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Large-Time Asymptotics for Periodic Solutions of a Generalized Burgers Equation

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Abstract: In this article, we construct large-time asymptotics for periodic solutions of a generalized Burgers equation (GBE) $u_t + uu_x + \lambda u = \frac{\delta}{2}u_{xx}$ for the parametric case $\lambda > \frac{\delta\pi^2}{l^2}$; here $\lambda > 0$, $\delta > 0$. A careful numerical study reveals that the asymptotic solution constructed here describes large time evolution for three different periodic initial conditions. Sachdev et al [1] studied the above GBE for $\lambda \leq \frac{\delta\pi^2}{l^2}$.

Keywords: generalized Burgers equation; balancing argument; periodic solutions

Mathematics Subject Classification (2000). 35B10, 35B40, 35B45.

1 Introduction

In this paper, we construct large-time asymptotics for the periodic solutions with period $2l$ of a generalized Burgers equation

$$u_t + uu_x + \lambda u = \frac{\delta}{2}u_{xx}, \tag{1}$$

where $\lambda > 0$ and $\delta > 0$ (small) and $\lambda > \frac{\delta\pi^2}{l^2}$. Sachdev et al [1] constructed large-time asymptotics for the periodic solutions of (1) when $\lambda \leq \frac{\delta\pi^2}{l^2}$. This paper together with Sachdev et al [1] gives the large-time asymptotics for the entire λ and δ parametric range.

The motivation for this work comes from the structure of the periodic solutions of the plane Burgers equation. Use of the Cole-Hopf transformation (see Sachdev [2]) gives the solution

$$u(x,t) = \frac{2\delta\pi}{l}\frac{\sum_{n=1}^{n=\infty} e^{-\frac{\delta n^2\pi^2 t}{2l^2}} n\sin\left(\frac{n\pi x}{l}\right)}{1 + 2\sum_{n=1}^{n=\infty} e^{-\frac{\delta n^2\pi^2 t}{2l^2}} \cos\left(\frac{n\pi x}{l}\right)} \tag{2}$$

for the plane Burgers equation ((1) with $\lambda = 0$) subject to the initial, boundary conditions

$$u(x,0) = A_1\sin\left(\frac{\pi x}{l}\right), \quad A_1 \text{ is an arbitrary constant,}$$
$$u(0,t) = u(l,t) = 0, \quad t > 0.$$

Expanding (2) as a series of descending exponenentials, we arrive at

$$u = \frac{2\delta\pi}{l}\left[e^{-\frac{\delta\pi^2 t}{2l^2}}\sin\left(\frac{\pi x}{l}\right) - e^{-\frac{2\delta\pi^2 t}{2l^2}}\sin\left(\frac{2\pi x}{l}\right) + e^{-\frac{3\delta\pi^2 t}{2l^2}}\left(\sin\left(\frac{3\pi x}{l}\right) + \sin\left(\frac{\pi x}{l}\right)\right) + \ldots\right] \text{, as } t \to \infty.$$

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We attempt to construct an asymptotic solution for (1) for $\lambda > \frac{\delta\pi^2}{l^2}$ with similar structure as above. We show numerically that the asymptotic solution of (1) so constructed gives the large time behavior of solutions of (1) for different initial conditions.

We construct the asymptotics for $\lambda > \frac{\delta\pi^2}{l^2}$ in such a manner that they agree with those of Sachdev et al [1] in the limit $\lambda \rightarrow \frac{\delta\pi^2}{l^2}$. For constructing large-time asymptotics, Sachdev et al [1] used a balancing argument (see Bender and Orszag [3], Grundy et al [4] and Sachdev [5]), wherein the solution

$$u(x,t) = A\mathrm{e}^{-kt}\sin(\frac{\pi x}{l}), k = \lambda + \frac{\delta\pi^2}{2l^2} \tag{3}$$

of the linearized form

$$u_t + \lambda u = \frac{\delta}{2}u_{xx} \tag{4}$$

of (1) is improvised for large values of t; however they did not impose the sinusoidal initial conditions while constructing the large-time asymptotics.

Parker [6] treated a generalization of (1) using a " Cole - Hopf " like transformation and linearized the resulting equation to a heat equation with damping. This equation was solved subject to the given initial and boundary conditions to arrive at an approximate solution.

Using a balancing argument, Sachdev et al [7] constructed large time asymptotics for the periodic solutions of the modified Burgers equations

$$u_t + u^n u_x = \frac{\delta}{2}u_{xx} \tag{5}$$

and validated their asymptotics numerically for $n = 2$ and $n = 3$. An interesting feature of these solutions is that the "zero" of the solution moves as it evolves under (5) when $n = 2$. An expression for this zero containing an undetermined constant was also found.

In the following section, we give large-time asymptotics for the periodic solutions of (1) when $\lambda > \frac{\delta\pi^2}{l^2}$. In section 3, we give comparison of analytic solution constructed, the numerical solution of the generalized Burgers equation (1) and the solution (3) of the linearized generalized Burgers equation (4).

2 Large-time asymptotics for the periodic solutions of (1)

In this section, we construct a large time asymptotic solution of (1), when $\lambda > 0$ and $\delta > 0$ (small). We expect that the solution of (1) behaves like the solution of the linearized equation

$$u_t + \lambda u = \frac{\delta}{2}u_{xx}, \tag{6}$$

satisfying

$$u(x,0) = A_1 \sin\left(\frac{\pi x}{l}\right), \tag{7}$$

$$u(0,t) = u(l,t) = 0, \qquad t > 0, \tag{8}$$

$$u(x,t) = u(x+2l,t), \qquad t > 0, \tag{9}$$

as $t \longrightarrow \infty$; here A_1 and l are positive constants. As the solution $u(x,t)$ of (1) is anti-symmetric and periodic (with period $2l$) in x, it suffices to find the solution on the interval $0 \leq x \leq l$. Sachdev et al [1] constructed the following (formal) asymptotic periodic solutions of (1) for large t when $\lambda \leq \frac{\delta\pi^2}{l^2}$.
For $\lambda < \frac{\delta\pi^2}{l^2}$ and λ sufficiently small, as $t \longrightarrow \infty$,

$$\begin{aligned} u(x,t) = {} & \mathrm{e}^{-kt}\left[B_1\sin\left(\frac{\pi x}{l}\right)\right] + \mathrm{e}^{-2kt}\left[B_2\sin\left(\frac{2\pi x}{l}\right)\right] \\ & + \mathrm{e}^{-3kt}\left[B_3\sin\left(\frac{\pi x}{l}\right) + B_4\sin\left(\frac{3\pi x}{l}\right)\right] + \cdots, \end{aligned} \tag{10}$$

where B_1 is the old age constant, $B_2 = \frac{B_1^2\pi}{2l(3\lambda-2k)}$, $B_3 = \frac{-B_1B_2\pi}{4kl}$, $B_4 = \frac{3B_1B_2\pi}{4l(4\lambda-3k)}$ and $k = \lambda + \frac{\delta\pi^2}{2l^2}$.
For $\lambda = \frac{\delta\pi^2}{l^2}$, as $t \longrightarrow \infty$,

$$\begin{aligned} u(x,t) &= A\mathrm{e}^{-kt}\sin\left(\frac{\pi x}{l}\right) + \mathrm{e}^{-2kt}\left[\frac{-A^2\pi}{2l}t\sin\left(\frac{2\pi x}{l}\right)\right] \\ &+\mathrm{e}^{-3kt}\left[\frac{A^3t}{12\delta}\sin\left(\frac{\pi x}{l}\right) + \frac{3A^3t}{4\delta}\sin\left(\frac{3\pi x}{l}\right)\right] + \cdots, \end{aligned} \tag{11}$$

where A is the old age constant. Motivated by the forms of solutions (10), (11) and analysis of Sachdev et al [1], we seek an asymptotic solution of (1) in the form

$$u(x,t) = \mathrm{e}^{-kt}f_1(x,t) + \mathrm{e}^{-2kt}f_2(x,t) + \mathrm{e}^{-3kt}f_3(x,t) + \cdots + \mathrm{e}^{-nkt}f_n(x,t) + \cdots \tag{12}$$

as $t \longrightarrow \infty$ (when $\lambda > \frac{\delta\pi^2}{l^2}$). As $t \longrightarrow \infty$, we expect that our (asymptotic) solution asymptotes to (3), solution of the linear equation (4) satisfying (7)-(9). Substitution of (12) into (1) and comparing the coefficients of e^{-kt}, e^{-2kt}, $\mathrm{e}^{-3kt}\cdots$ on both sides, we arrive at an infinite system of PDEs for $f_i, i = 1,2,3,\cdots$

$$f_{1,t} + (\lambda-k)f_1 - \frac{\delta}{2}f_{1,xx} = 0, \tag{13}$$

$$f_{2,t} + (\lambda-2k)f_2 - \frac{\delta}{2}f_{2,xx} = -f_1f_{1,x}, \tag{14}$$

$$f_{3,t} + (\lambda-3k)f_3 - \frac{\delta}{2}f_{3,xx} = -[f_1f_{2,x} + f_2f_{1,x}], ... \tag{15}$$

$$f_{n,t} + (\lambda-nk)f_n - \frac{\delta}{2}f_{n,xx} = -[f_1f_{n-1,x} + f_2f_{n-2,x} + \cdots + f_{n-1}f_{1,x}], ... \tag{16}$$

Motivated by the solution (3) of the linearized equation, we get

$$f_1(x,t) = A\sin\left(\frac{\pi x}{l}\right), \tag{17}$$

where A is the old age constant, which depends on the initial condition. Substituting the expression for f_1 in (14) , we obtain

$$f_{2,t} + (\lambda-2k)f_2 - \frac{\delta}{2}f_{2,xx} = -\frac{A^2\pi}{2l}\sin\left(\frac{2\pi x}{l}\right). \tag{18}$$

Multiplying (18) by $\mathrm{e}^{(\lambda-2k)t}$, we have

$$\left(f_2\mathrm{e}^{(\lambda-2k)t}\right)_t - \frac{\delta}{2}\left(f_2\mathrm{e}^{(\lambda-2k)t}\right)_{xx} = \frac{-A^2\pi}{2l}\mathrm{e}^{(\lambda-2k)t}\sin\left(\frac{2\pi x}{l}\right). \tag{19}$$

The RHS of above equation suggests the following form for $f_2\mathrm{e}^{(\lambda-2k)t}$.

$$f_2\mathrm{e}^{(\lambda-2k)t} = g(t)\sin\left(\frac{2\pi x}{l}\right). \tag{20}$$

Inserting (20) in (19) and simplifying, we get

$$g'(t) + 2\frac{\delta\pi^2}{l^2}g(t) = \frac{-A^2\pi}{2l}\mathrm{e}^{(\lambda-2k)t}. \tag{21}$$

Solving for g, we have

$$g(t) = \frac{-A^2\pi}{2l(2k-3\lambda)}\mathrm{e}^{(\lambda-2k)t} + c\mathrm{e}^{-4(k-\lambda)t}. \tag{22}$$

The arbitrary constant c appearing in (22) is to be found by assuming that $f_2\mathrm{e}^{-2kt}$ agrees with the second term $(-\frac{A^2\pi}{2l}t)\mathrm{e}^{-2kt}\sin(\frac{2\pi x}{l})$ in (11) as $\lambda \longrightarrow \frac{\delta\pi^2}{l^2}$. Thus, $c = \frac{-A^2\pi}{2l}\frac{1}{\lambda-2k_1}$ and we have

$$f_2(x,t) = B(1-\mathrm{e}^{(\lambda-2k_1)t})\sin\left(\frac{2\pi x}{l}\right), \tag{23}$$

where $k_1 = \frac{\delta\pi^2}{2l^2}$ and $B = \frac{A^2\pi}{2l}\frac{1}{\lambda-2k_1}$. Observe that first term of $\mathrm{e}^{-2\mathrm{kt}}\ f_2$ with f_2 as in (23) is dominant when $\lambda < \frac{\delta\pi^2}{l^2}$. Hence to this order, our asymptotics also contain that of the case of $\lambda < \frac{\delta\pi^2}{l^2}$. Equations (15), (17) and (23) give

$$f_{3,t} + (\lambda - 3k)f_3 - \frac{\delta}{2}f_{3,xx} = \left[1 - \mathrm{e}^{(\lambda-2k_1)t}\right]\left[-3D_1\sin\left(\frac{3\pi x}{l}\right) + D_1\sin\left(\frac{\pi x}{l}\right)\right], \tag{24}$$

where $D_1 = \frac{AB\pi}{2l}$. Motivated by the form of RHS of (24), we write

$$f_3(x,t) = g_1(t)\sin\left(\frac{\pi x}{l}\right) + g_2(t)\sin\left(\frac{3\pi x}{l}\right). \tag{25}$$

Substituting the above expression for f_3 in (24), we get

$$g_1'(t) - 2(\lambda + k_1)g_1(t) = D_1\left[1 - \mathrm{e}^{(\lambda-2k_1)t}\right], \tag{26}$$

$$g_2'(t) + 2(3k_1 - \lambda)g_2(t) = -3D_1[1 - \mathrm{e}^{(\lambda-2k_1)t}]. \tag{27}$$

Solving these first order ODEs, we arrive at

$$g_1(t) = c_1\mathrm{e}^{2(\lambda+k_1)t} - \frac{D_1}{2(\lambda+k_1)} + \frac{D_1}{4k_1+\lambda}\mathrm{e}^{(\lambda-2k_1)t} \tag{28}$$

$$g_2(t) = \begin{cases} -3D_1(t+c_3) + \frac{3D_1\mathrm{e}^{k_1t}}{k_1}, & \text{if } \lambda = 3k_1, \\ \frac{3D_1}{2k_1} + 3D_1(t+c_4)\mathrm{e}^{2k_1t}, & \text{if } \lambda = 4k_1, \\ 3D_1(\frac{-1}{6k_1-2\lambda} + \frac{\mathrm{e}^{(\lambda-2k_1)t}}{4k_1-\lambda}) + c_2\mathrm{e}^{-2(3k_1-\lambda)t}, & \text{otherwise.} \end{cases} \tag{29}$$

It is seen that the arbitrary constants c_1 and c_2 are zero when the most dominant terms of the limit (as $\lambda \to \frac{\delta\pi^2}{l^2}$) of the third term of (12) agree with that of (11). For the cases $\lambda = 3k_1, 4k_1$, we take the most dominant terms (as $t \longrightarrow \infty$) in each bracketed expression. Hence,

$$g_1(t) = -\frac{D_1}{2(\lambda+k_1)} + \frac{D_1}{4k_1+\lambda}\mathrm{e}^{(\lambda-2k_1)t}, \tag{30}$$

$$g_2(t) = \begin{cases} -3D_1t + \frac{3D_1\mathrm{e}^{k_1t}}{k_1}, & \text{if } \lambda = 3k_1, \\ \frac{3D_1}{2k_1} + 3D_1t\mathrm{e}^{2k_1t}, & \text{if } \lambda = 4k_1, \\ 3D_1(\frac{-1}{6k_1-2\lambda} + \frac{\mathrm{e}^{(\lambda-2k_1)t}}{4k_1-\lambda}), & \text{otherwise.} \end{cases} \tag{31}$$

Thus, for $\lambda > \frac{\delta\pi^2}{l^2}$,

$$\begin{aligned} u(x,t) = \mathrm{e}^{-kt}\left[A\sin\left(\frac{\pi x}{l}\right)\right] + \mathrm{e}^{-2kt}\left[B(1-\mathrm{e}^{(\lambda-2k_1)t})\sin\left(\frac{2\pi x}{l}\right)\right] + \\ \mathrm{e}^{-3kt}\left[g_1(t)\sin\left(\frac{\pi x}{l}\right) + g_2(t)\sin\left(\frac{3\pi x}{l}\right)\right] + \cdots \end{aligned} \tag{32}$$

as $t \mapsto \infty$, where A is the old age constant, $k = \lambda + k_1$, $k_1 = \frac{\delta\pi^2}{2l^2}$, $B = \frac{A^2\pi}{2l}\frac{1}{\lambda-2k_1}$, $g_1(t)$ and $g_2(t)$ are given in equations (30) and (31). Observe that the coefficient of e^{-2kt} in (32) has the dominant behavior $B\sin\left(\frac{2\pi x}{l}\right)$ when $\lambda < \frac{\delta\pi^2}{l^2}$ and has the dominant behavior $-B\mathrm{e}^{(\lambda-2k_1)t}\sin\left(\frac{2\pi x}{l}\right)$ when $\lambda > \frac{\delta\pi^2}{l^2}$ as $t \mapsto \infty$. Further, picking up the dominant behaviors (as $t \mapsto \infty$) of the coefficients of $\mathrm{e}^{-2kt},\ \mathrm{e}^{-3kt}$ in (32) gives (10) when $\lambda < \frac{\delta\pi^2}{l^2}$.

One may attempt a Fourier sine series of the form

$$u(x,t) = \Sigma_{n=1}^{\infty}A_n(t)\sin(\frac{n\pi x}{l}). \tag{33}$$

A substitution of this into (1) gives a system of ordinary differential equations for the functions $A_1(t),\ A_2(t),\ A_3(t)\ldots.$ The difficulty here is that every equation consists of all unknowns $A_1(t),\ A_2(t),\ A_3(t)\ldots.$ Observe the complication of solving this. However, with our form (12), we can solve one by one i.e., $f_1,\ f_2,\ f_3,\ldots$ respectively. It may be pointed out that truncating the series (33) and solving the equations numerically, is the method of finitely reproducing nonlinearities (see Bazley [8], Mittal and Singhal [9]).

3 Numerical study

In this section, we solve numerically the generalized Burgers equation (1) with $\lambda = 0.2,\ \delta = 0.08$ subject to each of the initial conditions

$$u(x,0) = \begin{cases} \frac{2x}{\pi}, & \text{if } 0 \le x \le \frac{\pi}{2}, \\ 2(1-\frac{x}{\pi}), & \text{if } \frac{\pi}{2} \le x \le \pi, \end{cases} \tag{34}$$

$$u(x,0) = \begin{cases} \frac{4x}{\pi}, & \text{if } 0 \le x \le \frac{\pi}{4}, \\ \frac{4}{3}(1-\frac{x}{\pi}), & \text{if } \frac{\pi}{4} \le x \le \frac{\pi}{2}, \\ \frac{4x}{3\pi}, & \text{if } \frac{\pi}{2} \le x \le \frac{3\pi}{4}, \\ 4(1-\frac{x}{\pi}), & \text{if } \frac{3\pi}{4} \le x \le \pi, \end{cases} \tag{35}$$

$$u(x,0) = \sin x, \tag{36}$$

with the boundary conditions

$$u(0,t) = u(\pi,t) = 0, \quad t > 0. \tag{37}$$

We chose $\triangle t = 0.0001,\ \triangle x = 0.0062$ and used Dawson [10]'s scheme for finding the numerical solution of plane Burgers equation with the initial-boundary conditions. Comparing with the exact solution (2) of plane Burgers equation, we see an accuracy of four decimal places at $t \approx 55$ when we take the initial profiles (34), (35) and (36).

As the initial profile evolves in time under the generalized Burgers equation (1), for sufficiently large time the solution behaves like a solution (3) of the linearized equation (4). We computed $\frac{u_{max}}{\sin(x_{max})}\mathrm{e}^{(\lambda+\delta/2)\mathrm{t}}$, where x_{max} is the value of x when u attains its maximum value u_{max} on $(0,\pi)$, at different times and chose the converged value for A. Convergence of A means that the nonlinear terms are negligible, i.e., we reached the linear regime. Table 1 gives the old age constants for different times and different initial profiles. Table 2 compares the numerical solution (u_{num}) of (1), subject to the initial-boundary conditions (34), (37) and the analytical solution (u_{ana}) computed using (32). Similarly, Table 3 corresponds to the solution of (1), (35), (37) and Table 4 to (1), (36), (37). The agreement between numerical and analytical solutions is very good.

Table 1: $\frac{u_{max}}{\sin(x_{max})}\mathrm{e}^{(\lambda+\delta/2)\mathrm{t}}$, where x_{max} is the value of x when u attains its maximum value u_{max} on $(0,\pi)$, is evaluated for different times for different initial profiles.

t	old age constant corresponding to the solution of (1), (34), (37)	old age constant corresponding to the solution of (1), (35), (37)	old age constant corresponding to the solution of (1), (36), (37)
5	1.2923	1.6052	1.5124
10	0.6665	0.8036	0.7496
15	0.5143	0.6062	0.5709
20	0.4639	0.5406	0.5118
25	0.4460	0.5175	0.4904
30	0.4398	0.5094	0.4830
35	0.4376	0.5065	0.4805
40	0.4369	0.5057	0.4797
45	0.4367	0.5054	0.4794
50	0.4367	0.5053	0.4793
55	0.4366	0.5053	0.4793
60	0.4366	0.5053	0.4793
65	0.4367	0.5053	0.4793
70	0.4367	0.5053	0.4793
75	0.4367	0.5053	0.4793
80	0.4367	0.5053	0.4793

Table 2: Numerical solution, u_{num}, of (1) subject to (34), (37) and analytical solution (32) at $t = 20$ with $\lambda = 0.2$, $\delta = 0.08$, old age constant$= 0.4367$ and $l = \pi$.

x	u_{num}	u_{ana}
0.1855	0.0005	0.0004
0.3742	0.0010	0.0009
0.5629	0.0015	0.0014
0.7516	0.0020	0.0019
0.9403	0.0025	0.0024
1.1290	0.0029	0.0028
1.3176	0.0032	0.0033
1.5063	0.0035	0.0036
1.6950	0.0036	0.0038
1.8837	0.0037	0.0038
2.0724	0.0036	0.0036
2.2611	0.0033	0.0033
2.4498	0.0028	0.0028
2.6384	0.0022	0.0021
2.8271	0.0015	0.0014
3.0158	0.0006	0.0006

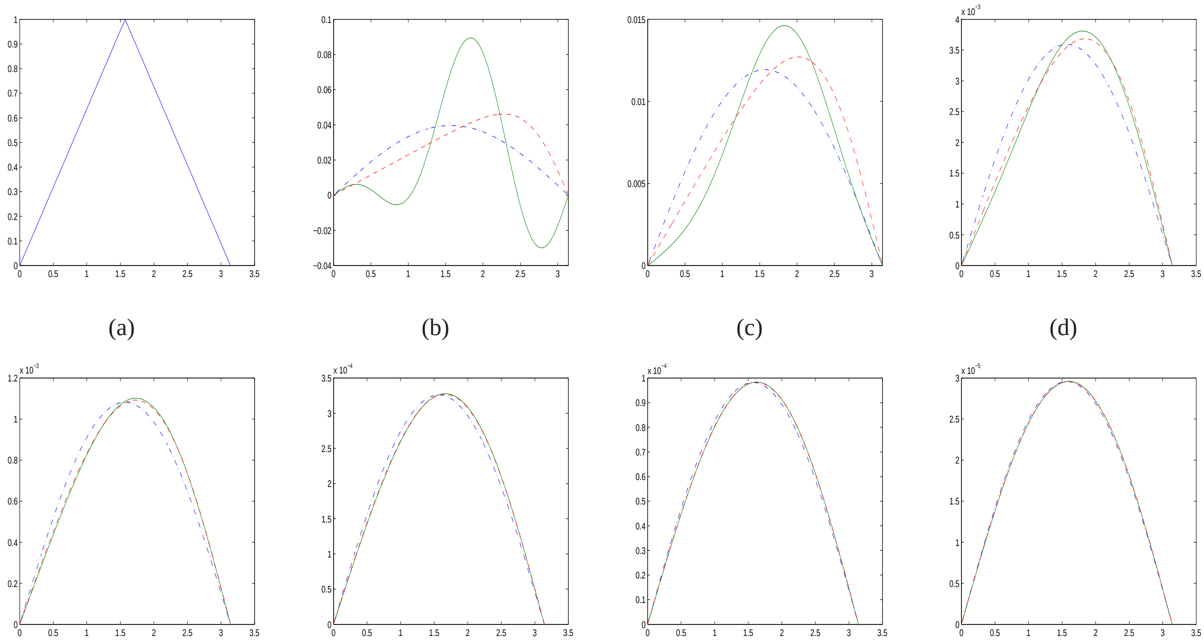
Table 3: Numerical solution, u_{num}, of (1) subject to (35), (37) and analytical solution (32) at $t = 20$ with $\lambda = 0.2$, $\delta = 0.08$, old age constant $= 0.5053$ and $l = \pi$.

x	u_{num}	u_{ana}
0.1855	0.0006	0.0005
0.3742	0.0012	0.0009
0.5629	0.0017	0.0015
0.7516	0.0023	0.0020
0.9403	0.0028	0.0026
1.1290	0.0033	0.0032
1.3176	0.0037	0.0038
1.5063	0.0040	0.0042
1.6950	0.0042	0.0044
1.8837	0.0043	0.0045
2.0724	0.0042	0.0043
2.2611	0.0038	0.0039
2.4498	0.0033	0.0033
2.6384	0.0026	0.0025
2.8271	0.0017	0.0016
3.0158	0.0007	0.0006

Table 4: Numerical solution, u_{num}, of (1) subject to (36), (37) and analytical solution (32) at $t = 20$ with $\lambda = 0.2$, $\delta = 0.08$, old age constant $= 0.4793$ and $l = \pi$.

x	u_{num}	u_{ana}
0.1855	0.0006	0.0004
0.3742	0.0011	0.0009
0.5629	0.0017	0.0014
0.7516	0.0022	0.0020
0.9403	0.0027	0.0025
1.1290	0.0031	0.0031
1.3176	0.0035	0.0036
1.5063	0.0038	0.0040
1.6950	0.0040	0.0042
1.8837	0.0040	0.0042
2.0724	0.0039	0.0040
2.2611	0.0036	0.0036
2.4498	0.0031	0.0031
2.6384	0.0024	0.0023
2.8271	0.0016	0.0015
3.0158	0.0007	0.0006

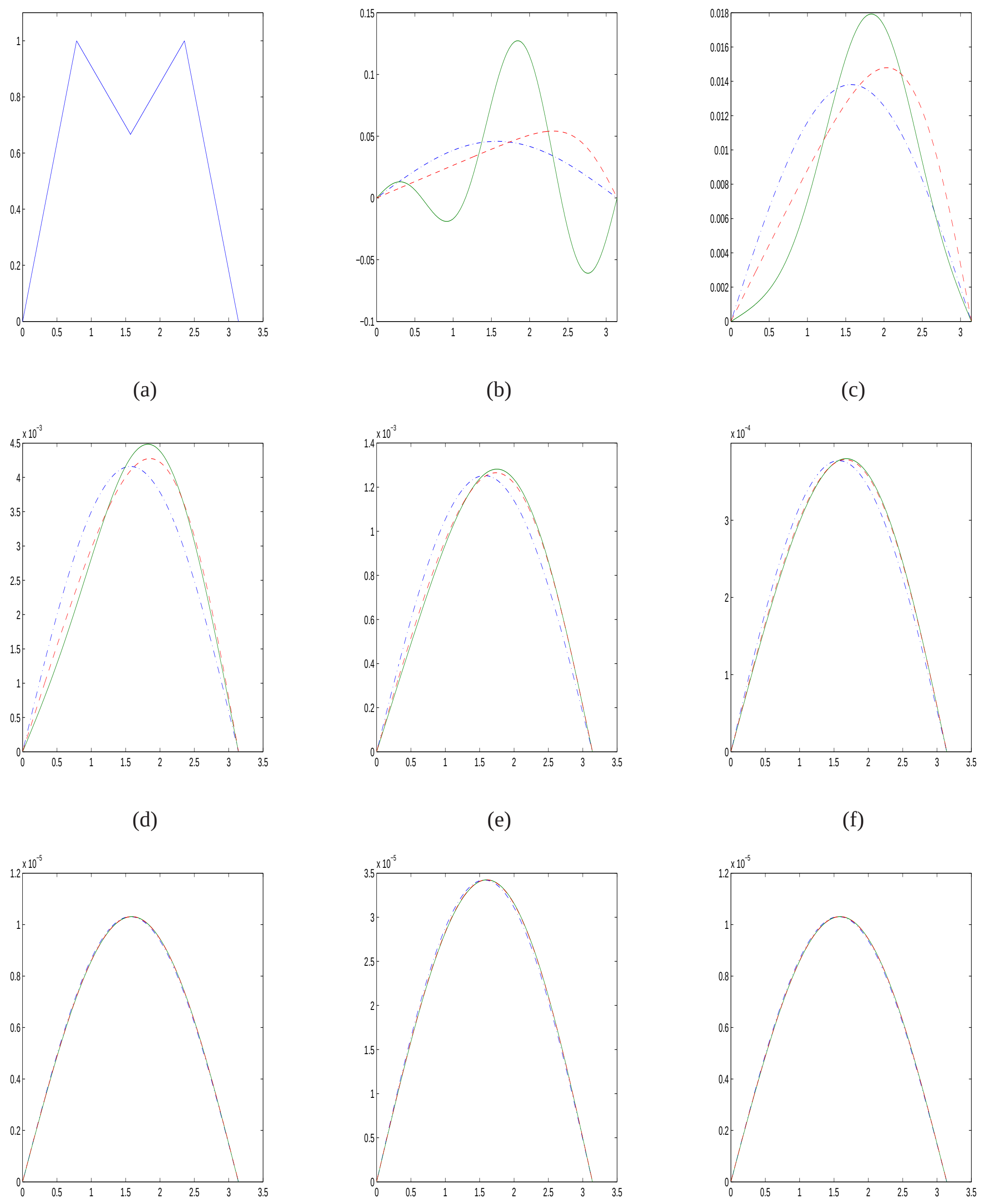
The following figures show the analytical solution (32), numerical solution of generalized Burgers equation (1), and the solution (3) of linearized form (4) at various times. We observe that our analytical solution agrees well much before the linear regime.



(a) (b) (c) (d)

(e) (f) (g) (h)

Figure 1: Numerical (dashed), analytical (solid) and linear (dashdotted) solutions of (1) subject to (34), (37) with $\lambda = 0.2,\ \delta = 0.08$ at times $t = 0, 10, 15, 20, 25, 30, 35, 40$ respectively.



(a) (b) (c) (d) (e) (f) (g) (h) (i)

Figure 2: Numerical (dashed), analytical (solid) and linear (dashdotted) solutions of (1) subject to (35), (37) with $\lambda = 0.2,\ \delta = 0.08$ at times $t = 0, 10, 15, 20, 25, 30, 55, 40, 45$ respectively.

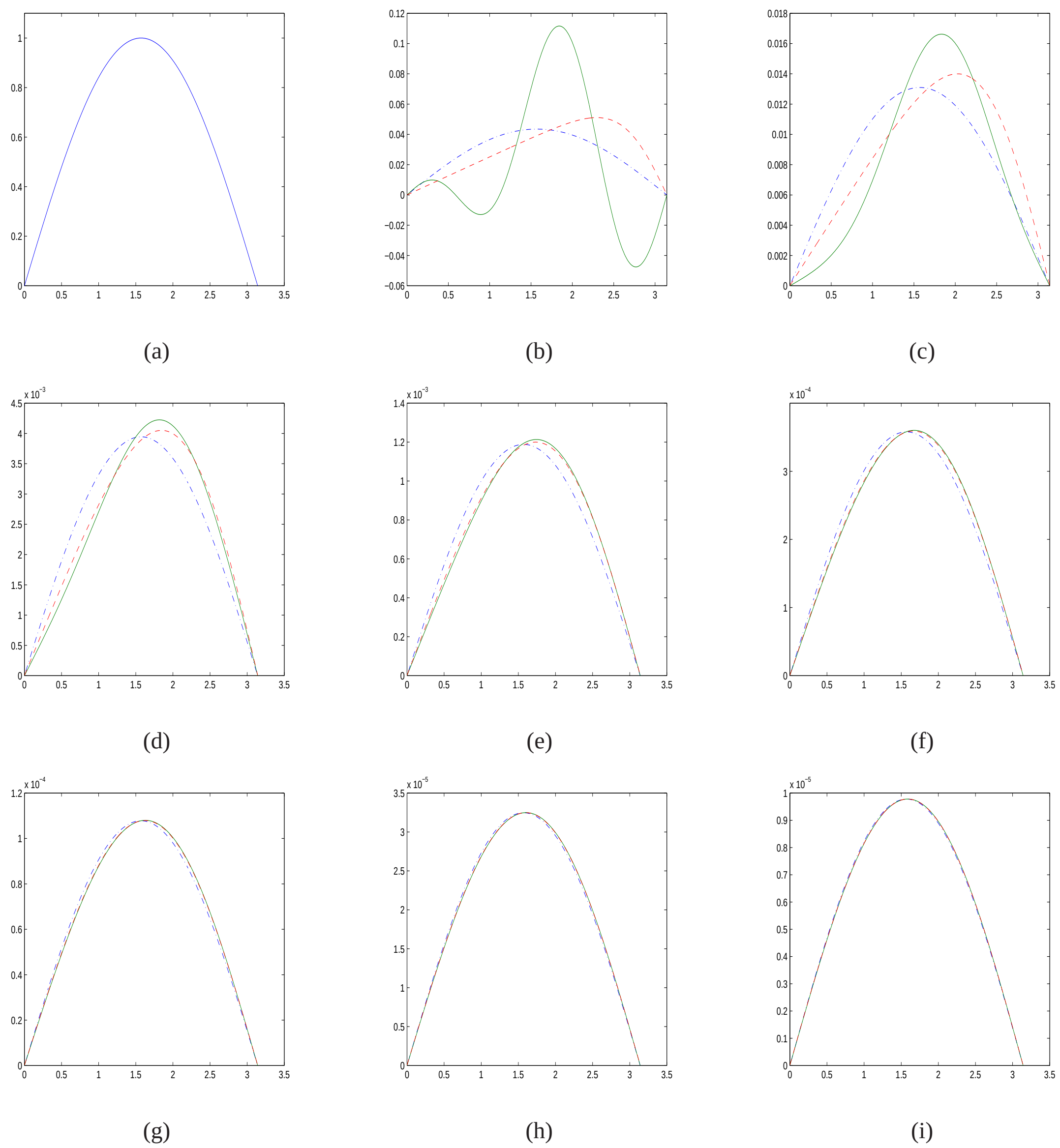


Figure 3: Numerical (dashed), analytical (solid) and linear (dashdotted) solutions of (1) subject to (36), (37) with $\lambda = 0.2,\ \delta = 0.08$ at times $t = 0, 10, 15, 20, 25, 30, 35, 40$ respectively.

4 Conclusions

In this article, we constructed periodic solutions for a generalized burgers equations with damping. These asymptotic solutions are compared with numerical solutions obtained by Dawson [10]'s scheme. The agreement between these two solutions is found to be very good.

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