Binary Darboux-Bäcklund Transformation and New Singular Soliton Solutions for the Nonisospectral Kadomtsev-Petviashvili Equation

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Abstract: Based on the elementary Darboux transformation, a further extended self-adjoint method for constructing Binary Darboux-Bäcklund transformation (BDBT) is applied to nonisospectral KP. And we obtain three new types singular soliton solutions for nonisospectral KP equation, such as singular soliton solution in the form of blow up waves with finite amplitude, another two types of singular soliton solution in the form of periodic blow up waves.

Keywords: binary Darboux-Bäcklund transformation; self-adjoint equation; the integral operator; singular soliton solution

1 Introduction

Darboux transformation (DT) method [1] based on Lax pairs has been proved to be one of the most fruitful algorithmic procedures to get explicit solutions of nonlinear evolution equations. The key for constructing Darboux transformation is to expose a kind of covariant properties that the corresponding spectral problems possess. In 1990, Matveev and Salle [2] first investigated the DT in integral form and presented binary Darboux transformation (BDT). Nimmo [3, 4] and Gu [5] has carried out a lot of excellent work about BDT: in Ref. [3], the general construction of BDT for KP hierarchy preserving certain properties of the operator, such as self-adjoint, is given; the BDT of two-dimensional Zakharov-Shabati/AKNS spectral problem [4] is obtained by composing the elementary transformation, for one solution matrix, with its inverse for another solution matrix.

In the past decade, a unified explicit form of Darboux transformation can be obtained for some isospectral equations, such as KdV, mKdV, KP, NNV, Euler, fractional nonlinear Schrödinger equation and Schrödinger-Boussinesq Equation [6–12]. Equations with nonisospectral eigenparameters may provide more realistic models, in the propagation of (small-amplitude) surface waves in straits or large channels of (slowly) varying depth and width and nonvanishing vorticity [13–15]. Recently, via Painlevé analysis, Hirota method, Wronskian technique and Darboux transformations, they have been a lot of conclusions [16–25, 30, 31]. These methods have been applied to find exact solutions some PDEs.

In this paper, based on the framework of Nimmo [4], we construct a Binary Darboux-Bäcklund transformation for the nonisospectral KP equation (1a) and get three new singular solutions of the nonisospectral KP equation (1b).

2 Binary Darboux-Bäcklund Transformation for nonisospectral KP equation

The nonisospectral KP equation are

\begin{align*}
&4u_t + y(u_{xxx} + 6uu_x + 3\partial^{-1}u_y) + 2ux_y + 4\partial^{-1}u_y = 0 \quad (1a) \\
&4u_t + y(u_{xxx} + 6uu_x + 3\partial^{-1}u_y) + 2ux_y + 4\partial^{-1}u_y + 2ux = 0 \quad (1b)
\end{align*}

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which describes water waves in \((x, y)\)-plane when the nonlinearity is higher than for the KP equation. Eq.\((1a)\) has been introduced in Darboux transformation \([26–29]\). In this section, we will employ the self-adjoint method to construct the BDBT for the nonisospectral KP equation\((1a)\) by using nonisospectral KP equation\((1b)\).

Consider the lax pairs of the nonisospectral KP equation \((1a)\) and \((1b)\)

\[
\phi_y = \phi_{xx} + u \phi, \quad (2a)
\]

\[
4 \phi_t = y A(u) \phi + x B(u) \phi + C(u) \phi, \quad (2b)
\]

and

\[
\phi_y = \phi_{xx} + u \phi, \quad (3a)
\]

\[
4 \phi_t = y A(u) \phi + x B(u) \phi + C(u) \phi - 2 \phi_x, \quad (3b)
\]

where

\[
A(u) = -4 \partial^3 - 6 u \partial - 3 (u_x + \partial^{-1} u_y),
\]

\[
B(u) = -2(\partial^2 + u), \quad C(u) = -2 \partial - \partial^{-1} u, \quad \partial = \frac{\partial}{\partial x}, \quad \partial^{-1} = \int_{-\infty}^{x}.
\]

### 2.1 BDBT of the \(t\)-independent Lax pair Eq.\((2a)\)

If \(u\) is a solution of nonisospectral KP equation \((1a)\) and \(\phi_1\) is a solution of Eqs.\((2a)\) and \((2b)\). Then the elementary DT for the nonisospectral KP equation \((1a)\) is defined by\([27]\)

\[
u[1] = u + 2 \partial^2 \ln \phi_1, \quad \phi[1] = \phi_x - \frac{\phi_{1x}}{\phi_1} \phi.
\]

And \(\phi[1]\) and \(u[1]\) satisfy the following equations

\[
\phi[1]_y = \phi[1]_{xx} + u[1] \phi[1], \quad (6)
\]

\[
4 \phi[1]_t = y A(u[1]) \phi[1] + x B(u[1]) \phi[1] + C(u[1]) \phi[1] - 2 \phi[1], \quad (7)
\]

\[
A(u[1]) = -4 \partial^3 - 6 u[1] \partial - 3 (u[1]_x + \partial^{-1} u[1]_y), \quad \partial = \frac{\partial}{\partial x}, \quad \partial^{-1} = \int_{-\infty}^{x},
\]

\[
B(u[1]) = -2(\partial^2 + u[1]), \quad C(u[1]) = -2 \partial - \partial^{-1} u[1], \quad (8)
\]

and \(u[1]\) defined by Eq.\((5)\) is a solution of nonisospectral KP equation \((1b)\).

Assuming that \(\theta\) is an arbitrary non-zero fixed solution of Eq.\((2a)\), from Eq.\((5)\), we can get the following equations

\[
u[1]_{\theta} = u + 2 \partial^2 \ln \theta, \quad \phi[1] = \phi_x - \frac{\theta_x}{\theta} \phi.
\]

We known \(u[1]_{\theta}\) and \(\phi[1]\) satisfy Eq.\((3a)\) (also Eq.\(2a)\))

\[
\phi[1]_y = \phi[1]_{xx} + u[1]_{\theta} \phi[1].
\]

Assuming that \(\hat{\phi}\) and \(\hat{u}\) are also solutions of Eq.\((3a)\)

\[
\hat{\phi}_y = \hat{\phi}_{xx} + \hat{u} \hat{\phi}, \quad (11)
\]

and \(\hat{\theta}\) is also an arbitrary non-zero fixed solution of Eq.\((11)\), from Eq.\((5)\), we can get the similar transformation to Eq.\((9)\)

\[
u[1]_{\theta} = \hat{u} + 2 \partial^2 \ln \hat{\theta}, \quad \phi[1] = \hat{\phi}_x - \frac{\hat{\phi}_{xx}}{\hat{\theta}} \hat{\phi}, \quad (12)
\]

then we can obtain the same equation

\[
\phi[1]_y = \phi[1]_{xx} + u[1]_{\theta} \phi[1],
\]

which is the equation \((10)\).
The self-adjoint of Eqs (2a), (10) and (11) are

\[-\phi_x^* = \phi_{xx}^* + u\phi^*,\]

\[-\phi[1]_x^* = \phi[1]_{xx}^* + u[1]\phi[1]^*,\] (14)

\[-\phi^*_y = \phi_{xy}^* + \hat{u}\phi^*.] (15)

For convenience, we define \(S, S_\theta, \hat{S}, S_*^+ \) and \(\hat{S}^+\) to be the sets of all solutions of Eqs. (2a), (10), (11), (13), (14) and (15).

As can be seen from above, \(u\) and \(\hat{u}\) have the same property and they are two solutions of Eq. (2a) and Eq. (11). The transformation between Eq. (2a) and Eq. (11) is called the BDBT, which is what we want to find in the present paper. We define the mappings between \(S\) and \(S_\theta\), \(\hat{S}\) and \(\hat{S}_\theta\) to be \(f_0\) and \(\hat{f}_0\)

\[f_0 : S \longrightarrow S_\theta, \quad \phi[1] = f_0(\phi),\] (16)

\[\hat{f}_0 : \hat{S} \longrightarrow \hat{S}_\theta, \quad \phi[1] = \hat{f}_0(\hat{\phi}).\] (17)

Through Eq. (9) \(\phi[1] = \phi_x - \partial_x\phi\), it is obvious that \(f_0 = \partial_x - \partial_x(\theta S)\) maps \(S\) into \(S_\theta\). Through (16) and (17), we can get the following equation

\[\hat{\phi} = f_0^{-1}f_0(\phi).\] (18)

Then the most important thing to construct the BDBT is to find the inverse of \(f_0\) and the relationship between \(\theta\) and \(\hat{\theta}\). In the following, we will employ the self-adjoint method to construct two integral operators.

For the arbitrary solution \(\phi\) of Eq. (2a) and \(\theta\) is the non-zero fixed solution of Eq. (2a), we can assume \(\phi = \theta M\). Substituting \(\phi = \theta M\) into Eq. (2a), we can get the corresponding equation \(2\theta M_x + \theta M_{xx} - \theta M_y = 0\). Introducing a new variable \(P\), we can put this equation into the following equation

\[M_x = \theta^{-1}P, \quad M_y = \theta^{-1}P_x - (\theta^{-1})_\theta P.\] (19)

According to the compatibility condition \(M_{xy} = M_{xx}\), Eq. (19) implies that

\[P_y = P_{xx} + (u + 2\partial^2 \ln \theta)P.\] (20)

where \(P\) is a solution of Eq. (10).

Because directly verify that \(\theta^{-1}\) satisfies

\[-(\theta^{-1})_y = (\theta^{-1})_{xx} + u[1]_\theta \theta^{-1}.\]

As \(\theta\) is the arbitrary non-zero fixed solution of Eq. (2a) and the definition of \(\phi\), there is \(\phi^{-1} \in S_\theta^+\), that is to say, \(\phi^{-1}\) satisfies Eq. (14).\n
\[-(\phi^{-1})_y = (\phi^{-1})_{xx} + u[1]_\theta \phi^{-1}.\]

With the Eq. (19), we can define an integral operator \(G\) by

\[G(P, P*) = C + \int_L PP^* dx + (P, P^* - PP^*)dy,\] (21)

for some constant \(C\) and a curve \(L\) in the plane \(S_\theta \times S_\theta^+\). We can make use of the integral operator \(G\) to express the solution of Eq. (2a), then every \(\phi \in S\) is \(\phi = \theta G(P, P^{-1})\), for some \(P \in S_\theta\).

It is obvious that for the arbitrary solution \(P\) of Eq. (20), there will be \(\phi = \theta G(P, \theta^{-1})\). Because of \(\phi \in S\), we can obtain \(f_\theta^{-1} = \theta G(\cdot, \theta^{-1})\).

Therefore, in order to find \(f_\theta^{-1}\), the following main task is to find the relationship between \(\theta\) and \(\hat{\theta}\). With the same method to construct the integral operator \(G\), for any \(P \in S_\theta^+\), we can get \(P^* = \theta^{-1}H\), where \(H\) satisfies the following equations

\[H_x = \theta_x \phi^* + \theta \phi_x^* - \theta \phi^*, \quad H_y = \theta_y \phi^* + \theta \phi_y^* - \theta \phi^*.\] (23)

According to the integrability condition \(H_{xy} = H_{xx}\), we can see \(\phi^* \in S^*\). We introduce another integral operator \(F\), as follows

\[F(\phi, \phi^*) = C + \int_L \phi \phi^* + \phi \phi^* dx + (\phi, \phi^* + \phi \phi^* - \phi \phi^* + \phi \phi^*)dy,\] (24)
for some constant $C$ and a curve $L$ in the plane $S \times S'$. We can make use of the integral operator $F$ to express the solution of Eq.(14), then every $\theta^* \in S_\theta^*$ is
\[
\theta^* = \theta^{-1} F(\theta, \phi^*),
\]
for some $\phi^* \in S^*$. $\theta$ and $\hat{\theta}$ have the same property and they are two solutions of Eq.(2a) and Eq.(11), there is $\hat{\theta}^{-1} \in S_{\hat{\theta}}$. Due to Eq.(25), it is seen that
\[
\hat{\theta}^{-1} = \theta^{-1} F(\theta, \theta').
\]
Via Eq.(21) and Eq.(24), we can get the relationship between integral operator $G$ and $F$
\[
F(\theta, \theta') + G(\theta, \theta') = \theta \theta'.
\]
With the help of Eq.(9) and Eq.(12), we can obtain $\hat{u}$,
\[
\hat{u} = u + 2 \hat{\theta}^2 \ln \theta - 2 \hat{\theta}^2 \ln \hat{\theta} = u + 2 \hat{\theta}^2 \ln \frac{\theta}{\hat{\theta}}, \tag{28}
\]
substituting $\hat{\theta} = \frac{\theta}{\theta'}$ into Eq.(28), we can obtain
\[
\hat{u} = u + 2 \hat{\theta}^2 \ln F(\theta, \theta'). \tag{29}
\]
With the help of Eqs.(18), (22) and (26), we can construct the BDBT for Eq.(2a)
\[
\hat{\phi} = \int_0^1 f_0(\phi) = \hat{\theta} G(f_0(\phi), \hat{\theta}^{-1}) = \hat{\theta} \left[C + \int_L f_0(\phi) \hat{\theta}^{-1} dx + \left(f_0(\phi) \hat{\theta}^{-1} - f_0(\phi) \hat{\theta} reverse \right) dy \right]. \tag{30}
\]
Substituting $\hat{\theta}^{-1} = \theta^{-1} F(\theta, \theta')$, $\hat{\theta} = \frac{\theta}{\theta'}$, $f_0(\phi) = \phi_x - \frac{\theta}{\theta'} \phi$ and $F(\theta, \theta') = C + \int_L \phi_x \theta_x + \theta \theta_x - \theta \theta_x + (\theta \theta_x - \theta \theta_x) \theta_x dy$, into Eq.(30). After an integration by parts, we can obtain the following equation
\[
\hat{\phi} = \phi - \frac{\theta F(\phi, \theta')}{F(\theta, \theta')}. \tag{31}
\]
As we known from above, the BDBT that we investigate the $t$-dependent Lax pair is Eq. (29) and (31). Eq.(2a) has been transformed into $\hat{\phi} = \phi_x + \hat{u} \phi_x$, where $\theta$ and $\theta'$ are arbitrary non-zero fixed solutions of Eq.(2a) and its self-adjoint Eq.(13). We can verify directly that via
\[
\hat{\phi}^* = \phi^* - \frac{\theta^* G(\theta, \phi^*)}{G(\theta, \theta')} \tag{32}
\]
Eq.(13) is also form invariable
\[
-\hat{\phi}^*_x = \phi^*_x + \hat{u} \phi^* \tag{33}
\]

### 2.2 BDBT of the $t$-dependent Lax pair Eq.(2b)

From above, we only consider the $t$-independent Lax pair Eq.(2a). Since Eq.(2b) should be form
\[
4\phi_t = yA(\hat{u})\phi + xB(\hat{u})\phi + C(\hat{u})\phi - 2\hat{\phi}, \tag{33}
\]
where
\[
A(\hat{u}) = -4\hat{u}^3 - 6u \hat{u} - 3(\hat{u}_x + \hat{u} \hat{u}_x), \tag{34}
\]
\[
B(\hat{u}) = -2(\hat{u}^2 + \hat{u}), \tag{35}
\]
and the self-adjoint of Eq.(2b) is
\[
4\phi^*_t = y\tilde{A}(u)\phi^* + x\tilde{B}(u)\phi^* + \tilde{C}(u)\phi^*, \tag{36}
\]
where
\[
\tilde{A}(u) = -4u^3 - 6u u - 3(u_x + u \hat{u}_x), \tag{37}
\]
\[
\tilde{B}(u) = 2(u^2 + u), \tag{38}
\]
by the transformation (29) and (31). Substituting Eqs. (29), (31) and (34) into Eq. (33) directly, with Eqs.
\[
\phi_t = \frac{1}{4} yA(u)\phi + \frac{1}{4} xB(u)\phi + \frac{1}{4} C(u)\phi, \tag{39}
\]
\[ \theta_t = \frac{1}{4} yA(u)\theta + \frac{1}{4} xB(u)\theta + \frac{1}{4} C(u)\theta, \]  

and Eq(4), we can see that there must be

\[ \partial_t F(\theta, \theta^*) = \frac{1}{4} y(-\theta_{xxx}\theta - 2\theta_{xx}\theta_x - \theta_x\theta + 3\theta_{xxy}\theta_x + 3\theta_{xxy}\theta_x - 3\theta_{xx}\theta_x + 3\theta_{xxy}\theta_x + u\theta_{xx}\theta_x + 3u\theta_{xx}\theta_x) \]

\[ + \frac{1}{2} x(\theta_{xx}\theta - \theta_x\theta + 2u\theta_{xx}\theta_x), \]  

\[ \partial_t G(\theta, \theta^*) = -\frac{1}{4} y(-\theta_{xxx}\theta - 2\theta_{xx}\theta_x - \theta_x\theta + 3\theta_{xxy}\theta_x + 3\theta_{xxy}\theta_x - 3\theta_{xx}\theta_x + 3\theta_{xxy}\theta_x + u\theta_{xx}\theta_x + 3u\theta_{xx}\theta_x) \]

\[ + \frac{1}{2} x(\theta_{xx}\theta - \theta_x\theta + 2u\theta_{xx}\theta_x). \]  

With the help of Eqs.(21), (24), (39) and (40), we should amend the operators \( F \) and \( G \) which can be defined as

\[ G(P, P^*) = C + \int \left[ PP_x dx + (PP_x + P_x P^*) dy + \left[ P_x P^* + PP_x - \frac{1}{4} y(-P_{xxx}P - 2P_{xx}P^*) \right. \right. \]

\[ \left. \left. - P_{xx}P + 3P_{xxy}P^* + 3P_{xxy}P^* + 3P_{xx}P^* + 3P_{xxy}P^* + u\theta_{xx}\theta_x + 3u\theta_{xx}\theta_x \right] \right] - \frac{1}{2} x(P_{xx}P^* + P_x P^* + 2u\theta_{xx}\theta_x) dt, \]  

\[ F(P, P^*) = C + \int \left[ P_x P^* dx + (P_x P^* - P^* P_x) dy + \left[ \frac{1}{4} y(-P_{xxx}P - 2P_{xx}P^*) - P_{xx}P^* + 3P_{xxy}P^* \right. \right. \]

\[ \left. \left. + 3P_{xxy}P^* + 3P_{xxy}P^* + 3P_{xx}P^* + u\theta_{xx}\theta_x + 3u\theta_{xx}\theta_x \right] + \frac{1}{2} x(P_{xx}P^* + P_x P^* + 2u\theta_{xx}\theta_x) \right] dt. \]  

(41)

(42)

It is easy to see

\[ \tilde{G}(P, P^*) + \tilde{F}(\phi, \phi^*) = PP^*. \]  

(43)

Through the analysis of \( t \)-independent lax pair Eq.(2a) and \( t \)-dependent lax pair Eq.(2b), the BDBT of Eq.(1a) is

\[ \hat{u} = u + 2\theta_{xx} \ln \tilde{F}(\theta, \theta^*), \]  

(44)

\[ \hat{\phi} = \phi - \frac{\theta \tilde{F}(\phi, \theta^*)}{\tilde{F}(\theta, \theta^*)}, \]  

(45)

\[ \hat{\phi}^* = \phi^* - \frac{\theta \tilde{G}(\theta, \phi^*)}{\tilde{G}(\theta, \theta^*)}, \]  

(46)

where \( \theta \) and \( \theta^* \) are arbitrary non-zero fixed solutions of Eqs.(2a), (2b), (4) and (13), (35), (36), \( \hat{u} \) and \( \hat{\phi} \) are solutions of nonisospectral KP equation (1b) and system (2a), (2b), respectively.

### 3 New singular soliton solutions of the nonisospectral KP equation (1b)

In this section, we will get some singular soliton solutions for the nonisospectral KP equation (1b) with the Binary Darboux-Bäcklund Transformation (44), (45) and (46). Taking \( u = 0 \) for the simplest starting solution of the nonisospectral KP equation (1a) and choosing the initial solution of Lax pairs, (2a), (2b) and adjoining Lax pairs (13), (35) are

\[ \phi = c_2(t + c_1)^{-1} \exp \frac{4t}{(t + c_1)^2} \]  

\[ \bar{\phi} = d_2(t + d_1) \exp \frac{4t}{(t + d_1)^2}, \]  

(47)

where \( c_1, c_2, d_1 \) and \( d_2 \) are arbitrary constants. With Eqs. (41) and (42), we can obtain

\[ \tilde{G}(\phi, \phi^*) = c + g(x,y) + \frac{2c_2d_2(t + d_1) \exp(\xi)}{c_1 - d_1}, \]

\[ \tilde{F}(\phi, \phi^*) = -c - g(x,y) + \frac{c_2d_2(t + d_1) \exp(\xi)}{t + c_1} - \frac{2c_2d_2(t + d_1) \exp(\xi)}{c_1 - d_1}. \]
where \( g(x, y) = \int \frac{c_2 y d_2 (t + d_1) \exp(\xi)}{(t + c_1)(t + d_1)}\, dt \), \( \xi = \frac{2x^2 d_1 + 2x \eta d_2 + 2x y d_1^2 + 2x y d_1 d_2 + 2x \eta d_2}{t + c_1} \) and \( f(x, y, t) = (1 - \xi)(t + c_1)^2(t + d_1) + (2x - t)(t + c_1)^2(t + d_1)^2 + (12y + 2yt + 2y d_1)(t + c_1)(t + d_1)^2 - (2x + 2x d_1 + 4y)(t + c_1)^2(t + d_1)^2 + 2y(t + c_1)^3(t + d_1) - 12y(t + d_1)^3. \)

By using BDBT (44), we arrive at new singular soliton solutions for the nonisospectral KP equation (1b) \( u = \left[ \ln \left( -c - g(x, y) + \frac{c_2 y d_2 (t + d_1) \exp(\xi)}{t + c_1} - \frac{2c_2 y d_2 (t + d_1) \exp(\xi)}{c_1 - d_1} \right) \right]_{x} \), (48)

Explain in detail as follows:

**Case 1.** Taking \( c = 0, c_1 = 0, c_2 = 1, d_1 = -1, d_2 = -2 \), we can obtain a type of singular soliton solution in the form of blow up waves with finite amplitude for nonisospectral KP equation (1b) and Fig.1 shows the detail structure

\[
\begin{align*}
\xi_1 &= \frac{-2x^2 + 2x \eta + 4y}{t + 1}, \\
\xi_1 &= \frac{-2x^2 + 2x \eta + 4y}{t + 1}
\end{align*}
\]

**Case 2.** Taking \( c = 2, c_1 = -1, c_2 = 1, d_1 = 0, d_2 = 3 \), a type of singular soliton solution in the form of periodic blow up waves is obtained for nonisospectral KP equation (1b) and given out directly in Fig.2

\[
\begin{align*}
\xi_2 &= \frac{2x^2 - 2x \eta + 4y}{t^2 - 1}, \\
\xi_2 &= \frac{2x^2 - 2x \eta + 4y}{t^2 - 1}
\end{align*}
\]

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Case 3. Taking $c = 3, c_1 = 1, c_2 = -2, d_1 = 0, d_2 = 3$, we can obtain another type singular soliton solution in the form of periodic blow up waves for nonisospectral KP equation (1b) and Fig.3 shows the detail structure

$$u = \left[ \ln \left( \frac{12t^2 \exp(\xi_3) + t(6 \exp(\xi_3) + g_3(x, y)) - 3t + g_3(x, y) - 3}{t + 1} \right) \right]_{xx},$$  \tag{51}

where $g_3(x, y) = -\int \frac{f_3(x, y, t) \exp(\xi_3)}{r(t+1)^3} dt$, $f_3(x, y, t) = (1 - \xi_3) r^2(t + 1)^4 + (2x - t) r(t + 1)^3 - 2y(t + 6) r(t + 1) - 2(x + 6) r(t + 1)^2 + 2y(t - 6)(t + 1)^3 - 12yt^2$ and $\xi_3 = \frac{-2x^2 - 2x - 8yt - 4y}{r(t+1)^3}$.

**Fig.3** Singular soliton solution in the form of periodic blow up waves for nonisospectral KP equation (1b) with Eq.(51) at $t = 1$ and $t = 2$.

4 Conclusion

As we well know, we can obtain some soliton solutions by bilinear method in Ref.[26, 27]. In our paper, we construct the Binary Darboux-Bäcklund transformation for the nonisospectral KP equation (1a). Then we obtained three new singular soliton solutions for the nonisospectral KP equation (1b), such as singular soliton solution in the form of blow up waves with finite amplitude, another two types of singular soliton solution in the form of periodic blow up waves. Selecting different initial values, we will have a wide range of singular multi-soliton solutions.

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