+ \frac{e^{-\frac{x}{\sqrt{2}}}(-4e^{-\frac{x}{\sqrt{2}}}+e^{-\sqrt{2}x}-1)(1-2a)^3}{8(1+e^{-\frac{x}{\sqrt{2}}})^4}\frac{t^3}{3!} + \cdots \\ &= \frac{1}{(1+e^{-\frac{x+ct}{\sqrt{2}}})}, \end{aligned}$$

which is exactly the same as obtained by Adomain decomposition method [11].

6 Conclusion

In this paper, the homotopy perturbation method has been successfully used to study Fisher's equation, generalized Fisher's equation and nonlinear diffusion equation of the Fisher type. An important conclusion can made here. The homotopy perturbation method avoids the need for calculating the Adomian polynomials which can be difficult in some cases. The results show that the homotopy perturbation method is a powerful mathematical tool for finding the exact and approximate solutions of nonlinear equations. In our work we use the MATLAB to calculate the series obtained from the homotopy perturbation method.

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