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Travelling Wave Solutions for the Generalized (2+1)-Dimensional ZK-MEW Equation

Alaattin Esen *, Selçuk Kutluay
Department of Mathematics, Inönü University, 44280 Malatya, Turkey
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Abstract: In this paper, we construct exact travelling wave solutions for the generalized (2+1)-dimensional ZK-MEW equation by using the solutions of an auxiliary ordinary differential equation given by Sirendaoreji [1]. It is shown that some solutions obtained in this study are new solutions which have not been reported yet.

Keywords:auxiliary equation method; travelling wave solutions; generalized ZK-MEW equation

1 Introduction

A large number of equations in many areas of applied mathematics, physics and engineering appear as nonlinear wave equations. One of the most important one-dimensional nonlinear wave equations is the KdV equation

$$u_t + auu_x + u_{xxx} = 0$$

which describes the evolution of weakly nonlinear and weakly dispersive wave used in various fields such as solid state physics, plasma physics, fluid physics and quantum field theory [2,3].

One of the best known 2-dimensional generalizations of the KdV equation is Zakharov-Kuznetsov (ZK) equation [4] in the form

$$u_t + auu_x + (u_{xx} + u_{yy})_x = 0$$

which governs the behaviour of weakly nonlinear ion-acoustic in a plasma comprising cold-ions and hot isothermal electrons in the presence of a uniform magnetic field [5-10]. The ZK equation is not integrable by the inverse scattering transform method. Shivamoggi [11] showed that the ZK equation has the Painleve property.

The modified equal width (MEW) equation is of the form

$$u_t + a\left(u^3\right)_x - u_{xxt} = 0$$

which appears in many physical applications [12-15]. The generalized form of the MEW equation in the ZK sense [16] is given by

$$u_t + a\left(u^n\right)_x + \left(bu_{xt} + ru_{yy}\right)_x = 0. \tag{1}$$

It is natural to call this equation as the ZK-MEW equation. Wazwaz [16] studied Eq. (1) using the sine-cosine method and the tanh technique.

In recent years, much efforts have been spent on the construction of exact travelling wave solutions of these types equations and many powerful methods have been presented such as the inverse scattering

***Corresponding author**. Tel.: +90-422-3410010; fax: +90-422-3410037. *E-mail address*: aesen@inonu.edu.tr

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transform [17], Backlund transform [18], Darboux transform [19], Hirota's bilinear method [20], Painlevé expansion [21], homogeneous balance method [22-24], Jacobi elliptic function expansion method [25,26], sine-cosine method [27], tanh-function method [28,29], F-expansion method [30-32], auxiliary equation method [33-41], exp-function method [42] and so on.

The aim of this paper is to obtain some new types exact travelling wave solutions for the generalized ZK-MEW equation with an arbitrary positive power n of the dependent variable $u(x,y,t)$ using the auxiliary equation method.

In order to use the auxiliary equation method we assume that a given nonlinear equation for $u(x,y,t)$ is of the form

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{xy}, u_{yy}, \ldots) = 0 \tag{2}$$

which involves higher order derivatives and nth power of u=$u(x,y,t)$

The basic steps of the method are listed as follows:

Step 1. Use the transformation $\xi = x + y - wt$ in Eq. (2), integrate the resulting equation as long as all terms contain derivatives and set the integration constant zero to reduce Eq. (2) into the following ordinary differential equation

$$Q(u, u_\xi, u_{\xi\xi}, \ldots) = 0. \tag{3}$$

Step 2. Assume that the solution $u(\xi)$ of Eq. (3) can be expressed in the finite series form

$$u(\xi) = \sum_{i=0}^{2M} a_i F^i(\xi) \tag{4}$$

where a_i's are constants to be determined, and M is a positive integer to be determined by balancing the highest order derivative term with the highest power nonlinear term in Eq. (3). In Eq. (4), $F(\xi)$ satisfies the auxiliary differential equation

$$F'^2 = AF^2 + BF^3 + CF^4 \tag{5}$$

where A, B and C are parameters to be determined. Eq. (5) admits several types of solutions as listed in Table 1 [1]. Some of the solutions in Table 1 is also given in [41].

Step 3. Substitute Eq. (4) along with Eq. (5) into Eq. (3) and equate to zero the coefficients of all powers of F yields a set of algebraic equations for A, B, C, $a_i (i = 0, 1, 2, \ldots, 2M)$ and w.

Step 4. Finally, solve the set of algebraic equations with the help of Mathematica or Maple and substitute the solutions obtained in this step back into ansatz (4), then obtain the exact travelling wave solutions for Eq. (2).

Table1: Solutions of Eq.(6) with $\Delta = B^2 - 4AC,\ \varepsilon = \pm 1$

No.	$F(\xi)$	No.	$F(\xi)$
1	$\frac{-AB\,\mathrm{sech}^2(\frac{\sqrt{A}}{2}\xi)}{B^2-AC(1+\varepsilon\tanh(\frac{\sqrt{A}}{2}\xi))^2},\ A>0$	8	$\frac{-A\sec^2(\frac{\sqrt{-A}}{2}\xi)}{B+2\varepsilon\sqrt{-AC}\tan(\frac{\sqrt{-A}}{2}\xi)},\ A<0,\ C>0$
2	$\frac{AB\,\mathrm{csch}^2(\frac{\sqrt{A}}{2}\xi)}{B^2-AC(1+\varepsilon\coth(\frac{\sqrt{A}}{2}\xi))^2},\ A>0$	9	$\frac{A\,\mathrm{csch}^2(\frac{\sqrt{A}}{2}\xi)}{B+2\varepsilon\sqrt{AC}\coth(\frac{\sqrt{A}}{2}\xi)},\ A>0,\ C>0$
3	$\frac{2A\,\mathrm{sech}(\sqrt{A}\xi)}{\varepsilon\sqrt{\Delta}-B\,\mathrm{sech}(\sqrt{A}\xi)},\ A>0,\ \Delta>0$	10	$\frac{-A\csc^2(\frac{\sqrt{-A}}{2}\xi)}{B+2\varepsilon\sqrt{-AC}\cot(\frac{\sqrt{-A}}{2}\xi)},\ A<0,\ C>0$
4	$\frac{2A\sec(\sqrt{-A}\xi)}{\varepsilon\sqrt{\Delta}-B\,\mathrm{sech}(\sqrt{-A}\xi)},\ A<0,\ \Delta>0$	11	$-\frac{A}{B}(1+\varepsilon\tanh(\frac{\sqrt{A}}{2}\xi)),\ A>0,\ \Delta=0$
5	$\frac{2A\,\mathrm{csch}(\sqrt{A}\xi)}{\varepsilon\sqrt{-\Delta}-B\,\mathrm{csch}(\sqrt{A}\xi)},\ A>0,\ \Delta<0$	12	$-\frac{A}{B}(1+\varepsilon\coth(\frac{\sqrt{A}}{2}\xi)),\ A>0,\ \Delta=0$
6	$\frac{2A\csc(\sqrt{-A}\xi)}{\varepsilon\sqrt{\Delta}-B\csc(\sqrt{-A}\xi)},\ A<0,\ \Delta>0$	13	$\frac{4A\exp(\varepsilon\sqrt{A}\xi)}{(\exp(\varepsilon\sqrt{A}\xi)-B)^2-4AC},\ A>0$
7	$\frac{-A\,\mathrm{sech}^2(\frac{\sqrt{A}}{2}\xi)}{B+2\varepsilon\sqrt{AC}\tanh(\frac{\sqrt{A}}{2}\xi)},\ A>0,\ C>0$	14	$\frac{4A\varepsilon\exp(\varepsilon\sqrt{A}\xi)}{1-4AC\exp(2\varepsilon\sqrt{A}\xi)},\ A>0,\ B=0$

2 The generalized ZK-MEW equation

The generalized (2+1)-dimensional ZK-MEW equation [16] is in the form of

$$u_t + a(u^n)_x + (bu_{xt} + ru_{yy})_x = 0,\ \ (n>1) \tag{6}$$

where a, b and r are arbitrary constants.

By substituting $u(x,y,t)=u(\xi)\,,\ \xi=x+y-wt$ into Eq. (6), integrating the resulting equation with respect to ξ and setting the integration constant to zero yield the ordinary differential equation

$$-wu+au^n+(r-bw)\,u_{\xi\xi}=0. \tag{7}$$

Inserting (4) into (7) and considering the homogeneous balance between $u_{\xi\xi}$ and u^n gives $M=\frac{1}{n-1}$. To obtain an analytic solution, M should be a positive integer. This requires the use of the transformation $u=v^{\frac{1}{n-1}}$ which reduces Eq. (7) to the equation

$$-w(n-1)^2v^2+a(n-1)^2v^3+(r-bw)(2-n)\,v_\xi^2+(r-bw)(n-1)\,vv_{\xi\xi}=0. \tag{8}$$

Balancing $vv_{\xi\xi}$ with v^3 gives M = 1. Therefore, we can choose the solution of Eq. (8) in the form

$$u=a_0+a_1F+a_2F^2 \tag{9}$$

Substituting (9) into (8) and setting each coefficient of F to zero, we obtain the following set of algebraic equations:

$F^0:(n-1)^2a_0^2(aa_0-w)=0,$

$F^1:(n-1)\,a_0a_1(w(Ab+2(n-1))+Ar+3a(n-1)\,a_0)=0,$

$F^2:\frac{1}{2}\Big(3B(n-1)\,ra_0a_1+bw\left(-3B(n-1)\,a_0a_1-2A\left(a_1^2+4(n-1)\,a_0a_2\right)\right)+2\Big(3a(n-1)^2a_0$
$\times\left(a_1^2+a_0a_2\right)-w(n-1)^2\left(a_1^2+2a_0a_2\right)+Ar\left(a_1^2+4(n-1)\,a_0a_2\right)\Big)\Big)=0,$

$F^3:2C(n-1)\,ra_0a_1+\frac{1}{2}Bra_1^2(n+1)+aa_1^3(n-1)^2+5Bra_0a_2(n-1)-2wa_1a_2(n-1)^2$
$+Ara_1a_2(n+3)+6aa_0a_1a_2(n-1)^2-\frac{1}{2}bw\left(4C(n-1)\,a_0a_1+2A(n+3)\,a_1a_2+B\left((n+1)\,a_1^2\right.\right.$
$\left.\left.+10(n-1)\,a_0a_2\right)\right)=0,$

$F^4:Cr\left(-6a_0a_2+n\left(a_1^2+6a_0a_2\right)\right)+\frac{1}{2}a_2\Big(B(5n+3)\,ra_1+2\left(4ar-w(n-1)^2\right)a_2+3a(n-1)^2$
$\times\left(a_1^2+a_0a_2\right)\Big)+\frac{1}{2}wk\left(a_2\left(B(5n+3)\,a_1+8Aa_2\right)\right)+2C\left(-6a_0a_2+n\left(a_1^2+6a_0a_2\right)\right)=0,$

$F^5:a_2\Big(4Cnra_1+\left(B(n+3)\,r+3a(n-1)^2a_1\right)a_2-bw\left(4Cna_1+B(n+3)\,a_2\right)\Big)=0,$

$F^6:a_2^2\left(-2bCw(n+1)+2C(n+1)\,r+a(n-1)^2a_2\right)=0.$

Solving the above system with the aid of Mathematica, we find the following three set of solutions:

$$\left\{\ a_0=0,\quad a_1=\tfrac{B(n+1)(bw-r)}{2a(n-1)^2},\quad a_2=0,\quad A=\tfrac{w(n-1)^2}{r-bw},\quad C=0,\right. \tag{10}$$

$$\left\{\ a_0=0,\quad a_1=0,\quad a_2=\tfrac{2C(n+1)(bw-r)}{a(n-1)^2},\quad A=\tfrac{w(n-1)^2}{4(r-bw)},\quad B=0,\right. \tag{11}$$

$$\left\{\ a_0=0,\quad a_1=\tfrac{B(n+1)(bw-r)}{a(n-1)^2},\quad a_2=-\tfrac{B^2(n+1)(bw-r)^2}{2aw(n-1)^4},\quad A=\tfrac{w(n-1)^2}{r-bw},\quad C=-\tfrac{B^2(bw-r)}{4w(n-1)^2}\right.\ . \tag{12}$$

Substituting (10) with $F(\xi)$ in Table 1 into (9) and noting that $u(x,y,t)=v^{\frac{1}{n-1}}$ we only obtain the following exact travelling wave solutions of Eq. (1) as follows:

$$u_1(x,y,t)=\left\{\frac{w(n+1)}{2a}\operatorname{sech}^2\left(\frac{n-1}{2}\sqrt{\frac{w}{r-bw}}\xi\right)\right\}^{\frac{1}{n-1}},\ w(r-bw)>0, \tag{13}$$

$$u_2(x,y,t)=\left\{-\frac{w(n+1)}{2a}\operatorname{csch}^2\left(\frac{n-1}{2}\sqrt{\frac{w}{r-bw}}\xi\right)\right\}^{\frac{1}{n-1}},\ w(r-bw)>0, \tag{14}$$

$$u_{3,4}(x,y,t)=\left\{\frac{w(n+1)}{a\left(1-\varepsilon\cosh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)\right)}\right\}^{\frac{1}{n-1}},\ \ w(r-bw)>0, \tag{15}$$

$$u_{5,6}(x,y,t)=\left\{\frac{w(n+1)}{a\left(1-\varepsilon\cos\left((n-1)\sqrt{-\frac{w}{r-bw}}\xi\right)\right)}\right\}^{\frac{1}{n-1}},\quad w(r-bw)<0, \tag{16}$$

$$u_{7,8}(x,y,t)=\left\{\frac{w(n+1)}{a\left(1-\varepsilon\sin\left((n-1)\sqrt{-\frac{w}{r-bw}}\xi\right)\right)}\right\}^{\frac{1}{n-1}},\quad w(r-bw)<0, \tag{17}$$

$$u_{9}(x,y,t)=\left\{\frac{w(n+1)}{2a}\sec^2\left(\frac{n-1}{2}\sqrt{-\frac{w}{r-bw}}\xi\right)\right\}^{\frac{1}{n-1}},\ w(r-bw)<0, \tag{18}$$

$$u_{10}(x,y,t)=\left\{\frac{w(n+1)}{2a}\csc^2\left(\frac{n-1}{2}\sqrt{-\frac{w}{r-bw}}\xi\right)\right\}^{\frac{1}{n-1}},\ w(r-bw)<0, \tag{19}$$

$$u_{11,12}(x,y,t)=\left\{-\frac{2Bw(n+1)\exp\left(\varepsilon(n-1)\sqrt{\frac{w}{r-bw}}\xi\right)}{a\left(\exp\left(\varepsilon(n-1)\sqrt{\frac{w}{r-bw}}\xi\right)-B\right)^2}\right\}^{\frac{1}{n-1}},\quad w(r-bw)>0, \tag{20}$$

where w and B are arbitrary constants, and $\xi=x+y-wt$.

From (11), we only obtain the following exact travelling wave solution of Eq.(1) to be different from the above solutions

$$u_{13,14}(x,y,t)=\left\{\frac{2C(n-1)^2(n+1)w^2(bw-r)\exp\left(\varepsilon(n-1)\sqrt{\frac{w}{r-bw}}\xi\right)}{a\left((r-bw)-C(n-1)^2w\exp\left(\varepsilon(n-1)\sqrt{\frac{w}{r-bw}}\xi\right)\right)^2}\right\}^{\frac{1}{n-1}}, \tag{21}$$

where $w(r-bw)>0$, w and C are arbitrary constants, and $\xi=x+y-wt$.

From (12), we get the exact travelling wave solution of Eq. (1) to be different from the above all solutions

$$u_{15,16}(x,y,t)=\left\{\frac{4w(n+1)}{a}\frac{\cosh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)-\varepsilon\sinh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)}{\left(2+\cosh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)-\varepsilon\sinh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)\right)^2}\right\}^{\frac{1}{n-1}}, \tag{22}$$

$$u_{17,18}(x,y,t)=\left\{-\frac{4w(n+1)}{a}\frac{\cosh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)-\varepsilon\sinh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)}{\left(2-\cosh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)+\varepsilon\sinh\left((n-1)\sqrt{\frac{w}{r-bw}}\xi\right)\right)^2}\right\}^{\frac{1}{n-1}}, \tag{23}$$

$$u_{19,20}(x,y,t)=\left\{-\frac{4Bw(n+1)\exp\left(\varepsilon(n-1)\sqrt{\frac{w}{r-bw}}\xi\right)}{a\left(\exp\left(\varepsilon(n-1)\sqrt{\frac{w}{r-bw}}\xi\right)-2B\right)^2}\right\}^{\frac{1}{n-1}}, \tag{24}$$

where $w(r-bw)>0$, w and B are arbitrary constants, and $\xi=x+y-wt$.

Among these solutions, (13), (14), (18) and (19) are identical to those obtained by Wazwaz [16] using the sine-cosine method. To our knowledge, the other solutions are new solutions for the generalized ZK-MEW equation (6) and have not been reported yet. It should be noted that all the solutions given in this paper have been verified with the aid of Mathematica by putting them back into the generalized (2+1)-dimensional ZK-MEW equation.

3 Conclusion

In this paper, the auxiliary equation and its solutions given by Sirendaoreji [1] have been successfully used to obtain some exact travelling wave solutions for the generalized ZK-MEW equation. It can be clearly seen that some solutions given in this paper are new solutions which have not been reported yet. The method can also be efficiently used to construct new and more exact travelling wave solutions for some other generalized nonlinear wave equations arising in mathematical physics.

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