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Fisher Information Measure Analysis of Earth's Apparent Resistivity

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Abstract: The observation of the natural surface variations of the electromagnetic field in a broad range of periods, one of the most important scientific challenges in the studies devoted to the geophysical monitoring, is performed by the magnetotelluric method (MT). The apparent resistivity is proportional to the modulus of the surface impedance, which is the tensorial relation between the horizontal components of the electric with the magnetic field. The subsurface resistivity distribution is obtained by inverting the apparent resistivity. The method of the Fisher Information Measure (FIM) has been applied to investigate the dynamics of the Earth's apparent resistivity during July 2003 in the area of Val d'Agri (southern Italy). The FIM is a quantitative methodology to characterize the complexity of a time series, identifying more or less ordered states of the underlying system. Our results show that the Earth's apparent resistivity is featured by complex behaviour, with periods characterized by less ordered dynamics and other by more ordered states. In particular, due to the relationship between periods and sounding depth, the Earth's apparent resistivity tends to behave more irregularly, indicating only random fluctuation, with the increase of the depth. Instead a relative more order states at higher periods suggests slight resistivity variation in the shallow layer.

Keywords: Fisher information measure; magnetotelluric method; Earth's apparent resistivity

1 Instroduction

The MT method is a geophysical technique used to image the subsurface electrical resistivity [1] by using as a source the Earth's natural varying electromagnetic field, whose range of periods is very broad. Under the assumption that the external sources are spatially uniform (plane wave) and neglecting the displacement current, the physical problem may be treated as one in pure diffusion, in which the skin depth is$\delta = 503(\rho T)^{1/2}$)(in meters), where ρ is the Earths resistivity and T the period [2]. Therefore, the investigation depth increases with period and can reach several tens of kilometers for longer periods, depending on the resistivity of the rocks. By means of the simultaneous measurements of the horizontal components of the electric and magnetic field, the frequency dependent impedance (called transfer function) of the subsoil can be estimated. Typical duration of a classical magnetotelluric measurements until period of 500 s not exceeds one day. By inverting the impedance tensor, which linearly relates the two horizontal electric components to the horizontal magnetic ones, the electrical resistivity distribution of the Earths interior can be defined.

The Earth impedance is temporal invariant until significant changes in the resistivity distribution underground do not occur: it means that the transfer function should be temporal invariant within its error bound in absence of resistivity variation and/or local electromagnetic noise sources.

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In order to study the long-term stability of MT tensor impedance estimates [3] both respect to the noise than to other possible causes of resistivity changes in depth, a monitoring MT data set provides a rare opportunity. And, in this framework, it is important to characterize the dynamics of apparent resistivity data by means of methodologies, which are able to deeply describe the temporal dynamics in order to understand the underlying geophysical mechanisms.

In this study we analysed the apparent resistivity data in the area of Val dAgri (southern Italy). We performed a magnetotelluric sounding, consisting of observing the natural electromagnetic signals along two orthogonal directions in a continuous monitoring for about one month during 2003, in order to gain insigth into the dynamical characterization of the Earths apparent resistivity.

2 Fisher information measure

The method used to analyse the time dynamics of the Earths apparent resistivity was the Fisher Information Measure (FIM), introduced by Fisher in 1925 in the context of statistical estimation [4]. In a seminal paper Frieden has shown FIM to be a versatile tool to describe the evolution laws of physical systems [5]. FIM permits to accurately describe the behavior of dynamic systems, and to characterize the complex signals generated by these systems [6]. This approach has been used by Martin et al. to characterize the dynamics of EEG signals [7]. Martin et al. have shown the informative content of FIM in detecting significant changes in the behavior of nonlinear dynamical systems [8], characterizing, thus, FIM as an important quantity involved in many aspects of the theoretical and observational description of natural phenomena. Following Ref. [8], high FIM values indicate the incoming of regular dynamics and more ordered state, while low FIM values denote the presence of chaos-like behavior in the system and less ordered state [7]. Therefore, FIM can be used as a good indicator of the complexity of a dynamical system.

Let us introduce the relevant Fisher-associated quantities. Let $f \equiv q^2$ be a probability density in $\Re^N (N > 1)$. Fishers quantity of information associated to f (or to the probability amplitude q) is defined as the (possibly infinite) non-negative number I

$$I(f) = \int_{\Re^N} dx \frac{|\nabla f|}{f} \tag{1}$$

or in terms of amplitudes

$$I(q) = \int_{\Re^N} dx \nabla q \cdot \nabla q \tag{2}$$

This formula defines a convex, isotropic functional I, which was first used by Fisher [4] for statistical purposes, and plays a fundamental role in information theory. Linnik used this functional to prove the central limit theorem by an information-theoretic approach [9]. It is clear that from Eq. 2 the integrand, being the scalar product of two vectors, is independent of the reference frame [8].

Let us focus the attention on one-dimensional case. Let us consider a random variable X whose probability density function is denoted as $f_X(x)$. Its FIM is defined as

$$I_X = \int (\frac{\partial}{\partial x} f_X(x))^2 \frac{dx}{f_X(x)} \tag{3}$$

Eq. 3 involves the calculation of the probability density function (pdf) $f_X(x)$.A rough approximation of the unknown probability density f is given by the histogram. Let us consider the statistical sample by $\{s_i\}_{i=1}^N$ where N is the length of the sample. We consider a finite interval [a, b] such that $a \leq min_i\{s_i\}$ and $b \geq max_i\{s_i\}$.

Next we divide the interval [a,b] into n nonintersecting subintervals of equal length $h = (b - a)/n$. A histogram is a function $f_{N,n}(x)$, constant on each of the subintervals $[x_k, x_{k+1}), k = 1, 2, , n$, defined as follows

$$f_{N,n}(x) = \frac{\#\{s_i \in [x_k, x_{k+1}) : x \in [x_k, x_{k+1})\}}{nh} \tag{4}$$

where $\#\{...\}$ counts the number of data values falling into the specified intervals of the signal. The best convergence to the searched density function is obtained if the number of subintervals n is proportional to the cube root of the number N of observations [10].

3 Data analysis

A classical MT sounding consists of simultaneous measurements of electric and magnetic fields to obtain, by means the impedance tensor, the apparent resistivity. In the case examined in the present study, the MT continuous monitoring station was equipped by a 24 bit analogical/digital conversion receiver, two induction coils to measure the two orthogonal components of the magnetic field, and two electric dipoles to measure the electric field.

The relationship between electric and magnetic fields as a function of frequency is given by the following tensorial equation [11]:

$$\begin{vmatrix} E_x(\omega) \\ E_y(\omega) \end{vmatrix} = \begin{vmatrix} Z_{xx}(\omega) & Z_{xy}(\omega) \\ Z_{yx}(\omega) & Z_{yy}(\omega) \end{vmatrix} = \begin{vmatrix} H_x(\omega) \\ H_y(\omega) \end{vmatrix} \tag{5}$$

where $\omega$is the angular frequency,(E_x, E_y) and (H_x, H_y) represent respectively the electric and magnetic components in an orthogonal reference and $Z(\omega)$ is a second rank 2x2 MT response tensor connecting them. As a simple linear system, the transfer function $Z(\omega)$ acts as a filter, while the magnetic and electric fields represent the input and output respectively.

The apparent resistivity of the ground is defined by the following equations:

$$\rho_{ij}(\omega) = \frac{1}{\mu_0\omega}|Z_{ij}(\omega)|^2 \tag{6}$$

where μ_0 is the permeability of the vacuum, Z_{ij} are the complex components of the tensor defined in Eq. 5, with $i, j = x$ or y

In this study the apparent resistivity, obtained by a continuous magnetotelluric (MT) monitoring from July 7 to 29, 2003, was analyzed. The sampling rate of data recording was set to 6.25Hz. Because of noise, the impedance tensor was statistically estimated. The whole series was divided into M short time windows (1 hour in length) and Fourier transformed. The Fourier coefficients for L periods in a band centred around T_i (i=1 to 25, in the present study) were used for the estimate of the transfer function $Z(\omega)$, for a total of K=ML complex data. The apparent resistivity curves ρ_{xy} and ρ_{yx} , related respectively to the off-diagonal components of impedance tensor Z_{xy} and Z_{yx}, were estimated using the procedure and the robust Transfer Function Estimation Program for data reduction described in [12]. In order to investigate the variability of the apparent resistivity in time, we constructed for each period Ti the time series of the hourly apparent resistivity values. With an observation window of 1 hour and in order to have a sufficient statistical accuracy of the estimate of the resistivity, the range of the investigated periods was shortened to [0.75s, 65.54s]. Therefore, the time series of the hourly apparent resistivity values are 20 and shown in Figs. 1 and 2. The total length of each time series is 536 hourly values, but there is a break of 80 hours due to technical problems of the measurement equipment. This gap does not affect the methodological procedure of the FIM, which is substantially based on the probability density function of the resistivity values for each period T.

Fig. 3 shows the FIM values for each resistivity time series (blue diamond). The FIM values decrease with the increase of the period (due to the relationship between period and sounding depth). This indicates that the Earths apparent resistivity shows a regular dynamics in the lower higher estimated periods and behaves more irregularly with the increase of the period indicating prevailing random variations.

A possible explanation of such results is that significant resistivity changes are confined in shallow layers: the diffusive character of the investigated field makes such changes gradually irrelevant as the skin depth (that increases as period increase) becomes significant higher than the depth involved in the resistivity variation. Furthermore superimposed to the decreasing trend, we observed scattering behaviour of FIM values probably due to man-made localized e.m. noise source.

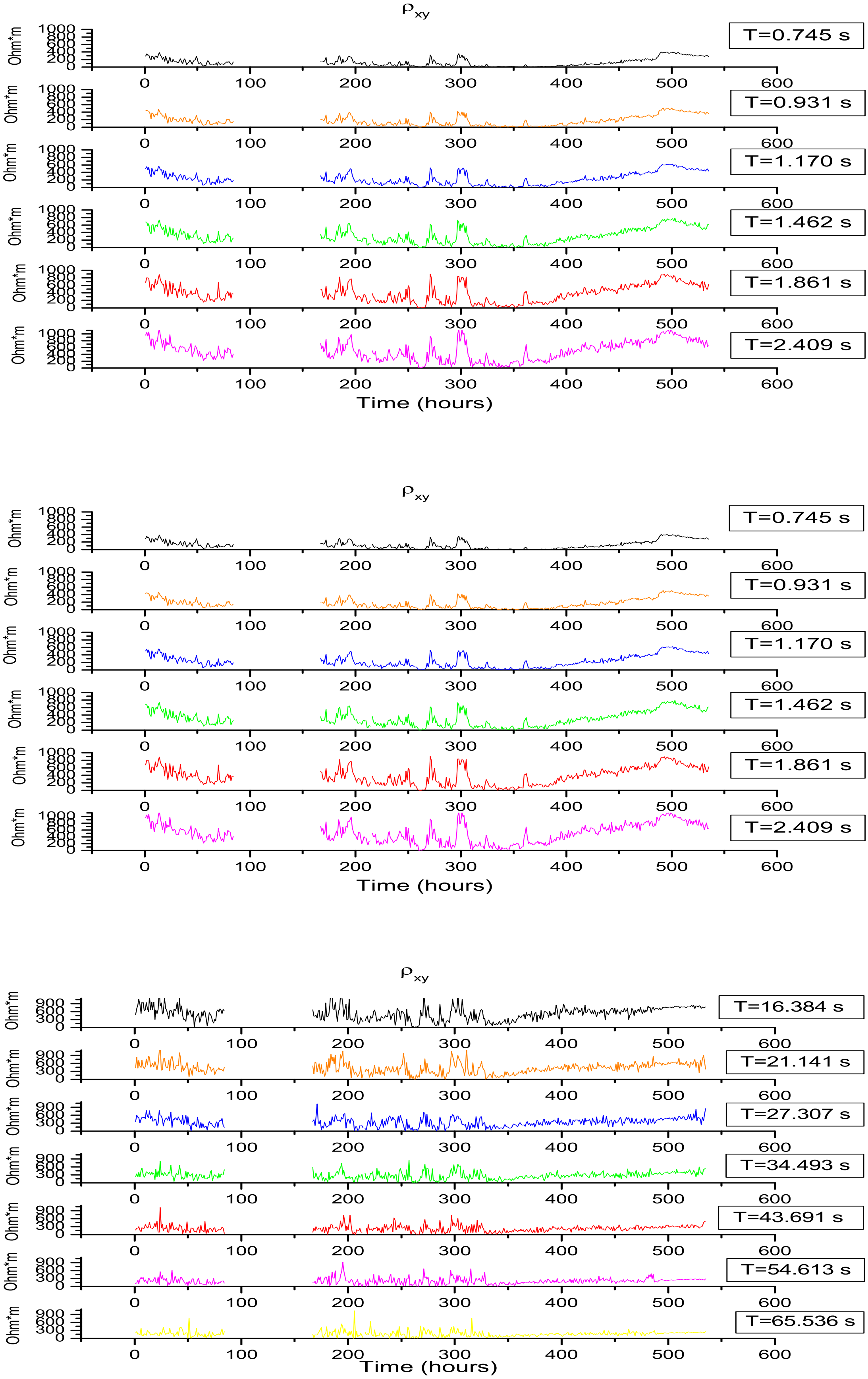

Figure 1: Time series of the hourly apparent resistivity values for each period T in xy direction.

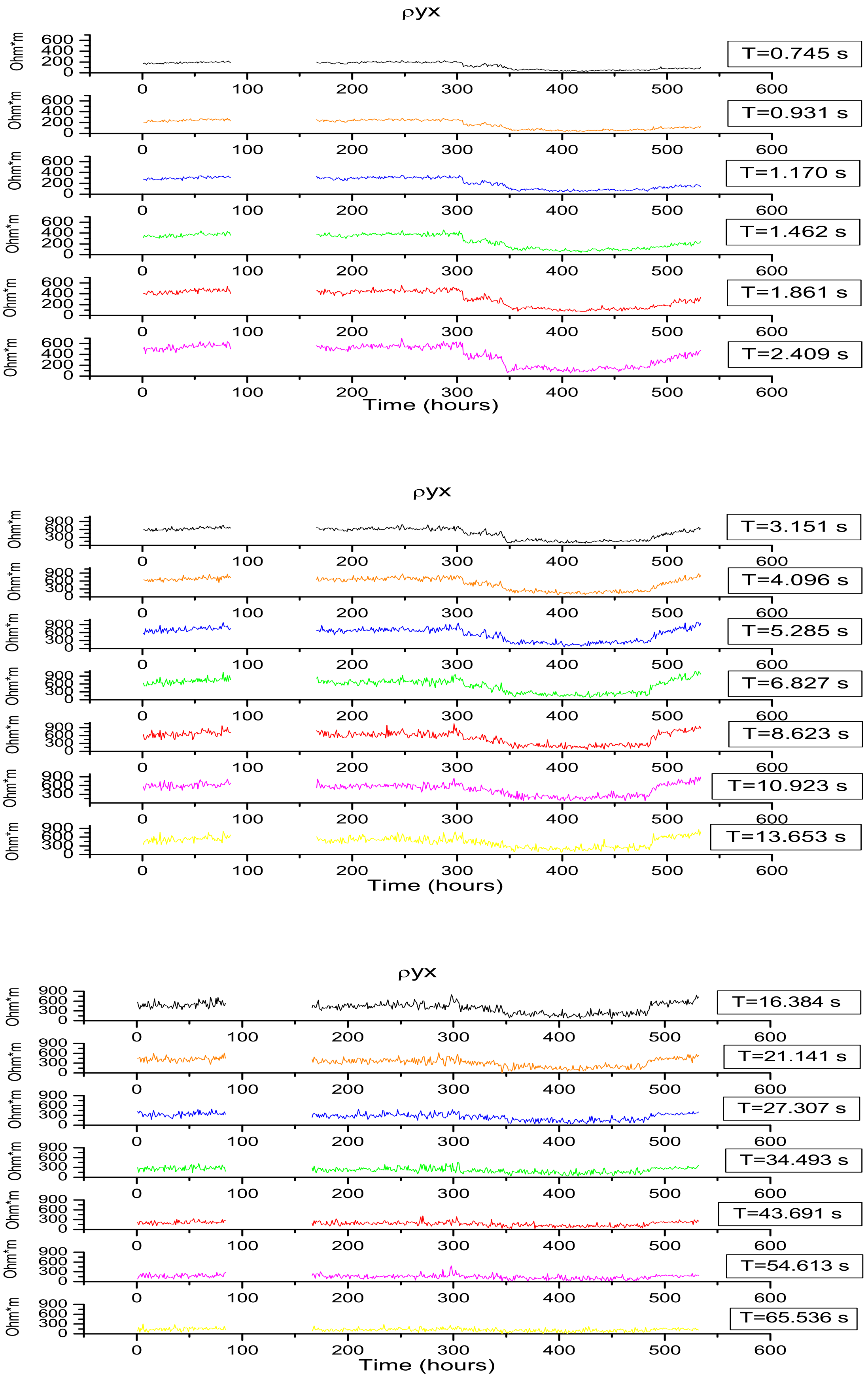

Figure 2: Time series of the hourly apparent resistivity values for each period T in yx direction.

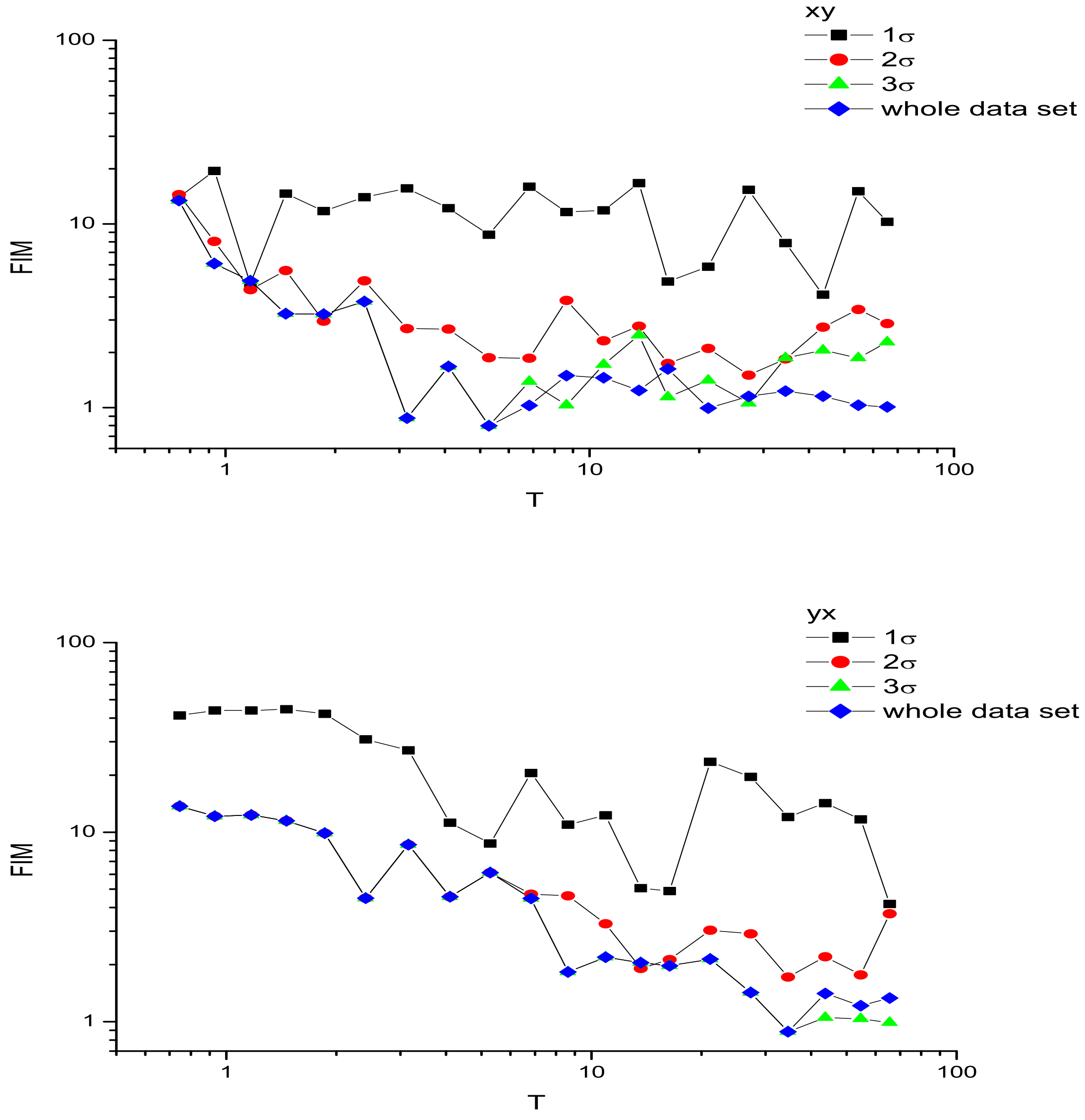

Figure 3: FIM values referred to a) xy and b) yx-component. In x-axis are plotted the periods of each apparent resistivity series.

More inference is recoverable by the analysis of the FIM restricted to different subsets. Therefore, we analysed the FIM truncating the data value at 1σ, 2σ and 3σ, where σ is the standard deviation. New hints are outlined only by the truncation at 1σ. It is evident that the FIM values for ρ_{yx} are higher, at highest periods, than that for ρ_{xy}, and this indicates that the resistivity in yx direction is characterized by more ordered states than resistivity in xy direction. Such occurrence can be due to anisotropic variation in the Earths resistivity distribution.

4 Conclusion

We suggested a new approach in investigating the dynamics of the temporal fluctuations of Earths apparent resistivity (period by period) signals on the basis of the FIM, which acts as a detector of changes in the dynamical behavior of geophysical systems. High FIM values indicate the onset of a more ordered dynamics, while low FIM values denote the presence of less regular behavior in the system.

The particular observed pattern suggests that the apparent resistivity is governed by more irregular dynamics at high sounding depths, for which only random fluctuation are present on the impedance estimates.

A variation of true resistivity at a specified layer implies a maximum variation on the apparent resistivity in that period range, whose skin depth is comparable with the depth of the layer. It means that the observed FIM pattern suggests that resistivity changes are confined in shallow layers.

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