ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.9(2010) No.2,pp.139-150

A Series of Traveling Wave Solutions for Nonlinear Evolution Equations Arising in Physics

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(Received 8 March 2009, accepted 23 November 2009)

Abstract: An extended algebraic method is devised to uniformly construct a series of complete exact solutions for some nonlinear evolution equations arising in physics. For illustration,we apply the proposed method to three nonlinear coupled physical system, namely, generalized Hirota-Satsuma coupled KdV system, coupled Maccaris equations and generalized-Zakharov equations.The solutions obtained include soliton solutions,rational solutions,triangular periodic solutions, Jacobi and Weierstrass doubly periodic wave solutions. Among them,the Jacobi elliptic periodic wave solutions exactly degenerate to the soliton solutions at a certain limit condition. It is shown that the extended algebraic method provides a very effective and powerful mathematical tool for solving other nonlinear evolution equations arising in physics.

Keywords:extended algebraic method; nonlinear evolution equations; new exact wave solutions

1 Introduction

Mathematical modelling of physical systems often leads to nonlinear evolution equations.Explicit solutions(or more precisely class of solutions) to such equations are of fundamental importance. In particular there is considerable interest in explicit traveling waves or solitary wave solutions.Nonlinear waves are encountered in numerous domains such as fluid mechanics,solid state physics,plasma physics and chemical physics.Names like solitons,kinks,breathers,etc.are commonly used in the vast literature dealing with this subject.A key step in investigating nonlinear problems is the derivation of traveling waves from the associated evolution equations.However,in contrast with linear wave theory where one can make use of the basic technique of Fourier analysis,we are here confronted with a large variety of methods.Several standard methods for obtaining such solutions are known $[1-20]$.

In this paper,we present an effective extension to the tanh method and develop a new extended algebraic method to uniformly construct a series of travelling wave solutions including soliton,rational,triangular periodic,Jacobi and Weierstrass doubly periodic solutions for general nonlinear equations.For illustration,we apply the proposed method to three nonlinear physical models,namely,generalized Hirota-Satsuma,coupled Maccaris system and generalized-Zakharov equations.

2 The proposed method

For a given nonlinear evolution equation

$$H(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, ...) = 0 \tag{1}$$

The main steps of our proposed method are given as follows:

Step 1. By using the travelling wave transformation $u(x,t) = u(\xi)$,$\xi = x - ct + l$,then Eq.(1) reduces to an ordinary differential equations

$$H(u, u^{'}, u^{''}, u^{'''}, ...) = 0 \tag{2}$$

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IJNS.2010.04.15/328

Step 2. We introduce a new variable $\phi = \phi(\xi)$ which is a solution of the following first order ODE

$$\phi^{'} = \sqrt{\sum_{j=0}^{r} h_j \phi^j} \tag{3}$$

Then the derivatives with respect to the variable ξ become the derivatives with respect to the variable ϕ as

$$\frac{d}{d\xi} = \sqrt{\sum_{j=0}^{r} h_j \phi^j}\frac{d}{d\phi}, \qquad \frac{d^2}{d\xi^2} = \frac{1}{2}\sum_{j=1}^{r} j h_j \phi^{j-1}\frac{d}{d\phi} + \sum_{j=0}^{r} j h_j \phi^j \frac{d^2}{d\phi^2} \tag{4}$$

Step 3. The next crucial step is that the solution we are looking for is expresed in the form

$$u(\xi) = a_0 + \sum_{i=1}^{n}[a_i\phi^i(\xi) + b_i\phi^{-i}(\xi) + c_i\phi^{i-1}(\xi)\phi^{'}(\xi) + d_i\phi^{-i}(\xi)\phi^{'}(\xi)], \tag{5}$$

where a_0, a_i,b_i,c_i,d_i are constants to be determined later.Balancing the higest derivative term with nonlinear term in (2),we obtain a relation between the positive integers n and r,from which the different possible values of n and r can be obtained.For example,in the case of KdV equation

$$u_t + 6uu_x + u_{xxx} = 0,$$

we have

$$r = n + 2 \tag{6}$$

If we take $n = 1$ and $r = 3$ in Eq.(6),we may use the following series expansion as a solution of KdV equation

$$u(\xi) = a_0 + a_1\phi(\xi) + b_1\phi^{-1}(\xi) + c_1\phi^{'}(\xi) + d_1\phi^{-1}(\xi)\phi^{'}(\xi),$$
$$\phi^{'}(\xi) = \sqrt{h_0 + h_1\phi(\xi) + h_2\phi(\xi)^2 + h_3\phi(\xi)^3}$$

Similarly,if we take $n = 2$,$r = 4$ in Eq.(6),we have

$$u(\xi) = a_0 + a_1\phi(\xi) + a_2\phi^2(\xi) + b_1\phi^{-1}(\xi) + b_2\phi^{-2}(\xi) + c_1\phi^{'}(\xi) + c_2\phi^{'}(\xi)\phi(\xi) + d_1\phi^{-1}(\xi)\phi^{'}(\xi) + d_2\phi^{-2}(\xi)\phi^{'}(\xi),$$
$$\phi^{'}(\xi) = \sqrt{h_0 + h_1\phi(\xi) + h_2\phi(\xi)^2 + h_3\phi(\xi)^3 + h_4\phi(\xi)^4} \tag{7}$$

Step 4. Inserting (3) into (2) and then setting the coefficients of all powers of ϕ^i and $\phi^i\sqrt{\sum_{j=0}^{r} h_j\phi^j}(i = 0, 1, ...n), (j = 0, 1, ..., r)$ to zero,we will get a system of algebraic equations, from which the constants a_0,a_i,b_i,c_i,d_i and ξ can be found explicitly.Step 5.Substituting the values a_0,a_i,b_i,c_i and d_i obtained in Step(4) back into Eq.(5) and then solving it,we may get its all possible solutions.In this paper, we only consider the case $r = 4$ and hence

$$\phi^{'}(\xi) = \sqrt{h_0 + h_1\phi(\xi) + h_2\phi(\xi)^2 + h_3\phi(\xi)^3 + h_4\phi(\xi)^4} \tag{8}$$

By considering different values of h_0,h_1,h_2,h_3 and h_4,we have the following results:
Case A.Eq.(7) admits two kinds of polynomials solutions as follows

$$\phi_1 = \sqrt{h_0}\xi, h_1 = h_2 = h_3 = h_4 = 0, h_0 > 0 \tag{9}$$

$$\phi_2 = -\frac{h_0}{h_1} + \frac{1}{4}h_1\xi^2, h_2 = h_3 = h_4 = 0, h_1 \neq 0 \tag{10}$$

Case B.Eq.(7) admits two kinds of exponential soliutions,namely,

$$\phi_3 = -\frac{h_1}{2h_2} + exp(\sqrt{h_2}\xi), h_3 = h_4 = 0, h_0 = \frac{h_1^2}{4h_2}, h_2 > 0 \tag{11}$$

$$\phi_4 = \frac{h_3}{2h_4}exp(\frac{h_3}{2\sqrt{-h_4}}\xi), h_0 = h_1 = h_2 = 0, h_4 > 0 \tag{12}$$

Case C.Eq.(7) admits two kinds of rationals, namely,

$$\phi_5 = -\frac{1}{\sqrt{h_4}}\xi, h_0 = h_1 = h_2 = h_3 = 0, h_4 > 0 \tag{13}$$

$$\phi_6 = -\frac{4h_3}{h_3^2\xi^2 - 4h_4}, h_0 = h_1 = h_2 = 0 \tag{14}$$

Case D.Eq.(7) admits the triangular solutions as follows

$$\phi_7 = -\frac{h_1}{2h_2} + \frac{h_1}{2h_2}sin(\sqrt{-h_2}\xi), h_0 = h_3 = h_4 = 0, h_2 < 0, \tag{15}$$

$$\phi_8 = \sqrt{\frac{-h_0}{h_2}}sin(\sqrt{-h_2}\xi), h_1 = h_3 = h_4 = 0, h_2 > 0, h_0 < 0, \tag{16}$$

$$\phi_9 = \sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi), h_0 = h_1 = h_3 = 0, h_2 < 0, h_4 > 0, \tag{17}$$

$$\phi_{10} = -\frac{h_2}{h_3}sec^2(\frac{\sqrt{-h_2}}{2}\xi), h_0 = h_1 = h_4 = 0, h_2 < 0, \tag{18}$$

$$\phi_{11} = \sqrt{\frac{h_2}{2h_4}}tan(\sqrt{h_2/2}\xi), h_1 = h_3 = 0, h_0 = \frac{h_2^2}{4h_4}, h_2 > 0, h_4 > 0 \tag{19}$$

Case E.Eq.(7) admits the kinds of hyperbolic solutions,namely,

$$\phi_{12} = -\frac{h_1}{2h_2} + \frac{h_1}{2h_2}sinh(2\sqrt{h_2}\xi), h_0 = h_3 = h_4 = 0, h_2 > 0, \tag{20}$$

$$\phi_{13} = \sqrt{\frac{h_0}{h_2}}sinh(\sqrt{h_2}\xi), h_1 = h_3 = h_4 = 0, h_2 > 0, h_0 > 0, \tag{21}$$

$$\phi_{14} = \sqrt{-\frac{h_2}{h_4}}sech(\sqrt{h_2}\xi), h_0 = h_1 = h_3 = 0, h_2 > 0, h_4 < 0, \tag{22}$$

$$\phi_{15} = -\frac{h_2}{h_3}sech^2(\frac{\sqrt{h_2}}{2}\xi), h_0 = h_1 = h_4 = 0, h_2 > 0, \tag{23}$$

$$\phi_{16} = \sqrt{-\frac{h_2}{2h_4}}tanh(\sqrt{-h_2/2}\xi), h_1 = h_3 = 0, h_0 = \frac{h_2^2}{4h_4}, h_2 < 0, h_4 > 0 \tag{24}$$

Case F.Eq.(7) admits three Jacobi elliptic function solutions as follows

$$\phi_{17} = \sqrt{-\frac{h_2m^2}{h_4(2m^2-1)}}cn(\sqrt{\frac{h_2}{(2m^2-1)}}\xi), h_4 < 0, h_2 > 0, h_1 = h_3 = 0, h_0 = \frac{m^2h_2^2(m^2-1)}{h_4(2m^2-1)^2}, \tag{25}$$

$$\phi_{18} = \sqrt{-\frac{h_2m^2}{h_4(m^2+1)}}sn(\sqrt{-\frac{h_2}{(m^2+1)}}\xi), h_4 > 0, h_2 < 0, h_1 = h_3 = 0, h_0 = \frac{h_2m^2}{h_4(m^2+1)^2}, \tag{26}$$

$$\phi_{19} = \sqrt{-\frac{h_2}{h_4(2-m^2)}}dn(\sqrt{\frac{h_2}{(2-m^2)}}\xi), h_4 < 0, h_2 > 0, h_0 = \frac{h_2^2(1-m^2)}{h_4(m^2-2)^2}, h_1 = h_3 = 0 \tag{27}$$

As $m \to 1$,the Jacobi elliptic periodic solutions degernerates to hyperbolic functions,i.e. $sn(\xi) = tanh(\xi)$, $cn(\xi) = sech(\xi)$ and $dn(\xi) = sech(\xi)$. When $m \to 0$, the Jacobi elliptic periodic solutions degernerate to triangular functions, i.e. $sn(\xi) = sin(\xi)$, $cn(\xi) = cos(\xi)$ and $dn(\xi) = 1$.

Case G.Eq.(7) admits a Weierstrass elliptic function solution

$$\phi_{20} = \psi(\frac{\sqrt{h_3}}{2}\xi, g_2, g_3), h_2 = h_4 = 0, h_3 > 0 \tag{28}$$

where $g_2 = -\frac{4h_1}{h_3}$ and $g_3 = -\frac{4h_0}{h_3}$.

3 New applications

In this section,we will demonstrate our proposed approach on three nonlinear evolution equations arising in physics ,namely,generalized Hirota-Satsuma coupled KdV system,coupled Maccaris equations and generalized-Zakharov equations.

3.1 Example(1).Generalized Hirota-Satsuma coupled KdV system

Let us first consider the Hirota-Satsuma coupled KdV system [13]

$$u_t = \frac{1}{4}u_{xxx} + 3uu_x + 3(w - v^2)_x, \qquad v_t = -\frac{1}{2}v_{xxx} - 3uv_x, \qquad w_t = -\frac{1}{2}w_{xxx} - 3uw_x \tag{29}$$

When $w = 0$,Eqs.(29) reduce to be the well-known Hirota-Satsuma coupled KdV system.We seek travelling wave solutions for Eqs.(29) in the form

$$u(x,t) = u(\xi), v(x,t) = v(\xi), w(x,t) = w(\xi), \xi = k(x - ct) \tag{30}$$

Substituting Eq.(30) into (29) yields an ODE as

$$-cku^{'} = \frac{1}{4}k^3u^{'''} + 3kuu^{'} + 3k(w - v^2)^{'}, \tag{31}$$

$$-ckv^{'} = -\frac{1}{2}k^3v^{'''} - 3kuv^{'}, \tag{32}$$

$$-ckw^{'} = -\frac{1}{2}k^3w^{'''} - 3kuw^{'} \tag{33}$$

Let

$$u = \alpha v^2 + \beta_0 v + \gamma, w = A_0 v + B_0, \tag{34}$$

where α,γ,β_0,A_0 and B_0 are constants.Inserting Eq.(34) into (32) and (33) and integrating once we know that (33) and (34) give rise to the same equation

$$k^2v^{''} = -2\alpha v^3 - 3\beta_0 v^2 + 2(c - 3\gamma)v + k_1, \tag{35}$$

where k_1 is an integration constant.Integrating (35) once again we have

$$k^2v^{'2} = -\alpha v^4 - 2\beta_0 v^3 + 2(c - 3\gamma)v^2 + 2k_1 v + k_2, \tag{36}$$

where k_2 is an integration constant.By means of Eqs.(34-36) we get

$$\begin{aligned} k^2u^{''} &= 2\alpha k^2 v^{'2} + k^2(2\alpha v + \beta_0)v^{''} = 2\alpha[-\alpha v^4 - 2\beta_0 v^3 \\ &+2(c - 3\gamma)v^2 + 2k_1 v + k_2] + (2\alpha v + \beta_0)[-2\alpha v^3 - 3\beta_0 v^2 + 2(c - 3\gamma)v + k_1] \end{aligned} \tag{37}$$

Integrating (31) once we have

$$\frac{1}{4}k^2u^{''} + \frac{3}{2}u^2 + cu + 3(w - v^2) + k_3 = 0, \tag{38}$$

where k_3 is an integration constant.Inserting (34) and (37) into (38) gives

$$\begin{gathered} 3\alpha c - 3\alpha\gamma + \frac{3}{4}\beta_0^2 - 3 = 0, \qquad \frac{1}{2}[\alpha k_1 + \beta_0 c + \gamma\beta_0) + A_0 = 0, \\ \frac{1}{4}(2\alpha k_2 + \beta_0 k_1) + \frac{3}{2}\gamma^2 + c\gamma + 3B_0 + k_3 = 0 \end{gathered} \tag{39}$$

Let

$$k_1 = \frac{1}{2\alpha^2}[\beta_0^3 + 2c\alpha\beta_0 - 6\alpha\beta_0\gamma), \qquad v(\xi) = aP(\xi) - \frac{\beta_0}{2\alpha} \tag{40}$$

Therefore from Eq.(39),we have

$$k^2P^{''}(\xi)-a(\frac{3\beta_0^2}{2\alpha}+2c-6\gamma)P(\xi)+2\alpha a^3P^3(\xi)=0 \tag{41}$$

By virtue of the technique of solution,we assume that the solution of Eq.(41) in the series form is

$$P(\xi)=a_0+a_1\phi(\xi)+b_1\phi^{-1}(\xi)+c_1\phi^{'}(\xi)+d_1\phi^{-1}(\xi)\phi^{'}(\xi), \tag{42}$$

where $\phi(\xi)$ satisfies Eq.(7),a_0,a_1,b_1,c_1 and d_1 are constants to be later determined .Substituting (42) along with (8) into (41) and setting the coefficients of all powers of ϕ^i and $\phi^i\sqrt{\sum_{j=0}^{r}h_j\phi^j}$,we will get a system of algebraic equations for a_0,a_1,b_1,c_1 and d_1.Solving the system of algebraic equations with the aid of MAPLE,we determine the coefficients as :

Case(1):

$$\begin{aligned}&c=\frac{12\gamma\alpha+3\beta_0^2}{4\alpha},a_0=0,h_1=0,d_1=d_1,h_0=\frac{b_1^2}{d_1^2},h_4=\frac{a_1^2}{d_1^2},\\&h_2=-\frac{6a_1b_1}{d_1^2},c_1=0,h_3=0,b_1=b_1,a_1=a_1,k=2\sqrt{-\alpha a d_1}\end{aligned} \tag{43}$$

Case(2):

$$\begin{aligned}&c=\frac{12\gamma\alpha+3\beta_0^2}{4\alpha},a_0=-\frac{k^2h_1}{4\alpha a^3b_1},h_1=h_1,d_1=0,h_0=-\frac{\alpha a^3b_1^2}{k^2},h_4=\frac{-1}{4096}\frac{k^6h_1^4}{b_1^6a^9\alpha^3},\\&h_2=-\frac{15}{32}\frac{k^2h_1^2}{\alpha a^3b_1^2},c_1=0,h_3=\frac{3}{64}\frac{k^4h_1^3}{b_1^4a^6\alpha^2},b_1=b_1,a_1=\frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3},k=k\end{aligned} \tag{44}$$

Case(3):

$$\begin{aligned}&c=\frac{12\gamma\alpha+3\beta_0^2}{4\alpha},a_0=-\frac{1}{4}\frac{k^2h_1}{\alpha b_1a^3},h_1=h_1,d_1=0,h_0=-\frac{\alpha a^3b_1^2}{k^2},h_4=h_4,\\&h_2=\frac{3}{8}\frac{k^2h_1^2}{\alpha a^3b_1^2},c_1=0,h_3=\frac{k^4h_1^3}{48b_1^4a^6\alpha^2},b_1=b_1,a_1=0,k=k\end{aligned} \tag{45}$$

From Case(1),substituting Eq.(43) into (34) and (40),we have a new exact travelling wave solutions fo Eqs.(29) as follows

$$\begin{aligned}&v(\xi)=a[a_1\phi(\xi)+\frac{b_1}{\phi(\xi)}+\frac{d_1\phi^{'}(\xi)}{\phi(\xi)}]-\frac{\beta_0}{2\alpha},\\&u(\xi)=\alpha[a[a_1\phi(\xi)+\frac{b_1}{\phi(\xi)}+\frac{d_1\phi^{'}(\xi)}{\phi(\xi)}]-\frac{\beta_0}{2\alpha}]^2+\beta_0[a[a_1\phi(\xi)+\frac{b_1}{\phi(\xi)}+\frac{d_1\phi^{'}(\xi)}{\phi(\xi)}]+\gamma,\\&w(\xi)=A_0[a[a_1\phi(\xi)+\frac{b_1}{\phi(\xi)}+\frac{d_1\phi^{'}(\xi)}{\phi(\xi)}]-\frac{\beta_0}{2\alpha}]+B_0\end{aligned} \tag{46}$$

Knowing Case (1) with Eqs.(46),admits to as series of travelling wave solutions of Eqs.(29):If $h_1=h_2=h_3=h_4=0$, $h_2>0$, admits to polynomial solution

$$v_1(\xi)=a[a_1\sqrt{h_0\xi}+\frac{b_1}{\sqrt{h_0\xi}}+\frac{d_1\frac{h_0}{2\sqrt{h_0\xi}}}{\sqrt{h_0\xi}}]-\frac{\beta_0}{2\alpha},u_1(\xi)=\alpha[v_1(\xi)]^2+\beta_0[v_1(\xi)]+\gamma,w_1(\xi)=A_0[v_1(\xi)]+B_0 \tag{47}$$

If $h_0=h_3=h_4=0$, $h_2<0$, admits to a triangular solution

$$\begin{aligned}&v_2(\xi)=a[a_1(\frac{-h_1}{2h_2}+\frac{h_1}{2h_2}sin(\sqrt{-h_2}\xi))+\frac{b_1}{-\frac{h_1}{2h_2}+\frac{h_1}{2h_2}sin(\sqrt{-h_2}\xi)}+(\frac{h_1}{2h_2})\frac{\sqrt{-h_2}d_1cos(\sqrt{-h_2}\xi)}{-\frac{h_1}{2h_2}+\frac{h_1}{2h_2}sin(\sqrt{-h_2}\xi)}]-\frac{\beta_0}{2\alpha},\\&u_2(\xi)=\alpha[v_2(\xi)]^2+\beta_0[v_2(\xi)]+\gamma,\qquad w_2(\xi)=A_0[v_2(\xi)]+B_0\end{aligned} \tag{48}$$

If $h_0 = h_3 = h_1 = 0$, $h_2 < 0$,$h_4 > 0$,admits to a triangular solution

$$v_3(\xi) = a[a_1\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi) + \frac{b_1}{\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi)} + \frac{d_1\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi)tan(\sqrt{-h_2}\xi)\sqrt{-h_2}}{\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi)}] - \frac{\beta_0}{2\alpha}, \tag{49}$$
$$u_3(\xi) = \alpha[v_3(\xi)]^2 + \beta_0[v_3(\xi)] + \gamma, \qquad w_3(\xi) = A_0[v_3(\xi)] + B_0$$

When $h_1 = h_3 = h_4 = 0$, $h_0 > 0$,$h_2 < 0$,admits to a triangular solution

$$v_4(\xi) = a[a_1\sqrt{\frac{-h_0}{h_2}}sin(\sqrt{-h_2}\xi) + \frac{b_1}{\sqrt{\frac{-h_0}{h_2}}sin(\sqrt{-h_2}\xi)} + \frac{d_1\sqrt{-h_2}\sqrt{\frac{-h_0}{h_2}}cos(\sqrt{-h_2}\xi)}{\sqrt{\frac{-h_0}{h_2}}sin(\sqrt{-h_2}\xi)}] - \frac{\beta_0}{2\alpha}, \tag{50}$$
$$u_4(\xi) = \alpha[v_4(\xi)]^2 + \beta_0[v_4(\xi)] + \gamma, \qquad w_4(\xi) = A_0[v_4(\xi)] + B_0$$

When $h_0 = h_1 = h_3 = 0$, $h_0 > 0$,$h_4 < 0$,admits to a hyperbolic solution

$$v_5(\xi) = a[a_1\sqrt{-\frac{h_2}{h_4}}sech(\sqrt{h_2}\xi) + \frac{b_1}{\sqrt{-\frac{h_2}{h_4}}sech(\sqrt{h_2}\xi)} + \frac{d_1\sqrt{h_2}\sqrt{-\frac{h_2}{h_4}}sech(\sqrt{h_2}\xi)tanh(\sqrt{h_2}\xi)}{\sqrt{-\frac{h_2}{h_4}}sech(\sqrt{h_2}\xi)}] - \frac{\beta_0}{2\alpha}, \tag{51}$$
$$u_5(\xi) = \alpha[v_5(\xi)]^2 + \beta_0[v_5(\xi)] + \gamma, \qquad w_5(\xi) = A_0[v_5(\xi)] + B_0$$

From Case(2),inserting Eq.(44) into (34) and (40),admits to the new exact traveling wave solutions fo Eqs.(29) as follows

$$v(\xi) = a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\phi(\xi) + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}, \tag{52}$$

$$u(\xi) = \alpha[a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\phi(\xi) + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}]^2 + \beta_0[a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\phi(\xi) + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}] + \gamma, \tag{53}$$

$$w(\xi) = A_0[a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\phi(\xi) + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}] + B_0 \tag{54}$$

With the aid of Case (2) with Eqs.(54),admits as series of traveling wave solutions of Eqs.(29) as follows:If $h_0 = h_1 = h_3 = 0$, $h_4 \neq 0$, admits to exponential solution

$$v_6(\xi) = a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}(\frac{h_3}{2h_4})exp(\frac{h_3}{2\sqrt{-h_4}}\xi) + \frac{b_1}{\frac{h_3}{2h_4}exp(\frac{h_3}{2\sqrt{-h_4}}\xi)}] - \frac{\beta_0}{2\alpha}, \tag{55}$$
$$u_6(\xi) = \alpha[v_6(\xi)]^2 + \beta_0[v_6(\xi)] + \gamma, \qquad w_6(\xi) = A_0[v_6(\xi)] + B_0$$

If $h_1 = h_3 = h_4 = 0$,$h_0 > 0$,$h_2 > 0$, admits to a triangular solution

$$v_7(\xi) = a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\sqrt{\frac{-h_0}{h_1}}sin(\sqrt{-h_2}\xi) + \frac{b_1}{\sqrt{\frac{-h_0}{h_1}}sin(\sqrt{-h_2}\xi)}] - \frac{\beta_0}{2\alpha}, \tag{56}$$
$$u_7(\xi) = \alpha[v_7(\xi)]^2 + \beta_0[v_7(\xi)] + \gamma, \qquad w_7(\xi) = A_0[v_7(\xi)] + B_0$$

If $h_0 = h_1 = h_3 = h_4 = 0$, $h_4 > 0$, admits to rational solution

$$v_8(\xi) = a[-\frac{k^2h_1}{4\alpha a^3b_1} - \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\frac{1}{\sqrt{h_4}}\xi + \frac{b_1}{\frac{1}{\sqrt{h_4}}\xi}] - \frac{\beta_0}{2\alpha}, \qquad u_8(\xi) = \alpha[v_8(\xi)]^2 + \beta_0[v_8(\xi)] + \gamma, \tag{57}$$
$$w_8(\xi) = A_0[v_8(\xi)] + B_0$$

If $h_2 = h_4 = 0$, $h_3 > 0$,we have admits to Weierstrass elliptic solution

$$v_9(\xi) = a[-\frac{k^2h_1}{4\alpha a^3b_1} + \frac{1}{64}\frac{k^4h_1^2}{\alpha^2a^6b_1^3}\psi(\frac{\sqrt{h_3}}{2}\xi, g_2, g_3) + \frac{b_1}{\psi(\frac{\sqrt{h_3}}{2}\xi, g_2, g_3)}] - \frac{\beta_0}{2\alpha}, \tag{58}$$
$$u_9(\xi) = \alpha[v_9(\xi)]^2 + \beta_0[v_9(\xi)] + \gamma, \qquad w_9(\xi) = A_0[v_9(\xi)] + B_0$$

When $h_0 = h_4 = h_1 = 0$,$h_2 > 0$,we have to a hyperbolic solution

$$\begin{aligned} v_{10}(\xi) &= a[-\frac{k^2h_1}{4\alpha a^3 b_1} - \frac{1}{64}\frac{k^4h_1^2}{\alpha^2 a^6 b_1^3}(\frac{h_2}{h_3})sech^2(\frac{\sqrt{h_2}}{2}\xi) - \frac{b_1}{(\frac{h_2}{h_3})sech^2(\frac{\sqrt{h_2}}{2}\xi)}] - \frac{\beta_0}{2\alpha}, \\ u_{10}(\xi) &= \alpha[v_{10}(\xi)]^2 + \beta_0[v_{10}(\xi)] + \gamma, \qquad w_{10}(\xi) = A_0[v_{10}(\xi)] + B_0 \end{aligned} \tag{59}$$

Proceeding as before,inserting Eq.(45)into (34) and (40),we have to the new exact travelling wave solutions fo Eqs.(29) as follows

$$\begin{aligned} v(\xi) &= a[-\frac{k^2h_1}{4\alpha b_1 a^3} + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}, \qquad w(\xi) = A_0[a[-\frac{1}{4}\frac{k^2h_1}{\alpha b_1 a^3} + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}] + B_0, \\ u(\xi) &= \alpha[a[-\frac{1}{4}\frac{k^2h_1}{\alpha b_1 a^3} + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}]^2 + \beta_0[a[-\frac{1}{4}\frac{k^2h_1}{\alpha b_1 a^3} + \frac{b_1}{\phi(\xi)}] - \frac{\beta_0}{2\alpha}] + \gamma, \end{aligned} \tag{60}$$

With the aid of Case (3) with Eqs.(60),admits as series of traveling wave solutions of Eqs.(29) as follows:If $h_1 = h_3 = h_4 = 0$, $h_0 > 0$,$h_2 > 0$,we have to a hyperbolic solution

$$\begin{aligned} v_{11}(\xi) &= a[-\frac{1}{4}\frac{k^2h_1}{\alpha b_1 a^3} + \frac{b_1}{\sqrt{\frac{h_0}{h_2}}sinh(\sqrt{h_2}\xi)}] - \frac{\beta_0}{2\alpha}, \qquad u_{11}(\xi) = \alpha[v_{11}(\xi)]^2 - \beta_0[v_{11}(\xi)] + \gamma, \\ w_{11}(\xi) &= A_0[v_{11}(\xi)] + B_0, \end{aligned} \tag{61}$$

if $h_0 = h_1 = h_4 = 0$, $h_2 < 0$, admits to a triangular solution

$$\begin{aligned} v(\xi) &= a[-\frac{k^2h_1}{4\alpha b_1 a^3} - \frac{b_1}{(\frac{h_2}{h_3})sec^2(\frac{\sqrt{-h_2}}{2}\xi)}] - \frac{\beta_0}{2\alpha}, \qquad u_{12}(\xi) = \alpha[v_{12}(\xi)]^2 - \beta_0[v_{12}(\xi)] + \gamma, \\ w_{12}(\xi) &= A_0[v_{12}(\xi)] + B_0, \qquad \xi = k(x - ct) \end{aligned} \tag{62}$$

3.2 Example(2).The coupled Maccaris equations

A second instructive model is the coupled Maccaris equations [21]

$$iQ_t + Q_{xx} + QR = 0, \qquad R_t + R_y + (|Q|^2)_x = 0. \tag{63}$$

In order to seek exact solutions of Eqs.(63),we suppose

$$Q(x, y, t) = u(x, y, t)exp[i(kx + \alpha y + \lambda t + l)], \tag{64}$$

where k,α and λ are constants to be later determined,l is an arbitrary constant.Substituting Eq.(64) into Eqs.(63) and yields the following PDEs

$$i(u_t + 2ku_x) + u_{xx} - (\lambda + k^2)u + uR = 0, \tag{65}$$

$$R_t + R_y + (u^2)_x = 0 \tag{66}$$

Using the transformation, we have

$$u = u(\xi), R = R(\xi), \xi = w(x + \beta y - 2kt + x_0), \tag{67}$$

where β and w are constants to be determined later,x_0 is an arbitrary constant,Eqs.(65) and (66) become the following ODEs

$$w^2u - (\lambda + k^2)u + uR = 0, \tag{68}$$

$$(\beta - 2k)R^{'} + (u^2)^{'} = 0, \tag{69}$$

where prime denotes the differential with respect to ξ.Integrating Eq.(69) with respect to ξ and taking the integration constant as zero yields

$$R = -\frac{1}{\beta - 2k}u^2(\xi) \tag{70}$$

Substituting Eq.(70) into (68),yields

$$\beta^2 u^{''}(\xi) - (\lambda + k^2)u(\xi) - \frac{1}{\beta - 2k}u^3(\xi) = 0, \tag{71}$$

According to the proposed method,balancing the term $u^3(\xi)$ with term $u^{''}(\xi)$ in Eq.(71) leads to $n = 1$.Therefore,we have

$$u(\xi) = a_0 + a_1\phi(\xi) + b_1\phi^{-1}(\xi) + c_1\phi^{'}(\xi) + d_1\phi^{-1}(\xi)\phi^{'}(\xi), \tag{72}$$

where a_0,a_1,b_1,c_1,d_1,c and k are constants to be later determined.Substituting (72) along with (8) into (71) and collecting coefficients of ϕ^i and $\phi^i\sqrt{\sum_{j=0}^{r} h_j\phi^j}$,then seting each coefficients to zero to derive a set of algebraic equations for a_0,a_1,b_1,c_1 and d_1.Solving the system of algebraic equations with the aid of MAPLE,we obtain three different cases of the coefficients as follows

Case(1):

$$\begin{aligned} a_0 &= \frac{h_1\beta^2(\beta - 2k)}{2b_2}, h_2 = 0, d_1 = 0, h_3 = \frac{h_1^3\beta^4(\beta^2 - 4k\beta + 4k^2)}{12b_1^4}, \\ k &= k, b_1 = b_1, , c_1 = 0, , h_0 = 0, a_1 = 0, \lambda = -k^2, , h_1 = h_1, h_4 = h_4, \end{aligned} \tag{73}$$

Case(2):

$$\begin{aligned} c_1 &= c_1, a_1 = a_1, h_1 = 0, h_4 = h_4, k = \frac{3\beta^3 h_4 - 4a_1^2}{6\beta^2 h_4}, b_1 = 0, h_2 = \frac{5a_1^2}{c_1^2}, d_1 = 0, h_3 = 0, a_0 = 0, \\ h_0 &= \frac{40a_1^4}{3h_4c_1^4}, \lambda = \frac{180\beta^6 a_1 h_4^2 + 9c_1\beta^6 h_4^2 - 24c_1a_1^2h_4\beta^3 + 16c_1a_1^4}{36\beta^4 c_1 h_4^2} \end{aligned} \tag{74}$$

Case(3):

$$\begin{aligned} c_1 &= 0, a_1 = a_1, h_1 = 0, h_0 = 0, k = \frac{\beta^3 - 2d_1^2}{2\beta^2}, h_4 = \frac{a_1^2}{d_1^2}, b_1 = 0, h_2 = 0, d_1 = d_1, \\ h_3 &= 0, a_0 = 0, \lambda = \frac{\beta^6 - 4d_1^2\beta^3 + 4d_1^4}{4\beta^4} \end{aligned} \tag{75}$$

With the aid of Case(1),inserting Eq.(73) into (70) and (64),we have a new exact traveling wave solutions fo Eqs.(63) as follows

$$u(\xi) = [\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{\phi(\xi)}], \qquad Q(\xi) = [\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{\phi(\xi)}]e^{i(kx+\alpha y+\lambda t+l)}, \tag{76}$$

$$R(\xi) = -\frac{1}{\beta - 2k}[\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{\phi(\xi)}]^2 \tag{77}$$

With the aid of Case (1) with Eqs.(70) and (64),admits as a series of travelling wave solutions of Eqs.(63) as follows:If $h_0 = h_2 = h_1 = 0$,$h_4 < 0$, admits to an exponential solution

$$\begin{aligned} u_1(\xi) &= [\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{\frac{h_3}{2h_4}exp(\frac{h_3}{2\sqrt{-h_4}}\xi)}], Q_1(\xi) = [\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{\frac{h_3}{2h_4}exp(\frac{h_3}{2\sqrt{-h_4}}\xi)}]e^{i(kx+\alpha y+\lambda t+l)}, \\ R_1(\xi) &= -\frac{1}{\beta - 2k}[\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{u_1(\xi)}]^2 \end{aligned} \tag{78}$$

If $h_0 = h_1 = h_2 = 0$,we have a rational solution

$$\begin{aligned} u_2(\xi) &= [\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{-\frac{4h_3}{h_3^2\xi^2 - 4h_4}}], \qquad Q_2(\xi) = [\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{-\frac{4h_3}{h_3^2\xi^2 - 4h_4}}]e^{i(kx+\alpha y+\lambda t+l)}, \\ R_2(\xi) &= -\frac{1}{\beta - 2k}[\frac{h_1\beta^2(\beta - 2k)}{2b_1} + \frac{b_1}{u_2(\xi)}]^2 \end{aligned} \tag{79}$$

By using Case(2),inserting Eq.(74) into (70) and (64),we have a new exact traveling wave solutions for Eqs.(63) as follows

$$u(\xi) = [a_1\phi(\xi) + c_1\phi^{'}(\xi)], \qquad Q(\xi) = [a_1\phi(\xi) + c_1\phi^{'}(\xi)]e^{i(kx+\alpha y+\lambda t+l)}, \tag{80}$$

$$R(\xi) = -\frac{1}{\beta - 2k}[a_1\phi(\xi) + c_1\phi^{'}(\xi)]^2 \tag{81}$$

From Case (2),Eqs.(80) and (81),yields a series of travelling wave solutions of Eqs.(63) as :If $h_1 = h_3 = h_4 = 0$,$h_0 > 0$,$h_2 > 0$, admits to a hyperbolic solution

$$\begin{aligned} u_3(\xi) &= [a_1\sqrt{\frac{h_0}{h_2}}sinh(\sqrt{h_2}\xi) + c_1\sqrt{\frac{h_0}{h_2}}sinh(\sqrt{h_2}\xi)\sqrt{h_2}cosh(\sqrt{h_2}\xi)], \\ Q_3(\xi) &= [a_1\sqrt{\frac{h_0}{h_2}}sinh(\sqrt{h_2}\xi) + c_1\sqrt{\frac{h_0}{h_2}}sinh(\sqrt{h_2}\xi)\sqrt{h_2}cosh(\sqrt{h_2}\xi)]e^{i(kx+\alpha y+\lambda t+l)}, \end{aligned} \tag{82}$$

$$R_3(\xi) = -\frac{1}{\beta - 2k}[u_3(\xi)]^2 \tag{83}$$

If $h_1 = h_3 = h_4 = 0$,$h_0 > 0$,$h_2 < 0$,admits to a triangular solution

$$\begin{aligned} u_4(\xi) &= [a_1\sqrt{\frac{h_0}{h_2}}sin(\sqrt{h_2}\xi) + c_1\sqrt{\frac{h_0}{h_2}}sin(\sqrt{h_2}\xi)\sqrt{h_2}cos(\sqrt{h_2}\xi)], \\ Q_4(\xi) &= [a_1\sqrt{\frac{h_0}{h_2}}sin(\sqrt{h_2}\xi) + c_1\sqrt{\frac{h_0}{h_2}}sin(\sqrt{h_2}\xi)\sqrt{h_2}cos(\sqrt{h_2}\xi)]e^{i(kx+\alpha y+\lambda t+l)}, \end{aligned} \tag{84}$$

$$R_4(\xi) = -\frac{1}{\beta - 2k}[u_4(\xi)]^2 \tag{85}$$

By using Case(3),with Eqs.(70) and (74),we have a new exact traveling wave solutions fo Eqs.(63) as

$$u(\xi) = [a_1\phi(\xi) + \frac{d_1\phi^{'}(\xi)}{\phi(\xi)}], \qquad Q(\xi) = [a_1\phi(\xi) + \frac{d_1\phi^{'}(\xi)}{\phi(\xi)}]e^{i(kx+\alpha y+\lambda t+l)}, \tag{86}$$

$$R(\xi) = -\frac{1}{\beta - 2k}[a_1\phi(\xi) + \frac{d_1\phi^{'}(\xi)}{\phi(\xi)}]^2, \tag{87}$$

From Case (3),If $h_0 = h_1 = h_2 = h_3 = 0$,$h_4 > 0$, admits to a rational solution

$$u_5(\xi) = [-a_1\frac{1}{\sqrt{h_4}}\xi + \frac{d_1\frac{1}{\sqrt{h_4}\xi^2}}{-\frac{1}{\sqrt{h_4}}\xi}], \qquad Q_5(\xi) = [-a_1\frac{1}{\sqrt{h_4}\xi} + \frac{d_1\frac{1}{\sqrt{h_4}\xi^2}}{-\frac{1}{\sqrt{h_4}\xi}}]e^{i(kx+\alpha y+\lambda t+l)}, \tag{88}$$

$$R_5(\xi) = -\frac{1}{\beta - 2k}[u_5(\xi)]^2, \qquad \xi = w(x + \beta y - 2kt + x_0) \tag{89}$$

3.3 Example(3).The generalized-Zakharov equations

The generalized Zakharov equations for the complex envelope $\psi(x,t)$ of the high-frequency wave and the real low-frequency field $v(x,t)$ in the form [14]

$$i\psi_t + \psi_{xx} - 2\lambda|\psi|^2\psi + 2\psi v = 0, \tag{90}$$

$$v_{tt} - v_{xx} + (|\psi|^2)_{xx} = 0, \tag{91}$$

where the cubic term in Eq.(90) describes the nonlinear-self interaction in the high frequency subsystem, such a term corresponds to a self-focusing effect in plasma physics. The coefficient λ is a real constant that can be a postive or negative number.Let us assume the traveling wave solution of Eqs.(90) and (91) in the form

$$\psi(x,t) = e^{i\eta}u(\xi), v = v(\xi), \qquad \eta = \alpha x + \beta t, \xi = k(x - 2\alpha t), \tag{92}$$

where $u(\xi)$ and $v(\xi)$ are real functions,the constants α,β and k are to be determined.Substituting (92) into Eqs.(90) and (91),we have

$$k^2u^{''} + 2uv - (\alpha^2 + \beta)u - 2\lambda u^3 = 0, \tag{93}$$

$$k^2(4\alpha^2 - 1)v^{''} + k^2(u^2)^{''} = 0 \tag{94}$$

In order to simplify ODEs (93) and (94),integrating Eq.(94) once and taking integration constant as zero, we integrate, which yields

$$v(\xi)=\frac{u^2}{(1-4\alpha^2)}+C_0, if \alpha^2\neq\frac{1}{4}, \tag{95}$$

where C_0-integration constant.Inserting Eq.(95) into (93),we have

$$k^2u^{''}+[2C-\alpha^2-\beta]u+2[\frac{1}{1-4\alpha^2}-\lambda]u^3=0 \tag{96}$$

Considering the homogeneous balance in Eq.(96),yields $n=1$.Thus the ansatz solutions of Eq.(96) can be expressed by

$$u(\xi)=a_0+a_1\phi(\xi)+b_1\phi^{-1}(\xi)+c_1\phi^{'}(\xi)+d_1\phi^{-1}(\xi)\phi^{'}(\xi) \tag{97}$$

Proceeding as before,substituting (97) and inserting (8) into (96) and using MAPLE,yields as set of algebraic equations for a_0,a_1,b_1,c_1,d_1,k and λ.Solving the system of algebraic equations,we have

$$\begin{aligned}
&h_0=0, b_1=0, h_2=-\frac{(\alpha^2-2C_0+\beta)^2}{3k^4}, c_1=0,\\
&h_1=\frac{4(32\lambda\alpha^4+32\lambda\beta\alpha^2+8\alpha^2-9\lambda\alpha^2-64\lambda C_0\alpha^2+8\beta-16C_0+18\lambda C_0-9\lambda)}{k^2\lambda},\\
&d_1=\sqrt{-\frac{1-4\alpha^2}{4-4\lambda+16\lambda\alpha^2}}k, a_0=\frac{-\alpha^2+4\beta\alpha^2+2C_0-8C_0\alpha^2+4\alpha^2-\beta}{12(1-\lambda+4\lambda\alpha^2)\sqrt{-\frac{1-4\alpha^2}{4-4\lambda+16\lambda\alpha^2}}k},\\
&h_4=\frac{(\beta^2+2\beta\alpha^2-4C_0\beta+\alpha^4-4C_0\alpha^2+4C_0^2)^2\lambda^2}{46656k^8(1-\lambda+4\lambda\alpha^2)^2},\\
&a_1=\frac{(\beta^2+2\beta\alpha^2-4C_0\beta+\alpha^4-4C_0\alpha^2+4C_0^2)\sqrt{-\frac{1-4\alpha^2}{4-4\lambda+16\lambda\alpha^2}}\lambda}{k^3(1-\lambda+4\lambda\alpha^2)},\\
&h_3=-\frac{(\alpha^2-2C_0+\beta)(\beta^2+2\beta\alpha^2-4C_0\beta+\alpha^4-4\alpha^2C_0+4C_0^2)\lambda}{108(1-\lambda+4\lambda\alpha^2)k^6}
\end{aligned} \tag{98}$$

Inserting Eq.(98) into Eqs.(92) and (95),admits to a new exact traveling wave solution as follows

$$\begin{aligned}
u(\xi)=[&\frac{-\alpha^2+4\beta\alpha^2+2C_0-8C_0\alpha^2+4\alpha^2-\beta}{12(1-\lambda+4\lambda\alpha^2)\sqrt{-\frac{1-4\alpha^2}{4-4\lambda+16\lambda\alpha^2}}k}\\
&+\frac{(\beta^2+2\beta\alpha^2-4C_0\beta+\alpha^4-4C_0\alpha^2+4C_0^2)\sqrt{-\frac{1-4\alpha^2}{4-4\lambda+16\lambda\alpha^2}}\lambda}{k^3(1-\lambda+4\lambda\alpha^2)}\phi(\xi)+\sqrt{-\frac{1-4\alpha^2}{4-4\lambda+16\lambda\alpha^2}}k\frac{\phi^{'}(\xi)}{\phi(\xi)}],
\end{aligned} \tag{99}$$

$$\psi(\xi)=u(xi)e^{i(\alpha x+\beta t)}, \tag{100}$$

$$v(\xi)=\frac{u^2(\xi)}{1-4\alpha^2}+C_0, \qquad \xi=k(x-2\alpha t) \tag{101}$$

From Eqs.(99) and (100),we obtain a series of a new exact traveling wave solutions as follows:If $h_0=h_3=h_4=0$,$h_2<0$, admits to a triangular solution

$$\begin{aligned}
&u_1(\xi)=[a_0+a_1(\frac{-h_1}{2h_2}+\frac{h_1}{2h_2}sin(\sqrt{-h_2}\xi))+d_1[-\frac{h_1}{2h_2}+\frac{h_1}{2h_2}sin(\sqrt{-h_2}\xi)]^{-1}\sqrt{-h_2}(\frac{h_1}{2h_2})cos(\sqrt{-h_2}\xi)],\\
&\psi_1(\xi)=u_1(\xi)e^{i(\alpha x+\beta t)}, \qquad v_1(\xi)=\frac{u_1^2(\xi)}{1-4\alpha^2}+C_0
\end{aligned} \tag{102}$$

If $h_0=h_3=h_4=0$,$h_2>0$, admits to a hyperbolic solution

$$\begin{aligned}
u_2(\xi)=a_0+a_1(\frac{-h_1}{2h_2}+\frac{h_1}{2h_2}sinh(2\sqrt{h_2}\xi))+d_1[\frac{-h_1}{2h_2}+\frac{h_1}{2h_2}sinh(2\sqrt{h_2}\xi)]^{-1}\sqrt{h_2}(\frac{h_1}{2h_2})\\
sinh(2\sqrt{h_2}\xi)cosh(2\sqrt{h_2}\xi), \qquad \psi_2(\xi)=u_2(\xi)e^{i(\alpha x+\beta t)}, \qquad v_2(\xi)=\frac{u_2^2(\xi)}{1-4\alpha^2}+C_0
\end{aligned} \tag{103}$$

If $h_0 = h_1 = h_4 = 0$,$h_2 > 0$,admits to a hyperbolic solution

$$\begin{aligned} u_3(\xi) &= a_0 - a_1\frac{h_2}{h_3}sech^2(\frac{\sqrt{h_2}}{2}\xi) + d_1[-\frac{h_2}{h_3}sech^2(\frac{\sqrt{h_2}}{2}\xi)]^{-1}\frac{h_2^{3/2}}{h_3}sech^2(\frac{\sqrt{h_2}}{2}\xi)tanh(\frac{\sqrt{h_2}}{2}\xi), \\ \psi_3(\xi) &= u_3(\xi)e^{i(\alpha x+\beta t)}, \qquad v_3(\xi) = \frac{u_3^2(\xi)}{1-4\alpha^2} + C_0 \end{aligned} \tag{104}$$

If $h_0 = h_1 = h_2 = 0$,$h_4 < 0$,admits to a exponentional solution

$$\begin{aligned} u_4(\xi) &= a_0 + a_1(\frac{h_3}{2h_4})exp(\frac{h_3}{2\sqrt{-h_4}}\xi) + d_1[\frac{h_3}{2h_4}exp(\frac{h_3}{2\sqrt{-h_4}}\xi)]^{-1}\frac{-h_3^2}{4h_4\sqrt{-h_4}\xi^2}exp(\frac{h_3}{2\sqrt{-h_4}}\xi) \\ \psi_4(\xi) &= u_4(\xi)e^{i(\alpha x+\beta t)}, \qquad v_4(\xi) = \frac{u_4^2(\xi)}{1-4\alpha^2} + C_0 \end{aligned} \tag{105}$$

If $h_0 = h_1 = h_2 = 0$,$h_4 < 0$,$h_2 < 0$,admits to a triangular solution

$$\begin{aligned} u_5(\xi) &= a_0 + a_1\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi) + d_1[\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi)]^{-1}\sqrt{-h_2}\sqrt{\frac{-h_2}{h_4}}sec(\sqrt{-h_2}\xi)tan(\sqrt{-h_2}\xi), \\ \psi_5(\xi) &= u_5(\xi)e^{i(\alpha x+\beta t)}, \qquad v_5(\xi) = \frac{u_5^2(\xi)}{1-4\alpha^2} + C_0 \end{aligned} \tag{106}$$

4 Conclusions

An extended algebraic method with a computerized symbolic computation has been proposed to obtain new exact solutions to three nonlinear evolution equations arising in nonlinear mathematical physics.The validity of this method has been tested by applying it successfully to generalized Hirota-Satsuma coupled KdV system,coupled Maccaris equations and generalized-Zakharov equations.As a results,the exact traveling wave solution are obtained include soliton solutions,rational solutions,triangular periodic solutions,Jacobi and Weierstrass doubly periodic wave solutions.Among them,the Jacobi elliptic periodic wave solutions exactly degenerate to the soliton solutions at a certain limit condition.It is worthwhile to mention that the method is straightforward and concise,and it can also be applied to other nonlinear evolution equations in physics.This is our task in future.

Acknowledgements

The author would like to express his sincere appreciations to Prof.Dr.S.A.El-Wakil for his continuous encouragement.

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