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New Explicit Exact Solutions of Nonlinear Evolution Equations Using the Generalized Auxiliary Equation Method Combined with Exp-function Method

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Abstract: In this paper, we utilize the Exp-function method with symbolic computational system Maple to constructing generalized solitary solutions of auxiliary equation. With the aid of auxiliary equation and its generalized solitary solutions, new exact solutions with three arbitrary functions of two nonlinear evolution equations in physics, namely, higher-order nonlinear Schrodinger equation describing propagation of ultrashort pulses in nonlinear optical fibers and generalized Zakharov equations are obtained. It is shown that the Exp-function method provides a straightforward and important mathematical tool for nonlinear evolution equations arising in physics.

Keywords: Exp-function method;Generalized auxiliary equation method;Nonlinear evolution equations;New exact solutions

1 Introduction

The investigation of exact solutions of nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena and gradually becomes one of the most important and significant tasks. In the past several decades, many effective methods for obtaining exact solutions of NLEEs have been presented $[1-30]$.

In recent years, the direct search for exact solutions of PDEs has become more and more attractive partly due to the availability of computer symbolic systems like Maple or Mathematica, which allows us to perform the complicated and tedious algebraic calculations on computer.

Very recently, He and Abdou [22], Abdou $[23-25]$ proposed a straightforward and concise method, called Exp-function method, to obtain generalized solitary solutions and periodic solutions of NLEEs. The solution procedure of this method, by the help of Maple, is of utter simplicity and this method can be easily extended to other nonlinear evolution equations.

The Exp-function is more general than the sinh-function and the tanh-function, so we can found more general solutions in the Exp-function method. The solution procedure, using Matab or Mathematica, is of utter simplicity. The Exp-function method can be employed in both the straightforward way and the subequation way. But we suggest that it is better to use this method directly, not only for its convenience, but also because it is sometimes possible to lose some information and solutions if we apply it in the subequation way. The Exp-function method is more convenient and effective than the extended Fan sub-equation method.

In the present paper is to extend the Exp-function method $[22-26]$ to generalized auxiliary equation. We consider the generalized auxiliary equation

$$\phi^{'2}(\xi) = E + P\phi^4(\xi) + R\phi^3(\xi) + Q\phi^2(\xi), \tag{1}$$

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where P, Q, R and E are constants to be determined later. Then employ the generalized auxiliary Eq.(1) and its generalized solitary solutions to find new and more general exact solutions of two nonliner evolution equations, namely, the higher-order nonlinear Schrodinger equation and generalized Zakharov equations [30].

2 Exp-function method for generalized auxiliary equation

By introducing a complex variation η defined as $\eta = k\xi + w$. Then Eq.(1) reduces to

$$k\phi^{''} - P\phi^3 - Q\phi - \frac{3}{2}R\phi^2 = 0, \tag{2}$$

where the prime denotes the derivative with respect to η. According to the Exp-function method, we assume that the solution of Eq.(2) can be expressed as

$$\phi(\eta) = \frac{\sum_{n=-c}^{d} a_n exp(n\eta)}{\sum_{m=-f}^{g} b_m exp(m\eta)}, \tag{3}$$

where c, d, f and g are positive integrs which are unknown to be determined later, a_n and b_m are unknown constants. Eq.(3) can be re-written as

$$\phi(\eta) = \frac{a_c exp(c\eta) + \cdots + a_{-d} exp(-d\eta)}{b_f exp(f\eta) + \cdots + b_{-g} exp(-g\eta)} \tag{4}$$

To determine values of c and f, we balance the linear term of the highest order in Eq.(2) with the highest order nonlinear term $\phi^{''}$ and ϕ^3, we have

$$\phi^{''} = \frac{c_1 exp[(2f + 3c)\eta] + ...}{c_2 exp[5f\eta] + ...}, \tag{5}$$

$$\phi^3 = \frac{c_3 exp[(c + 4f)\eta] + \cdots}{c_4 exp[5f\eta] + ...}, \tag{6}$$

where c_i are coefficients for simplicity.By balancing highest order of Exp-function in Eqs.(6) and (5), we have

$$3c + 2f = c + 4f, \tag{7}$$

which leads to the results $f = c$.Proceeding the same manner as illustrated above, we can determine values of d and g.Balancing the linear term of lowest order in Eq.(2)

$$\phi^{''} = \frac{d_1 exp[-(2g + 3d)\eta] + \cdots}{d_2 exp[-5g\eta] + ...}, \tag{8}$$

$$\phi^3 = \frac{d_3 exp[-(4g + d)\eta] + \cdots}{d_4 exp[-2g\eta] + ...}, \tag{9}$$

where d_i are coefficients for simplicity.By balancing highest order of Exp-function in Eqs.(8) and (9), we have

$$-(4g + d) = -(2g + 3d), \tag{10}$$

which leads to the result $g = d$.We can freely choose the values of c and d, but the final solution does not strongly depend upon the choice of values of c and d [22].For simplicity, we set $f = c = 1$ and $d = g = 1$, then Eq.(4) becomes

$$\phi(\eta) = \frac{a_1 exp(\eta) + a_0 + a_{-1} exp(-\eta)}{b_1 exp(\eta) + b_0 + b_{-1} exp(-\eta)} \tag{11}$$

Substituting Eq.(11) into Eq.(2), we have

$$[C_2exp(2\eta)+C_1exp(\eta)+C_0+C_{-1}exp(-\eta)+C_{-2}exp(-2\eta)+C_{-3}exp(-3\eta)+C_3exp(3\eta)]=0, \quad (12)$$

$$C_3=-Pa_1^3-Qa_1b_1^2-(3/2)Ra_1^2b_1,$$
$$C_{-3}=-Pa_{-1}^3-Qa_{-1}b_{-1}^2-(3/2)Ra_{-1}^2b_{-1},$$
$$C_2=-k^2a_1b_0b_1+k^2a_0b_1^2-3Pa_1^2a_0-Qa_0b_1^2-(3/2)Ra_1^2b_0-2Qa_1b_0b_1-3Ra_1a_0b_1,$$
$$C_{-2}=k^2a_0b_{-1}^2-3Pa_0a_{-1}^2-Qa_0b_{-1}^2-(3/2)Ra_{-1}^2b_0-k^2a_{-1}b_0b_{-1}-2Qa_{-1}b_0b_{-1}-3Ra_0a_{-1}b_{-1},$$
$$C_1=k^2a_1b_0^2+4k^2a_{-1}b_1^2-3Pa_1a_0^2-Qa_1b_0^2-(3/2)Ra_0^2b_1-k^2a_0b_1b_0-4k^2a_1b_{-1}b_1-2Qa_0b_1b_0-3Ra_1a_0b_0-3$$
$$Pa_1^2a_{-1}-Qa_{-1}b_1^2-(3/2_Ra_1^2b_{-1}-2Qa_1b_1b_{-1}-3Ra_1a_{-1}b_1,$$
$$C_{-1}=k^2a_{-1}b_0^2+4k^2a_1b_{-1}^2-3Pa_0^2a_{-1}-Qa_{-1}b_0^2-(3/2)Ra_0^2b_{-1}-3Pa_1a_{-1}^2-Qa_1b_{-1}^2-(3/2)Ra_{-1}^2b_1$$
$$-k^2a_0b_{-1}b_0-4k^2a_{-1}b_1b_{-1}-2Qa_0b_{-1}b_0-3Ra_0a_{-1}b_0-2Qa_{-1}b_1b_{-1}-3Ra_1a_{-1}b_{-1},$$
$$C_0=-Qa_0b_0^2-(3/2)Ra_0^2b_0-3Ra_1a_0b_{-1}-2Qa_{-1}b_0b_1-2Qa_0b_1b_{-1}+3k^2a_1b_0b_{-1}+3k^2a_{-1}b_0b_1$$
$$-6k^2a_0b_1b_{-1}-Pa_0^3-2Qa_1b_{[0}b_{-1}-6Pa_1a_0a_{-1}-3Ra_1a_{-1}b_0-3Ra_0a_{-1}b_1$$

Equating the coefficients of $exp(j\eta)(j=3,2,1,0,-3,-2,-1)$ to zero, we have

$$C_{-3}=0,C_3=0,C_{-2}=0,C_1=0,C_0=0,C_2=0,C_{-1}=0$$

Solving this system of algebraic equations with the aid of Maple, we obtain three different cases namely,

Case A

$$b_{-1}=-\frac{3Ra_{-1}}{2(1+2Q)},P=\frac{9(Q+1)R^2}{4(1+2Q)^2},b_1=b_1,$$
$$a_1=-\frac{2b_1(1+2Q)}{3R},b_0=\frac{2\sqrt{\frac{-3Ra_{-1}b_1}{Q-1}}}{1+2Q},$$
$$k=1,a_{-1}=a_{-1},a_0=-\frac{4(Q-1)\sqrt{\frac{-3Ra_{-1}b_1}{Q-1}}}{3R} \quad (13)$$

Case B

$$a_0=-\frac{2b_0(-Q+2Q^2-1)}{3R(Q+2)},b_1=\frac{(-Q+2Q^2-1)b_0^2}{8(Q^2+4Q+4)b_{-1}},$$
$$k=1,b_{-1}=b_{-1},P=\frac{9(Q+1)R^2}{4(1+2Q)^2},a_{-1}=-\frac{2(1+2Q)b_{-1}}{3R},$$
$$a_1=-\frac{b_0^2(4Q^3-3Q-1)}{12Rb_{-1}(Q^2+4Q+4)},b_0=b_0 \quad (14)$$

Case C

$$P=\frac{9(i^2+Q)R^2}{4(i^2+2Q)^2},b_{-1}=-\frac{3Ra_{-1}}{2(i^2+2Q)},b_0=\sqrt{\frac{3Ra_{-1}b_1}{(-Q+i^2)}},$$
$$a_0=\frac{4(i^2-Q)\sqrt{\frac{3Ra_{-1}b_1}{(-Q+i^2)}}}{R},b_1=b_1,a_{-1}=a_{-1},k=-i,a_1=-\frac{2b_1(i^2+2Q)}{3R}, \quad (15)$$

According to Case A, substituting Eq.(13) into (11), we obtain the following generalized solitary solution of Eq(1)

$$\phi(\eta)=\frac{-\frac{2b_1(1+2Q)}{3R}exp(\eta)-\frac{4(Q-1)\sqrt{\frac{-3Ra_{-1}b_1}{Q-1}}}{3R}+a_{-1}exp(-\eta)}{b_1exp(\eta)+\frac{2\sqrt{\frac{-3Ra_{-1}b_1}{Q-1}}}{1+2Q}-\frac{3Ra_{-1}}{2(1+2Q)}exp(-\eta)}, \quad (16)$$

$$\eta = \xi + w$$

With the aid of Case B, inserting Eq.(14) into (11), admits to the new generalized solitary solution of Eq.(1) as

$$\phi(\eta) = \frac{-\frac{b_0^2(4Q^3-3Q-1)}{12Rb_{-1}(Q^2+4Q+4)}exp(\eta) - \frac{2b_0(-Q+2Q^2-1)}{3R(Q+2)} - \frac{2(1+2Q)b_{-1}}{3R}exp(-\eta)}{b_1 exp(\eta) + b_0 + b_{-1}exp(-\eta)}, \tag{17}$$

$$\eta = \xi + w$$

Making use of Case C, using Eq.(15) into (11), we have a generalized solitary solution of Eq.(1) as follows

$$\phi(\eta) = \frac{-\frac{2b_1(i^2+2Q)}{3R}exp(\eta) + \frac{4(i^2-Q)\sqrt{\frac{3Ra_{-1}b_1}{(-Q+i^2)}}}{R} + a_{-1}exp(-\eta)}{b_1 exp(\eta) + \sqrt{\frac{3Ra_{-1}b_1}{(-Q+i^2)}} - \frac{3Ra_{-1}}{2(i^2+2Q)}exp(-\eta)}, \tag{18}$$

$$\eta = -i\xi + w$$

3 New applications

In order to illustrate the effectiveness and convenience of the method, we consider two nonlinear evolution equations arising in physics, namely, the generalized Zakharov equations and higher-order nonlinear Schrodinger equation .

3.1 Example (1).The generalized Zakharov equations

Let us first consider the generalized Zakharov equations for the complex envelope $\psi(x,t)$ of the high-frequency wave and the real low-frequency field $v(x,t)$ in [30]

$$i\psi_t + \psi_{xx} - 2\lambda|\psi|^2\psi + 2\psi v = 0,$$

$$v_{tt} - v_{xx} + (|\psi|^2)_{xx} = 0, \tag{19}$$

where the cubic term in Eqs.(19) describes the nonlinear-self interaction in the high frequency subsystem, such a term corresponds to a self-focusing effect in plasma physics. The coefficient λ is a real constant that can be a postive or negative number.Let us assume the travelling wave solution of Eqs.(19) in the form

$$\psi(x,t) = e^{[i\eta_0]}u(\xi), v = v(\xi), \xi = k_0(x - 2\alpha t), \eta_0 = (\alpha x + \beta t), \tag{20}$$

where $u(\xi)$ and $v(\xi)$ are real functions, the constants α, β and k_0 are to be determined.Substituting (20) into Eqs.(19), we have

$$k_0^2\phi^{''}(\xi) + 2uv - (\alpha^2 + \beta)u(\xi) - 2\lambda u^3(\xi) = 0, \tag{21}$$

$$k_0^2(4\alpha^2 - 1)v^{''}(\xi) + k_0^2(u^2(\xi))^{''} = 0 \tag{22}$$

In order to simplify ODEs (21) and (22), integrating Eq.(22) once and taking integration constant to zero, and integrating yields

$$v(\xi) = \frac{u^2(\xi)}{(1-4\alpha^2)} + C, if \alpha^2 \neq \frac{1}{4}, \tag{23}$$

where C is an integration constant.Inserting Eq.(23) into (21), we have

$$Au^{''}(\xi) + Bu(\xi) + Du^3(\xi) = 0, \tag{24}$$

$$A = k_0^2, B = [2C - \alpha^2 - \beta], D = 2[\frac{1}{1-4\alpha^2} - \lambda] \tag{25}$$

Our main goal is to solve Eq.(24) using the generalized auxiliary method, we suppose that the Eq.(24) has the formal solution can be expressed as

$$u(\xi) = c_0 + \sum_{i=1}^{M} c_i\phi^i(\xi) + d_i\phi^{-i}(\xi) + k_i\phi^{i-1}(\xi)\phi^{'}(\xi) + N_i\phi^{-i}(\xi)\phi^{'}(\xi), \tag{26}$$

where c_0, c_i, d_i, k_i and N_i are constants to be determined later.

Balancing $u^3(\xi)$ and $u''(\xi)$ in Eq.(24), we obtain $M = 1$ and suppose that Eq.(24) has the formal solutions

$$u(\xi) = c_0 + c_1\phi(\xi) + d_1\phi^{-1}(\xi) + k_1\phi'(\xi) + N_1\phi^{-1}(\xi)\phi'(\xi) \tag{27}$$

Substituting Eq.(27) into (24), and making use of Eq.(1), equating the coefficients of all power $\phi^j(\xi)\phi'^l(\xi)(l = 0, 1; j = 0, \pm1, \pm2, \pm3, ..)$ to zero, we we get a system of algebraic equations for c_0, c_1, d_1, k_1 and N_1.Solving this system by means of Maple, we have

$$c_0 = d_1 = k_1 = 0, N_1 = N_1, c_1 = c_1, k_0 = k_0, \alpha = \alpha, P = \frac{c_1^2}{N_1^2}, \beta = -\alpha^2 + 2C - \frac{k^2\alpha}{2},$$

$$\lambda = \frac{-k^2 - 4N_1^2 + 4k^2\alpha^2}{4N_1^2(2\alpha - 1)(2\alpha + 1)} \tag{28}$$

By using Eq.(28) and Eqs.(20), (23) and (16), we obtain the following new exact formal solutions of Eqs.(19)

$$\psi_1(\xi) = [c_1\phi(\xi) + N_1\phi'(\xi)\phi^{-1}(\xi)]e^{i\eta_0}, \tag{29}$$

$$v_1(\xi) = \frac{[c_1\phi(\xi) + N_1\phi'(\xi)\phi^{-1}(\xi)]^2}{(1 - 4\alpha^2)} + C,$$

where $\phi(\xi)$ is defined by Eq.(16).

By means Eq.(28) and Eqs.(20), (23) and (17), we get the following new exact formal solutions of Eqs.(19)

$$\psi_2(\xi) = [c_1\phi(\xi) + N_1\phi'(\xi)\phi^{-1}(\xi)]e^{i\eta_0}, \tag{30}$$

$$v_2(\xi) = \frac{[c_1\phi(\xi) + N_1\phi'(\xi)\phi^{-1}(\xi)]^2}{(1 - 4\alpha^2)} + C,$$

where $\phi(\xi)$ is defined by Eq.(17).

From Eq.(28) and Eqs.(20), (23) and (18), we obtain the new exact solutions of Eqs.(19)

$$\psi_3(\xi) = [c_1\phi(\xi) + N_1\phi'(\xi)\phi^{-1}(\xi)]e^{i\eta_0}, \tag{31}$$

$$v_3(\xi) = \frac{[c_1\phi(\xi) + N_1\phi'(\xi)\phi^{-1}(\xi)]^2}{(1 - 4\alpha^2)} + C,$$

where $\phi(\xi)$ is defined by Eq.(18).

3.2 Example(2).The higher-order nonlinear Schrodinger equation

A second instructive model is the higher-order nonlinear Schrodinger equation [30]

$$\frac{\partial\psi}{\partial z} = i\alpha_1\frac{\partial^2\psi}{\partial t^2} + i\alpha_2\psi|\psi|^2 + \alpha_3\frac{\partial^3\psi}{\partial t^3} + \alpha_4\frac{\partial\psi|\psi|^2}{\partial t} + \alpha_5\psi\frac{\partial|\psi^2|}{\partial t}, \tag{32}$$

where ψ is slowly varying envelop of the electric field, the subscripts z and t are spatial and temporal partial derivative in retard time coordinates, and α_1, α_2, α_3, α_4 and α_5 are the real parameters related to the group velocity, self phase modulation, thrid order dispersion, self steepening and self frequency shift arising from stimulated Raman scattering respectively.

To look for the travelling wave solution of Eq.(32), we use the gauge transformation

$$\psi(z, t) = \phi(\xi)e^{[i(k_0z - wt)]}, \xi = t - \lambda z + \xi_0, \tag{33}$$

where k_0, w, λ are constants to be determined, ξ_0 is an arbitrary constant.Substituting Eq.(33) into (32) yields the real system

$$(\alpha_1 - 3\alpha_3 w)u''(\xi) + (\alpha_3 w^3 - \alpha_1 w^2 - k_0)u(\xi) + (\alpha_2 - \alpha_4 w)u^3(\xi) = 0,$$

$$\alpha_3 u^{'''}(\xi) + (2\alpha_1 w - 3\alpha_3 w^2 + \lambda)u^{'}(\xi) + (3\alpha_4 + 2\alpha_5 w)u^2(\xi)u^{'}(\xi) = 0, \tag{34}$$

under the constraint conditions

$$w = \frac{\alpha_1(3\alpha_4 + 2\alpha_5) - 3\alpha_2\alpha_3}{6\alpha_3(\alpha_4 + \alpha_5)}, \tag{35}$$

$$k_0 = -\frac{1}{\alpha_3}[(\alpha_1 - 3\alpha_3 w)(2w\alpha_1 - 3w^2\alpha_3 + \lambda)] - w^2\alpha_1 + w^3\alpha_3 \tag{36}$$

Eqs.(34) becomes

$$Au^{''}(\xi) + Bu(\xi) + Du^3(\xi) = 0, \tag{37}$$

$$A = 1, B = \frac{2w\alpha_1 + \lambda - 3w^2\alpha_3}{\alpha_3},$$

$$D = \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3} \tag{38}$$

By virtue of the technique of solution of Eq.(37), we assume that the solution of Eq.(37) in the series form

$$u(\xi) = c_0 + \sum_{i=1}^{M} c_i\phi^i(\xi) + d_i\phi^{-i}(\xi) + k_i\phi^{i-1}(\xi)\phi^{'}(\xi) + N_i\phi^{-i}(\xi)\phi^{'}(\xi)], \tag{39}$$

where c_0, c_i, d_i, k_i and N_i are constants to be determined later.

Consider the balance between $u^3(\xi)$ and $u^{''}(\xi)$ in Eq.(37), we obtain $M = 1$.Thus the we suppose of Eq.(37) can expressed as

$$u(\xi) = [c_0 + c_1\phi(\xi) + d_1\phi^{-1}(\xi) + k_1\phi^{'}(\xi) + N_1\phi^{-1}(\xi)\phi^{'}(\xi)] \tag{40}$$

Substituting Eq.(40) into (37) with Eq.(1), equating the coefficients of all power of $\phi^j(\xi)\phi'^l(\xi)(l = 0, 1; j = 0, \pm1, \pm2, \pm3, ..)$ to zero, we get a system of algebraic equations for c_0, c_1, d_1, k_1 and N_1.Solving this system by means of Maple, we have

$$c_0 = d_1 = k_1 = 0, N_1 = \sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}, c_1 = \sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}},$$

$$\lambda = 2w\alpha_1 + 3w^2\alpha_3 + (1/2)\alpha_3, w = w, k = k \tag{41}$$

From Eq.(41) and Eqs.(16) and (33), admits to new exact solutions of Eqs.(32) as follows

$$\psi_1(\xi) = [\sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}\phi(\xi) + \sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}\phi^{-1}(\xi)\phi^{'}(\xi)]e^{[i(kz - wt)]}, \tag{42}$$

where $\phi(\xi)$ is defined by Eq.(16),

$$\xi = t - [2w\alpha_1 + 3w^2\alpha_3 + (1/2)\alpha_3]z + \xi_0$$

Using Eq.(41) and Eqs.(17) and (33), admits to new exact solutions of Eqs.(32) as follows

$$\psi_2(\xi) = [\sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}\phi(\xi) + \sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}\phi^{-1}(\xi)\phi^{'}(\xi)]e^{[i(kz - wt)]}, \tag{43}$$

where $\phi(\xi)$ is defined by Eq.(17),

$$\xi = t - [2w\alpha_1 + 3w^2\alpha_3 + (1/2)q\alpha_3]z + \xi_0,$$

From Eq.(41) and Eqs.(18) and (33), admits to new exact solutions of Eqs.(32) in the folowing from

$$\psi_3(\xi) = [\sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}\phi(\xi) + \sqrt{\frac{-3\alpha_3}{6\alpha_4 + 4\alpha_5}}\phi^{-1}(\xi)\phi^{'}(\xi)]e^{[i(kz - wt)]}, \tag{44}$$

where $\phi(\xi)$ is defined by Eq.(18),

$$\xi = t - [2w\alpha_1 + 3w^2\alpha_3 + (1/2)\alpha_3]z + \xi_0$$

4 Conclusion

In this paper, Exp-function method with a computerized symbolic computation Maple is used for constructing the new exact travelling wave solutions for the generalized Zakharov equations and higher-order nonlinear Schrodinger equation.The main idea of this method is to take full advantage of the generalized auxiliary equation which has more new solutions.It seems that the Exp-function method is more effective and simple than other methods and a lot of solutions can be obtained in the same time.In addition, this method is also comuterizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

It is worthwhile to mention that this method is straightforward and concise, and it can also be applied to other nonlinear evolution equations arising in mathematical physics.

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