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The Cauchy Problem of the Degasperis-Procesi Equation with the Dispersive Term

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Abstract: This paper is concerned with several aspects of the Cauchy problem and the local well-posedness for the Degasperis-Procesi equation with the dispersive term.

Keywords: Cauchy problem; shallow water equation; dispersive term; well posedness

1 Introduction

Our aim is to study the Cauchy problem and the local well-posedness of the D-P equation with the dispersive term:

$$\partial_t u - \partial^3_{txx} u + \partial^3_{xxx} u - \partial^5_{xxxxx} u + 4u\partial_x u = 3\partial_x u \partial^2_{xx} u + u\partial^3_{xxx} u \tag{1.1}$$

with an initial data $u(x,0) = u_0$.

For the existence and uniqueness of the Cauchy problem of (1.1), [5,6] used the method introduced by Bourgain to deal with the periodic KdV equation and the Schödinger equation; Kenig, Ponce and Vega improved this method [7,8]to deal with the non-periodic case.

2 Preliminaries

As above and henceforth, we denote the convolution by $*$. We write $\widehat{f}$ as the Fourier transform of f. $\psi, \sigma \in C_0^\infty(\mathbb{R})$, supp $\psi \in (-1,1)$ and $\psi = 1$ in $[-1/2, 1/2]$, $\sigma = 1$ in supp ψ and supp $\sigma \in (-1,1), 0 < \sigma < 1$. $\|\cdot\|_s$ stand for the norm in the classical Sobolev space. Let $\xi \in \mathbb{R}$, $\langle \xi \rangle = 1 + |\xi|$, $X^{s,b}$ denotes as the completion of the Schwartz space with respect to the norm:

$$\|f\|^2_{X^{s,b}} = \int_{\mathbb{R}} \int_{\mathbb{R}} \langle \tau - \xi^3 \rangle^{2b} \langle \xi \rangle^{2s} |\widehat{f}(\xi,\tau)|^2 d\xi d\tau \tag{2.1}$$

Rewrite D-P equation

$$\begin{cases} \partial_t u + \partial^3_{xxx} u + p = 0 \\ u(x,0) = u_0 \end{cases} \tag{2.2}$$

where the nonlinear term p is given by:

$$p = \frac{1}{2}\partial_x\left(u^2\right) + \left(1 - \partial^2_{xx}\right)^{-1}\left(\partial_x\left(\frac{3}{2}u^2\right)\right) \tag{2.3}$$

First we consider the corresponding homogeneous linear equation

$$\begin{cases} \partial_t v + \partial^3_{xxx} v = 0, \qquad x, t \in \mathbb{R} \\ v(x,0) = v_0 \end{cases} \tag{2.4}$$

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$W(t)$ denotes the corresponding unitary group; the solution of (2.4) can be represented by:

$$v(x,t) = W(t)\, v_0(x) = S_t * v_0(x),$$

where $S_t(\cdot)$ is defined as the oscillatory integral equation:

$$S_t(x) = c \int e^{ix\xi} e^{it\xi^3} d\xi$$

Then (2.2) is equivalent to the integral equation:

$$u(x,t) = W(t)\, u_0(x) - \int_0^t W(t-t')\, g(x,t')\, dt'$$

We briefly collect the needed results from [2] in order to pursue our goal.

Lemma 2.1 *Let* $\widehat{H}_\rho(\xi,\tau) = \frac{h(\xi,\tau)}{\langle\tau\rangle^\rho}$. *There for* $\rho > 1/2$, $\|H_\rho\|_{L_x^2 L_t^\infty} \le C\|h\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.2 *Let* $\widehat{\mathscr{F}_\rho}(\xi,\tau) = \lambda(\xi)\widehat{F}_\rho(\xi,\tau)$, *where* $\widehat{F}_\rho(\xi,\tau) = \frac{f(\xi,\tau)}{\langle\tau-\xi^3\rangle^\rho}$, $\lambda \in C_0^\infty(\mathbb{R})$, $\lambda(\xi) = 1$ *if* $|\xi| > 1$, $\lambda(\xi) = 0$ *if* $|\xi| > 2$. *Then for* $\rho < 1/2$, $\|F_\rho\|_{L_x^2 L_t^\infty} \le C\|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.3 *If* $|\xi| > 3/8$, $0 < \theta < 1/8$, *then* $\|D_x^\theta \mathscr{F}_\rho\|_{L_x^4 L_t^4} \le C\|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.4 *If* $\rho > 5/12$, *then* $\|F_\rho\|_{L_x^4 L_t^6} \le C\|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.5 *If* $\rho > \theta/12$, *where* $\theta \in [0,1]$, *then* $\|D_x^\theta F_\rho\|_{L_x^{2/(1-\theta)} L_t^2} \le C\|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.6 *If* $\rho > 1/3$, *then* $\|D_x^{1/4} F_\rho\|_{L_x^4 L_t^3} \le C\|f\|_{L_\xi^2 L_\tau^2}$.

Lemma 2.7 $\|\psi(\delta^{-1}t)\, W(t)u_0\|_{X^{s,t}} \le C_0 \delta^{(1-2b)/2}\|u_0\|_{H^s}$.

Lemma 2.8 *If* $1/2 < b < 1$, *then* $\|\psi(\delta^{-1}t)\, h\|_{X^{s,t}} \le C_0\delta^{(1-2b)/2}\|h\|_{X^{s,b}}$.

Lemma 2.9 *If* $1/2 < b < 1$, *then* $\|\psi(\delta^{-1}t) \int_0^t W(t-t')\omega(t')\|_{X^{s,b}} \le C\delta^{(1-2b)/2}\|\omega\|_{X^{s,b-1}}$.

Lemma 2.10 $\|\psi(\delta^{-1}t) \int_0^t W(t-t')\omega(t')\|_{H^s} \le C\delta^{(1-2b)/2}\|\omega\|_{X^{s,b-1}}$.

3 Nonlinear estimates

We now describe an estimate needed to handle the nonlinear term in the KP equation. We first recall from some $[3,4]$ the following statement:

Lemma 3.1 (3, Lemma 3.4) *Let* $q(x,t) = \sigma^2(\delta^{-1}t)\left(1-\partial_{xx}^2\right)^{-1}\partial_x\left((\partial_x u(x,t))^2\right)$. *Suppose*

$$\begin{cases} 1/2 < \varepsilon < 5/8, 1/2 < b < 7/12, b \le \varepsilon, \varepsilon + b \le 9/8, \\ \gamma_0/4b + 1 - 2b = \theta_0, where \gamma_0 = 1/2 - \beta_0, \\ 3/8 < \beta_0 < 1/2. \end{cases}$$

then $\|q\|_{X^{-\varepsilon,b-1}} \le C\delta^{\theta_0}\|u\|^2_{X^{-\varepsilon,b}}$.

Lemma 3.2 (4, Theorem 3.1) *Assume*

$$\begin{cases} 3/8 < s < 1/2, 1/2 < b < 7/12, 0 < b - s \le 1/8,, \\ \gamma_0/4b + 1 - 2b = \theta_0 > 0, where \gamma_0 = 1/2 - \beta_0, \\ 3/8 < \beta_0 < 1/2. \end{cases}$$

then $\|q\|_{X^{s,b-1}} \le C\delta^{\theta_0}\|u\|^2_{X^{s,b}}$.

Theorem 3.1 *Let $P_1(x,t) = \sigma^2(\delta^{-1}t)\partial_x\left(u^2(x,t)\right)$. Suppose $s \geq 0, 1/2 < b < 7/12$, then there exists $\theta_0 > 0$, such that $\|p_1\|_{X^{s,b-1}} \leq C\delta^{\theta_0}\|u\|^2_{X^{s,b}}$.*

Proof. Set $v(x,t) = \sigma\left(\delta^{-1}t\right)u\left(x,t\right)$. Then we have

$$\widehat{p_1}\left(\xi,\tau\right) = \xi\int\int\widehat{v}\left(\xi-\xi_1,\tau-\tau_1\right)d\xi_1 d\tau_1 = \xi\widehat{v}*\widehat{v}(\xi,\tau).$$

Let $h\left(\xi,\tau\right) = \langle\tau-\xi\rangle^{b-1}\langle\xi\rangle^s\widehat{p_1}\left(\xi,\tau\right)$, then $\|p_1\|_{X^{s,b-1}} \leq \|h\|_{L^2_\xi L^2_\tau}$. For $g \geq 0$ with $\|g\|_{L^2_\xi L^2_\tau} \leq 1$

$$\|h\|_{L^2_\xi L^2_\tau} = \sup_g\left|\int\int g(\xi,\tau)\langle\tau\xi^3\rangle^{b-1}\langle s\rangle^s\xi\widehat{v}*\widehat{v}\left(\xi,\tau\right)d\xi d\tau\right|$$

Let $f(\xi,\tau) = \langle\tau-\xi^3\rangle^b\langle\xi\rangle^s\widehat{v}(\xi,\tau)$.

$$\|p_1\|_{X^{s,b-1}} = \sup_g\left|\int_{R^4}\frac{g(\xi,\tau)\xi\langle\xi\rangle^s}{\langle\tau-\xi^3\rangle^{1-b}}\frac{f(\xi_1,\tau_1)}{\langle\tau_1-\xi_1^3\rangle^b\langle\xi_1\rangle^s}\frac{f(\xi-\xi_1,\tau-\tau_1)}{\langle\tau-\tau_1-(\xi-\xi_1)^3\rangle^b\langle\xi-\xi_1\rangle^s}d\mu\right|$$

where $d\mu = d\xi d\tau d\xi_1 d\tau_1$. Let

$$X = \left|\int_{R^4}\frac{g(\xi,\tau)\xi\langle\xi\rangle^s}{\langle\tau-\xi^3\rangle^{1-b}}\frac{f(\xi_1,\tau_1)}{\langle\tau_1-\xi_1^3\rangle^b\langle\xi_1\rangle^s}\frac{f(\xi-\xi_1,\tau-\tau_1)}{\langle\tau-\tau_1-(\xi-\xi_1)^3\rangle^b\langle\xi-\xi_1\rangle^s}d\mu\right|$$

Since $\langle\xi\rangle^{s+\varepsilon} \leq \langle\xi_1\rangle^{s+\varepsilon}\langle\xi-\xi_1\rangle^{s+\varepsilon}$, where ε satisfies Proposition 3.1,

$$X \leq \int\frac{g(\xi,\tau)|\xi|}{\langle\tau-\xi^3\rangle^{1-b}\langle\xi\rangle^\varepsilon}\frac{|f(\xi_1,\tau_1)|\langle\xi_1\rangle^\varepsilon}{\langle\tau_1-\xi_1^3\rangle^b}\frac{|f(\xi-\xi_1,\tau-\tau_1)|\langle\xi-\xi_1\rangle^\varepsilon}{\langle\tau-\tau_1-(\xi-\xi_1)^3\rangle^b}d\mu$$

By Lemma 3.1, $X \leq C\delta^{\theta_0}\|u\|_{X^{-\varepsilon,b}}$, we have

$$\|p_1\|_{X^{s,b-1}} \leq C\delta^{\theta_0}\|u\|^2_{X^{-\varepsilon,b}} \leq C\delta^{\theta_0}\|u\|^2_{X^{s,b}}$$

■

Theorem 3.2 *Let $p_2(x,t) = \sigma^2\left(\delta^{-1}t\right)\left(1-\partial^2_{xx}\right)^{-1}\partial_x\left(u^2(x,t)\right)$. Suppose $s \geq 0, 1/2 < b < 7/12$, then there exists $\theta_0 > 0$, such that $\|p_2\|_{X^{s,b-1}} \leq C\delta^{\theta_0}\|u\|^2_{X^{s,b}}$.*

Proof.

$$|\widehat{p_2}(\xi,\tau)| = \frac{|\xi|}{1+\xi^2}\left|\widehat{v}*\widehat{v}(\xi,\tau)\right| \leq |\xi|\left|\widehat{v}*\widehat{v}(\xi,\tau)\right| = \left|\widehat{p_1}(\xi,\tau)\right|,$$

and then proceed as in the proof of Theorem 3.1. ■

Proposition 3.1 (4, Corollary 3.1) *Given $s \in (1/4, 3/8]$, there exists $b \in (1/2, 1)$, such that, for $b' \in (1/2, b)$ satisfying $b - b' \leq \min\{1/4 + s/3, s - 1/6\}$, then*

$$\|B(F,F)\|_{X^{s,b-1}} \leq \|F\|_{X^{s,b'}},$$

where constant C depends on ρ, b and $b - b'$.

Proposition 3.2 (4, Theorem 3.5) *For $s \geq 0, b \in (1/2, 3/4), b' \in (1/2, b)$, there exists $c > 0$, such that*

$$\|\partial_x(p^2)\|_{X^{s,b-1}} \leq C\|P\|^2_{X^{s,b}}$$

and

$$\|\left(1-\partial^2_{xx}\right)^{-1}\partial_x(p^2)\|_{X^{s,b-1}} \leq C\|P\|^2_{X^{s,b}}$$

4 Results

Theorem 4.1 *Let $s \geq 0$, then there exists $b > \frac{1}{2}$ such that, for any $u_0 \in H^s(\mathbb{R})$, there are $T = T(\|u_0\|_{H^s(\mathbb{R})}) > 0$ and a unique solution $u(t)$ of (2.4) in the time interval $[0,T]$, satisfying*

$$u \in C\left([0,T]; H^s\left(\mathbb{R}\right)\right), \tag{4.1}$$

$$u \in X^{s,b}, \tag{4.2}$$

$$\partial_x(u^2), (1-\partial_{xx}^2)^{-1}(\partial_x(\partial_x u)^2) \in X^{s,b-1},$$
$$\partial_t u \in X^{s-3,b-1}.$$

Moreover, for any $T' \in (0,T)$, there exists a neighborhood $\mathfrak{V}$ of u_0 in $H^s(\mathbb{R})$, such that the map, $\tilde{u}_0 \to \tilde{u}(t)$ from $\mathcal{V}$ into the class defined by (4.1) and (4.2) with $\tilde{T}$ instead of T, is Lipschitz.

Proof. Let $u_0 \in H^s(\mathbb{R}), s \in (3/8, 1/2)$. For $p \in X^{s,b-1}$, we define a map

$$\begin{aligned}\Phi_{u_0}(p) = \Phi(p) &= \psi(\delta^{-1}t)W(t-t')u_0 - \psi(\delta^{-1}t)\int_0^t W(t-t')p(t')dt' \\ &= \psi(\delta^{-1}t)W(t-t')u_0 - \psi(\delta^{-1}t)\int_0^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'\end{aligned}$$

Let p be defined by (2.3). It suffices to prove $\Gamma(u) = \Phi(p)$ is a contraction in $\mathscr{B}$:

$$\mathscr{B} = \{u \in X^{s,b} : \|u\|_{X^{s,b}} \leq 2c_0\delta^{(1-2b)/2}\|u_0\|_{H^s}\}.$$

We have

$$\begin{aligned}\|\Gamma(u)\|_{X^{s,b}} &= \|\Phi(p)\|_{X^{s,b}} \\ &\leq \|\phi(\delta^{-1}\cdot)W(\cdot)u_0\|_{X^{s,b}} \\ &+c\|\phi(\delta^{-1}t)\int_0^t W(t-t')\sigma^2(t')\partial_x(u^2(t')dt'\|_{X^{s,b}} \\ &+c\|\phi(\delta^{-1}t)\int_0^t W(t-t')\sigma^2(t')(1-\partial_{xx}^2)^{-1}\partial_x(u^2(t'))dt'\|_{X^{s,b}} \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + c\delta^{(1-2b)/2}\|p_1\|_{X^{s,b-1}} + c\delta^{(1-2b)/2}\|p_2\|_{X^{s,b-1}} \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + c\delta^{(1-2b)/2}\delta^{\theta_0}\|u\|_{X^{s,b}}^2 \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + 4c_0^2c\delta^{(1-2b)/2}\delta^{\theta_0}\delta^{(1-2b)}\|u\|_{H^s}^2 \\ &\leq c_0\delta^{(1-2b)/2}\|u_0\|_{H^s} + 4c_0^2c\delta^{(1-2b)/2}\delta^{r_0}\|u\|_{H^s}^2\end{aligned}$$

follows from Lemma 2.7, Lemma 2.9, Theorem 3.1 and Theorem 3.2, where $r_0 = \theta_0 + (1-2b) > 0$. Choosing δ sufficiently small, such that $4c_0c\delta^{r_0}\|u\|_{H^s} < \frac{1}{2}$, then

$$\|\Gamma(u)\|_{X^{s,b}} \leq \frac{3}{2}\delta^{(1-2b)/2}\|u_0\|_{H^s}$$

so $\Gamma(\mathscr{B}) \subset \mathscr{B}$. By same arguments as in Section 3 we have

$$\begin{aligned}&\|\Gamma(u)-\Gamma(\tilde{u})\|_{X^{s,b}} \\ &\leq c\delta^{(1-2b)/2}(\|\sigma^2(\delta^{-1}t)\partial_x(u^2)\|_{X^{s,b-1}} \\ +\|\sigma^2(\delta^{-1}t)(1-\partial_{xx}^2)^{-1}\partial_x(u^2)\|_{X^{s,b-1}} &\leq c\delta^{(1-2b)/2}\delta^{\theta_0}\|u+\tilde{u}\|_{X^{s,b}}\|u-\tilde{u}\|_{X^{s,b}} \\ &\leq c\delta^{(1-2b)/2}\delta^{\theta_0}4c_0\delta^{(1-2b)}\|u_0\|_{H^s}\|u-\tilde{u}\|_{X^{s,b}} \\ &= 4cc_0\delta^{r_0}\|u_0\|_{H^s}\|u-\tilde{u}\|_{X^{s,b}} \\ &\leq \frac{1}{2}\|u-\tilde{u}\|_{X^{s,b}}\end{aligned}$$

Γ is a contraction on $\mathscr{B}$, so there exists $u \in \mathscr{B}$ such that

$$u(t) = \psi(\delta^{-1}t)W(t)u_0(x) - \psi(\delta^{-1}t)\int_0^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'$$

By the definition of ψ and σ, for $t \in [-\delta/2, \delta/2]$, i.e. $T = \frac{\delta}{2}$, we have ,

$$u(t) = W(t)u_0 - \int_0^t W(t-t')p(t')dt'$$

where δ depends only on $\|u_0\|_{H^s}$. Let u and $\tilde{u}$ be solutions corresponding to the initial u_0 and $\tilde{u_0}$, respectively. By similar arguments, we have

$$\|u - \tilde{u})|_{X^{s,b}} \leq c\|u_0 + \tilde{u}_0\|_{X^{s,b}} + \frac{1}{2}\|u - \tilde{u}\|_{X^{s,b}}$$

so

$$\|u - \tilde{u})|_{X^{s,b}} \leq c\|u_0 + \tilde{u}_0\|_{X^{s,b}}$$

Now we prove $u \in C([-T,T]; H^s(\mathbb{R}))$. For $0 \leq \tilde{t} < t < T, t - \tilde{t} < \Delta t$, by

$$u(x,t) = \psi(\delta^{-1}t)W(t-\tilde{t})u(x,\tilde{t}) - \psi(\delta^{-1}t)\int_{\tilde{t}}^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'$$

and the definition of σ, we have

$$\begin{aligned}
&\|\Gamma(t) - \Gamma(\tilde{t})\|_{H^s} \\
&\leq \|W(t-\tilde{t})u(\tilde{t})\|_{H^s} + \|\psi(\delta^{-1}t)\int_{\tilde{t}}^t W(t-t')\sigma^2(\delta^{-1}t')p(t')dt'\|_{H^s} \\
&\leq \|W(t-\tilde{t})u(\tilde{t})\|_{H^s} + \|\psi(\delta^{-1}t)\int_{\tilde{t}}^t W(t-t')\sigma^2(\frac{t'-\tilde{t}}{2\Delta t})p(t')dt'\|_{H^s} \\
&\leq \|W(t-\tilde{t})u(\tilde{t})\|_{H^s} + c\|\sigma^2(\frac{\cdot - \tilde{t}}{2\Delta t})p\|_{X^{s,b-1}} \\
&\leq \|W(t-\tilde{t})u(\tilde{t})\|_{H^s} + c(\Delta t)^{\theta_0}\|u\|^2_{X^{s,b}}
\end{aligned}$$

so when $\Delta t \to 0$, we have $\|u(t) - u(\tilde{t})\| \to 0$, which completes the proof of the theorem. ■

Theorem 4.2 *Let $s \in (1/4, 3/8]$, then there exists $b > 1/2$, for any $u_0 \in H^s(\mathbb{R})$, whose norm is sufficiently small, and then the Cauchy problem (2.2) has a unique solution $u(t)$ in (0,1) satisfying*

$$\begin{aligned}
&u \in BC((0,1); H^s(\mathbb{R})), \\
&u \in X^{s,b}, \\
&\partial_x(u^2), (1-\partial^2_{xx})^{-1}(\partial_x(\partial_x u)^2) \in X^{s,b-1}, \\
&\partial_t u \in X^{s-3,b-1}.
\end{aligned}$$

Proof. For $u_0 \in H^s(\mathbb{R}), s \in (1/4, 1)$, and $\|u_0\|_{H^s} = r$ sufficiently small, define a map

$$\Phi_{u_0}(p) = \Phi(p) = \psi(t)W(t-t')u_0 - \psi(t)\int_0^t W(t-t')\sigma^2(t')p(t')dt'$$

where $p = p(t)$ is given by (2.3). Let $\Gamma(u) = \Phi(p)$. Now we prove that Γ is a contraction on $\mathscr{B}(2c_0r) = \{u \in X^{s,b} : \|u\|_{X^{s,b}} \leq 2c_0r\}$. By Lemma 2.7, Proposition 3.1-3.2, we have

$$\begin{aligned}
&\|\Gamma(u)\|_{X^{s,b}} \leq \|\phi(t)W(t)u_0\|_{X^{s,b}} \\
&+\|\phi(t)\int_0^t W(t-t')\sigma^2(t')(\frac{1}{2}\partial_x(u^2(t') + (1-\partial^2_{xx})^{-1}\partial_x(\frac{3}{2}u^2(t')dt'\|_{X^{s,b}} \\
&\leq c_0\|u_0\|^s_H + c\|\phi(t)u\|_{X^{s,b}} \\
&\leq c_0r + c(2c_0r)^2.
\end{aligned}$$

Choosing r satisfying $4c_0cr < 1$, then $\|\Gamma(u)\|_{X^{s,b}} \leq 2c_0r$. By the same argument, letting $u, \tilde{u} \in \mathscr{B}(2c_0r)$, we have

$$\begin{aligned} &\|\Gamma(u) - \Gamma(\tilde{u})\|_{X^{s,b}} \\ \leq &\frac{1}{2}\|\phi(t)\int_0^t W(t-t')\sigma^2(t')\partial_x(u^2(t') - \tilde{u}^2(t'))dt'\|_{X^{s,b}} \\ +&\frac{3}{2}\|\phi(t)\int_0^t W(t-t')\sigma^2(t')(1-\partial_{xx}^2)^{-1}\partial_x(u^2(t') - \tilde{u}^2(t'))dt'\|_{X^{s,b}} \\ \leq &c\|u+\tilde{u}\|_{X^{s,b}}\|u-\tilde{u}\|_{X^{s,b}} \\ \leq &4c_0cr\|u-\tilde{u}\|_{X^{s,b}}. \end{aligned}$$

Γ is is a contraction on $\mathscr{B}(2c_0c)$, so there exists a unique $u \in \mathscr{B}(2c_0c)$ such that

$$u(t) = \psi(t)W(t)u_0 - \psi(t)\int_0^t W(t-t')\sigma^2(t')p(t')dt'$$

By the definition of ψ, σ, we have ,on $[-1/2, 1/2]$,

$$u(t) = W(t)u_0 - \int_0^t W(t-t')\sigma^2(t')p(t')dt'$$

where $p = p\,(u)$ is given by (2.3). ■

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