

Optimal Switching Sequence of a Class of Switched Systems with Parameter Uncertainty

Zhiming Fang *,Zhengrong Xiang,Qingwei Chen,Jing Hua

School of Automation, Nanjing University of Science & Technology, Nanjing 210094, Jiangsu,P.R.China

(Received 27 September 2009, accepted 21 January 2010)

Abstract: The problem of optimal switching for a class of switched systems with parameter uncertainty is investigated in the paper. The conditions for robust stabilization of this class of switched systems with parameter uncertainty are presented based on multi-Lyapunov function technique and Linear matrix inequality technique. The cost functional with respect to the number of switchings, the order of switching and the switching instants is presented. An approach is presented based on genetic algorithms, this approach imposes no restriction on the switching order or the number of switchings. This is in contrast to existing algorithms where a fixed number of switchings is set a priori. In this approach, the optimal solution can be determined by solving the cost functional. Results of numerical simulation are provided to support the proposed method.

Keywords: Switched systems; Optimal control; Genetic algorithms; multi-Lyapunov function

1 Introduction

Switched systems consists of a collection of subsystems and a switching law orchestrating the active subsystem at each time instant. Many real-world processes such as chemical processes, air traffic management, manufacturing systems,automotive systems,etc.can be modeled as switched systems.

The problems of optimal control and optimal switching for switched systems,which require the solutions of the optimal switching sequences,have attracted the attention of many researchers in recent years. Branicky [1]discuss general conditions for the existence of optimal control laws for hybrid systems and compare several algorithms for optimal control. Sussmann [2] present necessary optimality conditions for a trajectory of a switched system with a fixed sequence of finite length by using Maximum Principle. Riedinger [3] use a similar approach to solve linear quadratic cost functionals but considering both autonomous and controlled switches. Bengea [4] apply the Maximum Principle to an embedded system governed by a logic variable and a continuous control, some necessary and sufficient conditions are introduced for optimality. Hedlund [5] use convex dynamic programming to approximate hybrid optimal control laws and to compute lower and upper bounds of the optimal cost. Gokbayrak [6] use a hierarchical decomposition approach to break down the overall optimal control problem into smaller ones. Xu [7] present two-stage method to solve the optimal control problems of switched systems with fixed switching sequence. Shaikh [8] consider a hybrid optimal control problem and give necessary optimality conditions for a fixed sequence by using Maximum Principle and nonfixed sequence by using a suboptimal result based on the Hamming distance permutations of an initial given sequence. Egerstedt [9] consider an optimal control problem for switched systems, a gradient-descent algorithm is proposed based on an especially simple form of the gradient of the cost functional. Seatzu [10] apply master-slave procedure and switching table procedure to the optimal control of continuous-time switched affine systems that the number of switching is finite length, a state feedback control law is computed.

For a class of switched systems with parameter uncertainty, based on arbitrary switching in this paper we explore the design of optimal switching sequence by treating the order of switching, number of switchings and switching instants, all as decision variables. In deviation from the traditional approach, we solve the optimization problem as an optimal control problem by describing the switching action using multiple control inputs. We construct a cost functional based on arbitrary switching. Linear matrix inequality(LMI) and multi-Lyapunov function technique are used to ensure the asymptotically

*Corresponding author. E-mail address: fzm7889@hotmail.com

stable of system in the sense of Lyapunov. By using genetic algorithms(GA), the cost functional is solved to obtain the optimal the order of switching, number of switchings and switching instants.

The paper is organized as follows: The problem is formulated in Section 2. The asymptotically stable of system in the sense of Lyapunov is proved in Section 3. We solve the cost functional based on genetic algorithms in Section 4. Numerical examples are presented in Section 5 to illustrate the theoretical results. The conclusions are drawn in Section 6.

2 Problem formulation

2.1 Switched systems

In this paper, we consider switched systems consisting of the subsystems

$$\begin{cases} \dot{x} = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)f_i(\sigma_i) \\ \sigma_i = C_i^T x(t) \end{cases} \quad (1)$$

where $i : Z^+ = [0, K] \rightarrow \{1, 2, \dots, N\}$ is the switching signal which is assumed to be a piecewise continuous from the right function depending on time or state or both, N is the total number of subsystems. $x(t) \in R^n$ is the state, $f_i(\sigma_i) \in R^n$, $f_{ji}(\sigma_i) \in W[0, \infty) = \{f_{ji}(\sigma_i) | f_{ji}(0) = 0, \sigma_i f_{ji}(\sigma_i) > 0 (\sigma_i \neq 0)\}$ is in infinite sector or $f_{ji}(\sigma_i) \in W[0, w] = \{f_{ji}(\sigma_i) | f_{ji}(0) = 0, 0 < \sigma_i f_{ji}(\sigma_i) \leq w \sigma_i^2 (\sigma_i \neq 0)\}$ is in finite sector, $f_{ji}(\sigma_i) \in f_i(\sigma_i)$.

$A_i \in R^{n \times n}$, $B_i \in R^{n \times m}$, $C_i \in R^{n \times m}$ are known real constant matrices. $\Delta A_i(t)$, $\Delta B_i(t)$ are system uncertainties satisfying

$$[\Delta A_i(t) \quad \Delta B_i(t)] = H_i F_i(t) [E_{ia} \quad E_{ib}]$$

where H_i , E_{ia} and E_{ib} are given constant matrices which characterize the structure of the uncertainty, $F_i(t)$ are uncertainties satisfying $F_i^T(t) F_i(t) \leq I$.

The following lemmas play an important role in our later development.

Lemma 1 ([11]) Let H, E and F are matrices with appropriate dimensions. Suppose $F^T F \leq I$, then for any scalar $\lambda > 0$, we have

$$HFE + E^T F^T H^T \leq \lambda^{-1} H H^T + \lambda E^T E$$

Lemma 2 (Schur complement[12]) Let M, P, Q be given matrices such that $Q > 0$. Then

$$\begin{bmatrix} P & M \\ M^T & -Q \end{bmatrix} < 0 \Leftrightarrow P + M Q^{-1} M^T < 0$$

Lemma 3 ([13]) Suppose we have candidate Lyapunov functions $V_i, i = 1, 2, \dots, N$ and vector fields $\dot{x} = f_i(x)$ with $f_i(0) = 0$ for all i . Let S be the set of all switching sequence associated with the system. If for each $s \in S$ we have that for all i , V_i is Lyapunov-like for f_i and $x_s(\bullet)$ over $s|i$, then the system is stable in the sense of Lyapunov.

2.2 Optimal control problem based on arbitrary switching

Problem 1 For a switched system, given a fixed time interval $[t_0, t_f]$, find a switching sequence such that the cost functional

$$J = \psi(x_{t_f}) + \int_{t_0}^{t_f} L(x(t)) dt = x^T P x + \int_{\sigma_0}^{\sigma_f} f(\sigma) d\sigma \quad (2)$$

is minimized, where t_0, t_f and $x(t_0) = x_0$ are given, $\psi : R^n \rightarrow R, L : R^n \times R^m \rightarrow R$. Switching sequence includes the order of subsystems switching U , the switching instants (t_1, t_2, \dots, t_k) and the number of switchings k .

The order of subsystems switching U is the discrete input of switched systems, so system (1) can be written as

$$\begin{cases} \dot{x} = F(x(t), U) = \sum_{i=1}^N u_i ((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)f_i(\sigma_i)) \\ \sigma_i = C_i^T x(t) \end{cases}, \quad \sum_{i=1}^N u_i = 1 \quad (3)$$

where u_i refers to the i -th component of U and U is a Boolean vector so that there is one and only one component of U equal to 1 at any time, the others equal to 0. That is to say one and only one subsystem is active at any time, other subsystems are inactive. We can use U to denote the order of subsystems switching. Given a fixed time interval $[t_0, t_f]$, (t_1, t_2, \dots, t_k) is a set of switching instants satisfying $t_0 < t_1 < \dots < t_k < \dots < t_f$. For any finite $t_f > t_0$, there exists a positive integer K , which may depend on t_f , such that during the time interval $[t_0, t_f]$ the system(1) switches no more than K times. Every subsystem must be active one time during the time interval $[t_0, t_f]$ at least. If the number of subsystems is N , k is the number of switchings satisfying $k \geq N - 1$. That is to say the number of switchings k satisfies $N - 1 \leq k \leq K$.

As usually assumed in the literature, this assumption means that only finite many switchings occur during any finite time interval and the number of switchings is bounded by an integer that depends on the finite time. That is to say, switched system in this paper is not Zeno system.

Based on arbitrary switching, equation(2) can be written as

$$J = [U_1 \ U_2 \ \dots \ U_{k+1}][J_1 \ J_2 \ \dots \ J_{k+1}]^T$$

where

$$U_j = [u_{j1} \ u_{j2} \ \dots \ u_{jN}]^T, \quad J_j = \begin{bmatrix} \psi(x(t_f)) + \int_{t_{j-1}}^{t_j} L_{j1}(x(t))dt \\ \psi(x(t_f)) + \int_{t_{j-1}}^{t_j} L_{j2}(x(t))dt \\ \vdots \\ \psi(x(t_f)) + \int_{t_{j-1}}^{t_j} L_{jN}(x(t))dt \end{bmatrix} = \begin{bmatrix} x(t_j)^T P_{j1} x(t_j) + \int_{\sigma_{j-1}}^{\sigma_j} f_{j1}(\sigma_{j1})dt \\ x(t_j)^T P_{j2} x(t_j) + \int_{\sigma_{j-1}}^{\sigma_j} f_{j2}(\sigma_{j2})dt \\ \vdots \\ x(t_j)^T P_{jN} x(t_j) + \int_{\sigma_{j-1}}^{\sigma_j} f_{jN}(\sigma_{jN})dt \end{bmatrix}$$

k is the number of switchings, N is the total number of subsystems, $j \in \{1, 2, \dots, k\}$.

So we have a optimal problem of switched systems based on arbitrary switching:

Problem 2 For a switched system, given a fixed time interval $[t_0, t_f]$, find a switching sequence includes the order of subsystems switching U , the switching instants (t_1, t_2, \dots, t_k) and the number of switchings k such that the cost functional

$$\begin{aligned} & J = [U_1 \ U_2 \ \dots \ U_{k+1}][J_1 \ J_2 \ \dots \ J_{k+1}]^T \\ \text{s.t. } & U \\ & t_0 < t_1 < \dots < t_k < t_f \\ & N - 1 \leq k \leq K \end{aligned} \tag{4}$$

is minimized.

3 Robust stabilization analyse based on LMI and multi-Lyapunov function

Theorem 1 For system (1), when $f_{ji}(\sigma_i) \in W[0, \infty) = \{f_{ji}(\sigma_i) | f_{ji}(0) = 0, \sigma_i f_{ji}(\sigma_i) > 0 (\sigma_i \neq 0)\}$ is in the infinite sector. System (1) with arbitrary switching is asymptotically stable, if there exist matrices $P_i > 0$ with appropriate dimensions and scalars $\varepsilon_{i1} > 0, \varepsilon_{i2} > 0$ and λ_{ij} , such that the following linear matrix inequalities hold:

$$\begin{bmatrix} S_1 & S_2 & P_i H_i & 0 & P_i H_i & 0 \\ * & S_3 & 0 & \sqrt{\frac{1}{2}} C_i^T H_i & 0 & \sqrt{\frac{1}{2}} C_i^T H_i \\ * & * & -\varepsilon_{i1} I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{i1} I & 0 & 0 \\ * & * & * & * & -\varepsilon_{i2} I & 0 \\ * & * & * & * & * & -\varepsilon_{i2} I \end{bmatrix} < 0 \tag{5}$$

$$P_j - P_i < \lambda_{ij} I \tag{6}$$

where $S_1 = A_i^T P_i + P_i A_i + \frac{3}{2} \varepsilon_{i1} E_{ia}^T E_{ia}$

$$S_2 = P_i B_i + \frac{1}{2} A_i C_i$$

$$S_3 = C_i^T B_i + \frac{3}{2} \varepsilon_{i2} E_{ib}^T E_{ib}$$

Proof. Construct candidate Lyapunov function of every subsystem $V_i(x)$ as

$$V_i(x) = x^T P_i x + \int_{\sigma_i(t_0)}^{\sigma_i(t)} f_i(\sigma_i) d\sigma_i \tag{7}$$

where $P_i = P_i^T > 0$.

Along the solution of (1), we have

$$\begin{aligned} \dot{V}_i(x) = & x^T ((A_i + \delta A_i)^T P_i + P_i (A_i + \delta A_i)) x + x^T P_i (B_i + \delta B_i) f_i(\sigma_i) + f_i^T(\sigma_i) (B_i + \delta B_i) P_i x \\ & + f_i^T(\sigma_i) C_i^T ((A_i + \delta A_i) x + (B_i + \delta B_i) f_i(\sigma_i)) \end{aligned}$$

Due to lemma 1, we have

$$\begin{aligned} \dot{V}_i(x) \leq & \begin{bmatrix} x \\ f_i(\sigma_i) \end{bmatrix}^T \begin{bmatrix} A_i^T P_i + P_i A_i + \frac{3}{2} \varepsilon_{i1} E_{ia}^T E_{ia} & P_i B_i + \frac{1}{2} A_i C_i \\ (P_i B_i + \frac{1}{2} A_i C_i)^T & C_i^T B_i + \frac{3}{2} \varepsilon_{i2} E_{ib}^T E_{ib} \end{bmatrix} \begin{bmatrix} x \\ f_i(\sigma_i) \end{bmatrix} \\ & + \begin{bmatrix} x \\ f_i(\sigma_i) \end{bmatrix}^T \begin{bmatrix} (\varepsilon_{i1}^{-1} + \varepsilon_{i2}^{-1}) P_i H_i H_i^T P_i & 0 \\ 0 & \frac{1}{2} (\varepsilon_{i1}^{-1} + \varepsilon_{i2}^{-1}) C_i^T H_i H_i^T C_i \end{bmatrix} \begin{bmatrix} x \\ f_i(\sigma_i) \end{bmatrix} \end{aligned}$$

Due to lemma 2 and (5), we have $\dot{V}_i(x) < 0$. So $V_i(x)$ is Lyapunov function of every subsystem.

At switching instant t_{k+1} , system switches from i -th subsystem to j -th subsystem, we have

$$V_j(t_{k+1}) = x(t_{k+1})^T P_j x(t_{k+1}) + \int_{\sigma_j(t_{k+1})}^{\sigma_j(t_{k+1})} f_j(\sigma_j) d\sigma_j \tag{8}$$

$$V_i(t_{k+1}) = x(t_{k+1})^T P_i x(t_{k+1}) + \int_{\sigma_i(t_k)}^{\sigma_i(t_{k+1})} f_i(\sigma_i) d\sigma_i \tag{9}$$

(8) minus (9) is

$$\begin{aligned} V_j(t_{k+1}) - V_i(t_{k+1}) &= x(t_{k+1})^T P_j x(t_{k+1}) - x(t_{k+1})^T P_i x(t_{k+1}) - \int_{\sigma_i(t_k)}^{\sigma_i(t_{k+1})} f_i(\sigma_i) d\sigma_i \\ &\leq x(t_{k+1})^T P_j x(t_{k+1}) - x(t_{k+1})^T P_i x(t_{k+1}) - x(t_{k+1})^T \lambda_{ij} x(t_{k+1}) \\ &= x(t_{k+1})^T (P_j - P_i - \lambda_{ij} I) x(t_{k+1}) \tag{10} \end{aligned}$$

Substitute (6) into (10), we have $V_j(t_{k+1}) - V_i(t_{k+1}) \leq x(t_{k+1})^T (P_j - P_i - \lambda_{ij} I) x(t_{k+1}) < 0$

Due to lemma 3, System (1) with arbitrary switching is asymptotically stable in the sense of Lyapunov. ■

Theorem 2 For system (1), when $f_{ji}(\sigma_i) \in f_i(\sigma_i)$ is in the finite sector. System (1) with arbitrary switching is asymptotically stable, if there exist matrices $P_i > 0$ with appropriate dimensions and scalars $\varepsilon_{i1} > 0, \varepsilon_{i2} > 0, \lambda_{ij}$ and w_i , such that the following linear matrix inequalities hold:

$$\begin{bmatrix} S_1 & S_2 & P_i H_i & 0 & P_i H_i & 0 \\ * & S_3 & 0 & \sqrt{\frac{1}{2}} C_i^T H_i & 0 & \sqrt{\frac{1}{2}} C_i^T H_i \\ * & * & -\varepsilon_{i1} I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{i1} I & 0 & 0 \\ * & * & * & * & -\varepsilon_{i2} I & 0 \\ * & * & * & * & * & -\varepsilon_{i2} I \end{bmatrix} < 0$$

$$P_j - P_i < \lambda_{ij} I$$

$$\text{where } S_1 = A_i^T P_i + P_i A_i + \frac{3}{2} \varepsilon_{i1} E_{ia}^T E_{ia}$$

$$S_2 = P_i B_i + \frac{1}{2} A_i C_i$$

$$S_3 = C_i^T B_i + \frac{3}{2} \varepsilon_{i2} E_{ib}^T E_{ib}$$

Proof. The proof of theorem 2 is similar to the proof of theorem 1. It is omitted. ■

4 Solving optimal problem using genetic algorithms

To solve Problem 2, genetic algorithms(GA) are used. Genetic algorithms search for an optimum solution based on the mechanics of natural selection and genetic. A population of chromosomes is formed initially from a random set of solutions. New generations are then produced, and some measure of fitness for the chromosomes is used to guide the selection. Offsprings are formed by merging two chromosomes from the current generation using a crossover operator, and modifying a chromosome using a mutation operator. A new generation is formed by selecting some of the parents and offsprings, based on the fitness values, and rejecting others to keep the population size constant. Fitter chromosomes have higher probabilities of being selected. After several generations, the algorithm converges to the best chromosome, which represents the optimal solution to the problem provided by the GA.

Genetic algorithms

Stage 1: Decision variable

In problem 2, the order of subsystems switching U , the switching instants (t_1, t_2, \dots, t_k) and the number of switchings k are optimization variables.

Stage 2: Optimization problem

$$\begin{aligned} \min \quad & J = [U_1 \ U_2 \ \dots \ U_{k+1}][J_1 \ J_2 \ \dots \ J_{k+1}]^T \\ \text{s.t.} \quad & U \\ & t_0 < t_1 < \dots < t_k < t_f \\ & N - 1 \leq k \leq K \end{aligned}$$

Stage 3: The number of switchings k is real-coded, the switching instants (t_1, t_2, \dots, t_k) and the order of subsystems switching U is serial code and construct one unit v_i , every unit of the switching instants (t_1, t_2, \dots, t_k) is ten-bit binary-coded, every unit of the order of subsystems switching U is N -bit binary-coded, the length of v_i is $10K + N(K + 1)$ -bit binary-coded. Every v_i and corresponding k_i construct a binary group (k_i, v_i) . Where $1 \leq i \leq n$, n is the number of sample.

stage 4: The switching instants (t_1, t_2, \dots, t_k) can be got by decoding from the 1-th bit to the $10k_i$ -th bit code depending on corresponding k_i . The order of subsystems switching U can be got by decoding from the $10k + 1$ -th bit to the $10K + N(k_i + 1)$ -th bit code.

stage 5: In the paper, the fitness function $F(t, U, k)$ is the cost functional J .

Stage 6: Based on optimal problem, we design appropriate selection operation, crossover operation and mutation operation. We hold the best chromosome of the parents generation in the offsprings generation.

Stage 7: Based on optimal problem, we design appropriate the size of group M , the generation of evolution G , crossover operator P_c and mutation operator P_m .

5 Numerical simulation

A. The order of subsystems switching U and the number of switchings k have been confirmed beforehand.

Example 1 : Given a switched system

$$\begin{aligned} \text{Subsystem1 :} & A_1 = \begin{bmatrix} -2 & 1 \\ -2 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.01 & 1 \\ 1 & 0.01 \end{bmatrix}, \\ & E_{1a} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix}, \quad E_{1b} = \begin{bmatrix} 0.02 \\ 0 \end{bmatrix}, \quad f_1(\sigma_1) = 0.5\sigma_1 - 0.2 \sin \sigma_1; \\ \text{Subsystem2 :} & A_2 = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.01 & 1 \\ 1 & 0.01 \end{bmatrix}, \\ & E_{2a} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad E_{2b} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \quad f_2(\sigma_2) = 0.5\sigma_2 - 0.2 \sin \sigma_2; \end{aligned}$$

assume that $t_0 = 0, t_f = 2$ and the system switches once at $t = t_1 (0 < t_1 < 2)$ from subsystem 1 to 2. We want to find optimal swtiching instant t_1 such that the cost functional J is minimized. Here $x(0) = [4 - 3]^T$.

For this problem, we use Theorem 1 and genetic algorithms to obtain the values that are showed as follows and in TABLE 1.

Table1: Results of Example 1

	t_1	J
1	0.0665	27.8080
2	0.0665	27.8135
3	0.0665	27.8155

$$P_1 = \begin{bmatrix} 0.7372 & -0.2022 \\ -0.2022 & 1.2675 \end{bmatrix}, \varepsilon_{11} = 1.7331, \varepsilon_{12} = 1.7329;$$

$$P_2 = \begin{bmatrix} 0.7733 & -0.0071 \\ -0.0071 & 0.5280 \end{bmatrix}, \varepsilon_{21} = 1.7330, \varepsilon_{22} = 1.7330;$$

$$\lambda_{12} = 1.4042$$

We select the best solution. The system switches once at $t_1 = 0.0665$ from subsystem 1 to 2. Fig.1 shows that the system is asymptotically stable.

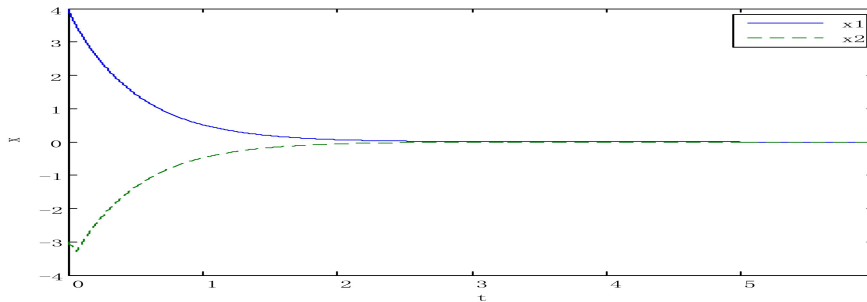


Figure 1: Response of state variable

B. The number of switchings k has been confirmed beforehand.

Example 2 Consider the same switched system as in example 1 except that the system switches once at $t = t_1 (0 < t_1 < 2)$. We want to find optimal switching instant t_1 and the order of subsystems switching U such that the cost functional $J = [U_1 \ U_2] [J_1 \ J_2]^T$ is minimized. Here $x(0) = [4 \ -3]^T$.

For this problem, we use Theorem 1 and genetic algorithms to obtain the values that are showed as follows and in TABLE 2.

Table2: Results of Example 2

	U	t_1	J
1	2,1	1.8441	0.2793
2	2.1	1.8112	0.2810
3	2.1	1.8368	0.2797

We select the best solution. The system switches once at $t_1 = 1.8441$ from subsystem 2 to 1. Fig.2 shows that the system is asymptotically stable.

C. The switching sequence has no restriction beforehand.

Example 3 : Consider the same switched system as in example 1 except that the switching sequence has no restriction beforehand. Now we want to find the optimal switching sequence such that the cost functional $J = [U_1 \ U_2 \ \dots \ U_k] [J_1 \ J_2 \ \dots \ J_k]^T$ is minimized. Here $x(0) = [4 \ -3]^T$ and $0 < t_1 < \dots < t_k < \dots < 2, 1 \leq k \leq 5$.

For this problem, we use Theorem 1 and genetic algorithms to obtain the values that are showed as follows and in TABLE 3.

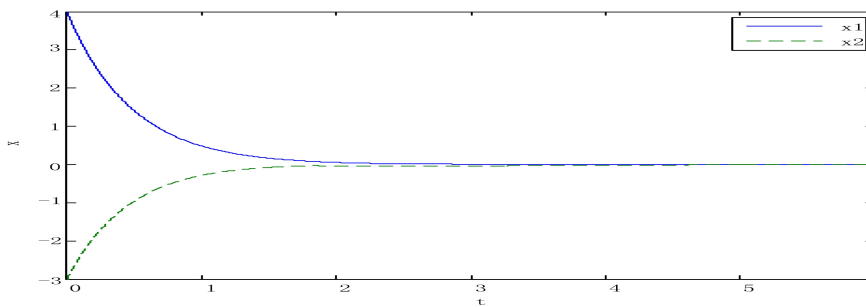


Figure 2: Response of state variable

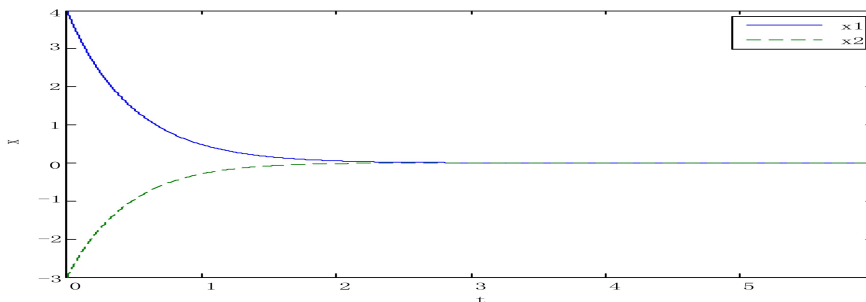


Figure 3: Response of state variable

Table3: Results of Example 3

	k	U	t_1, t_2	J
1	2	2,1,2	1.9296,1.9980	0.2790
2	2	2,1,2	1.9978,1.9994	0.2771
3	2	2,1,2	1.9472,1.9531	0.2787

We select the best solution. The system switches once at $t_1 = 1.9978$ from subsystem 2 to 1, at $t_2 = 1.9994$ from subsystem 1 to 2. Fig.3 shows that the system is asymptotically stable.

From example 1 to example 3, By restrictive conditions including switching instants, the order of switching and number of switchings are relaxed, the minimum of cost functional that we get by using genetic algorithms to approach the real minimum of cost functional.

6 Conclusions

We addressed the optimal switching problem for a class of switched systems with parameter uncertainty. By treating the order of switching, number of switchings and switching instants, all as decision variables, we construct a cost functional based on arbitrary switching. Linear matrix inequality(LMI) and multi-Lyapunov function technique are used to ensure the asymptotically stable of system in the sense of Lyapunov. By using genetic algorithms(GA), the cost functional is solved to obtain the optimal the order of switching, number of switchings and switching instants. Because of parameter uncertainty and localization of genetic algorithms, the solution is not always optimal solution, sometimes we only get suboptimal solution. Next, how to ensure that the solution is optimal is our work.

References

[1] M. S. Branicky, V. S. Borkar, and S. K. Mitter. A unified framework for hybrid control: Model and optimal control theory. *IEEE Trans. Automatic Control.*, 43(1):(1998),31-45.
 [2] H. J. Sussmann. A maximum principle for hybrid optimal control problems. *Proc. 38th IEEE Conf. on Decision and Control*, Phoenix, Arizona, USA (1999).

- [3] P. Riedinger, C. Zanne, and F. Kratz. Time optimal control of hybrid systems. *Proc. American Contr. Conf.*, San Diego, California, USA (1999).
- [4] S. C. Bengea, R.A. DeCarlo. Optimal control of switching systems. *Automatica.*, 41:(2005), 11-27.
- [5] S. Hedlund, A. Rantzer. Convex dynamic programming for hybrid systems. *IEEE Trans. Automatic Control*, 47(9):(2002), 1536-1540.
- [6] K. Gokbayrak, C. G. Cassandras. A hierarchical decomposition method for optimal control of hybrid systems. *Proc. 38th IEEE Conf. on Decision and Control*. Phoenix, Arizona, USA (1999).
- [7] X. P. Xu, P. J. Antsaklis. Optimal control of switched systems based on parameterization of the switching instants. *IEEE Trans. Automatic Control*, 49(1):(2004), 2-16.
- [8] M. S. Shaikh, P. E. Caines. On the optimal control of hybrid systems: optimization of switching times and combinatoric schedules. *Proc. American Contr. Conf.*, Denver, USA (2003).
- [9] M. Egerstedt, Y. Wardi, and H. Axelsson. Transition-time optimization for switched-mode dynamical systems. *IEEE Trans. Automatic Control*, 51(1):(2006), 110-115.
- [10] C. Seatzu, D. Corona, A. Giua, and A. Bemporad. Optimal control of continuous-time switched affine systems. *IEEE Trans. Automatic Control*, 51(5):(2006), 726-741.
- [11] Y. Wang, L. Xie, and C. E. DeSouza. Robust control of a class of uncertain nonlinear systems. *Syst. Control Lett.*, 19:(1992), 139-149.
- [12] S. Boyd, L. ElGhaoui, E. Feron, and V. Balakrishnan. Linear matrix inequalities in system and control theory. *SIAM.*, Philadelphia, PA (1994).
- [13] M. S. Branicky. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Trans. Automatic Control*, 43(4):(1998), 475-482.