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Adaptive Backstepping Control of the Uncertain Unified Chaotic System

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Abstract: In this paper, an improved adaptive backstepping design is proposed to control the uncertain unified chaotic system. With the method, parameters identification and control can be achieved simultaneously with only one controller within finite steps based on Lyapunov stability theory. This paper also proves that the adaptive backstepping design method with only one controller presented in [12] is unavailable. Numerical stimulations are provided to demonstrate the effectiveness of the method discussed in this paper and the unavailability of the method with only one controller presented in [12].

Keywords: uncertain unified chaotic system, adaptive backstepping, chaotic control

1 Introduction

In recent years, there has been considerable interest in the control and application of chaos in nonlinear dynamical systems. For the past years, many different techniques have been proposed to control chaos, such as OGY method, differential geometric approach, linear state space feedback, adaptive control, fuzzy control and backtepping design [1-12]. In this present paper, we further develop the adaptive control technique introduced in Ref. [11]. With the developed method, parameter identification and control of the uncertain unified chaotic system can be achieved simultaneously with only one controller.

In the following section, the dynamics of the unified chaotic system is presented. In Section 3 the control of the uncertain unified chaotic system is discussed. In Section 5 we prove that the adaptive backstepping design method with only one controller presented by Ref. [12] is unavailable. Numerical simulations in Section 4 and Section 5 are provided to show the effectiveness of the method discussed in Section 3 and the unavailability of the method with only one controller presented by Ref. [12], respectively. Finally a concluding remark is given.

In 1963, Lorenz found the first canonical chaotic attractor [13]. In 1999, Chen found another similar but not topological equivalent chaotic attractor [14], as the dual of Lorenz system, in a sense defined by Vanecek and Celikovsky [15]: the Lorenz system satisfies the condition $a_{12}a_{21} > 0$ while Chen system satisfies $a_{12}a_{21} < 0$, where a_{12} and a_{21} are the corresponding elements in the constant matrix $A = (a_{ij})_{3\times3}$ for the linear part of the system. In 2002, Lü and Chen found a new chaotic system [16], bearing the name of the Lü system, which satisfies the condition $a_{12}a_{21} = 0$, thereby bridging the gap between the Lorenz and Chen attractors [17]. In the same year, Lü et al. unified above the three chaotic systems into one chaotic system which is called unified chaotic system[18].

The nonlinear differential equations that describe the unified chaotic system are

$$\begin{cases} \dot{x}_1 = (25\theta + 10)(x_2 - x_1), \\ \dot{x}_2 = (28 - 35\theta)x_1 - x_1x_3 + (29\theta - 1)x_2, \\ \dot{x}_3 = x_1x_2 - \frac{\theta+8}{3}x_3, \end{cases} \tag{1}$$

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where $\theta \in [0,1]$. It was shown in Ref. [19] that system (1) exhibits chaotic behavior for any $\theta \in [0,1]$. When $\theta \in [0,0.8)$, sysyem (1) is called the generalized Lorenz system; when $\theta = 0.8$, it becomes the generalized Lü system; when $\theta \in (0.8,1]$, system(1) is called the generalized Chen system. In fact, system (1) also shows chaotic behavior for $\theta = 1.1$ [20] with the initial condition $[x_1(0), x_2(0), x_3(0)] = [20, 20, 20]$.

2 The improved adaptive backstepping control with only one controller

The controlled system of system (1) is described as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = bx_1 - x_1x_3 + cx_2, \\ \dot{x}_3 = x_1x_2 - kx_3 + u, \end{cases} \tag{2}$$

where a, b, c and k are unknown parameters to be identified, and $a = 25\theta + 10$, $b = 28 - 35\theta$, $c = 29\theta - 1$, $k = \frac{\theta+8}{3}$, u is a controller to be designed. Since the system parameter θ is unknown, the parameters a, b, c and k are also unknown. Here we suppose that a is positive. Define three error variables

$$\begin{cases} z_1 = x_1, \\ z_2 = x_2 - \alpha_1, \\ z_3 = x_3 - \alpha_2, \end{cases} \tag{3}$$

where α_1 and α_2 are virtual feedback functions to be defined [21].

Step 1: The derivative of z_1 is expressed as

$$\dot{z}_1 = \dot{x}_1 = a(x_2 - x_1) = az_2 - az_1 + a\alpha_1. \tag{4}$$

Using α_1 as a control to stabilize the z_1-subsystem defined by Eq. (4), we choose the following Lyapunov function: $V_1 = \frac{1}{2}z_1^2$. Calculating the derivative of V_1 along system (2), we have

$$\dot{V}_1 = z_1\dot{z}_1 = z_1(az_2 - az_1 + a\alpha_1) = az_1z_2 + a\alpha_1z_1 - az_1^2. \tag{5}$$

We can choose

$$\alpha_1 = c_0z_1 - c_1z_2, \tag{6}$$

where $c_0 \in R$and $c_1 \in R$. Then we have

$$\dot{V}_1 = a(1-c_1)z_1z_2 - a(1-c_0)z_1^2. \tag{7}$$

We will cancel the first term in the next step. According to Eq. (6), Eq. (4) can be written in the following form

$$\dot{z}_1 = a(1-c_1)z_2 - a(1-c_0)z_1. \tag{8}$$

Step 2: In this step, we deal with the singularity problem caused by the term $-x_1x_3$ in the second equation of system (2).

Define $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{k}$ as the estimates of a, b c and k, and introduce the parameters errors

$$\bar{a} = \hat{a} - a, \bar{b} = \hat{b} - b, \bar{c} = \hat{c} - c, \bar{k} = \hat{k} - k. \tag{9}$$

Then the derivative of z_2 is

$$\begin{aligned} &\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = bx_1 - x_1x_3 + cx_2 - \dot{\alpha}_1 \\ &= bz_1 - z_1z_3 - \alpha_2z_1 + cz_2 + c(c_0z_1 - c_1z_2) - c_0\dot{z}_1 + c_1\dot{z}_2 \\ &(1-c_1)\dot{z}_2 = -z_1z_3 - \alpha_2z_1 + cz_2 + bz_1 + cc_0z_1 - cc_1z_2 - ac_0(1-c_1)z_2 + ac_0(1-c_0)z_1 \\ &+ \bar{b}z_1 - \bar{b}z_1 + \bar{c}c_0z_1 - \bar{c}c_0z_1 + \bar{a}c_0(1-c_0)z_1 - \bar{a}c_0(1-c_0)z_1. \end{aligned} \tag{10}$$

Letting $c_1 \neq 1$, we have

$$\begin{aligned} \dot{z}_2 = \tfrac{1}{1-c_1}[&-z_1z_3 - \alpha_2z_1 - (1-c_1)(ac_0 - c)z_2 - \bar{b}z_1 - c_0\bar{c}z_1 - c_0(1-c_0)\bar{a}z_1 + (\bar{b}+b)z_1 \\ &+ c_0(\bar{c}+c)z_1 + c_0(1-c_0)(\bar{a}+a)z_1]. \end{aligned} \tag{11}$$

According to Eq. (8) and Eq. (11), we obtain the (z_1, z_2)-subsystem:

$$\begin{cases} \dot{z}_1 = a(1-c_1)z_2 - a(1-c_0)z_1, \\ \dot{z}_2 = \frac{1}{1-c_1}[-z_1z_3 - \alpha_2 z_1 - (1-c_1)(ac_0 - c)z_2 - \bar{b}z_1 - c_0\bar{c}z_1 - c_0(1-c_0)\bar{a}z_1 \\ \quad + (\bar{b}+b)z_1 + c_0(\bar{c}+c)z_1 + c_0(1-c_0)(\bar{a}+a)z_1]. \end{cases} \tag{12}$$

Using α_2 as a control to stabilize the (z_1, z_2)-subsystem (12), we choose the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\bar{a}^2 + \frac{1}{2}\bar{b}^2 + \frac{1}{2}\bar{c}^2. \tag{13}$$

Its time derivative is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 + \bar{a}\dot{\bar{a}} + \bar{b}\dot{\bar{b}} + \bar{c}\dot{\bar{c}} = -a(1-c_0)z_1^2 - (ac_0 - c)z_2^2 - \tfrac{1}{1-c_1}z_1z_2z_3 \\ &+ \bar{a}(\dot{\bar{a}} - \tfrac{c_0(1-c_0)}{1-c_1}z_1z_2) + \bar{b}(\dot{\bar{b}} - \tfrac{1}{1-c_1}z_1z_2) + \bar{c}(\dot{\bar{c}} - \tfrac{c_0}{1-c_1}z_1z_2) \\ &- \tfrac{1}{1-c_1}[\alpha_2 - a(1-c_1)^2 - b - \bar{b} - c_0(\bar{c}+c) - c_0(1-c_0)(\bar{a}+a)]. \end{aligned} \tag{14}$$

Choose

$$\begin{cases} \dot{\bar{a}} = \frac{c_0(1-c_0)}{1-c_1}z_1z_2 - m\bar{a}, \\ \dot{\bar{b}} = \frac{1}{1-c_1}z_1z_2 - n\bar{b}, \\ \dot{\bar{c}} = \frac{c_0}{1-c_1}z_1z_2 - r\bar{c}, \end{cases} \tag{15}$$

where $m > 0$,$n > 0$ and $r > 0$. Choose

$$\alpha_2 = a(1-c_1)^2 + b + \bar{b} + c_0(\bar{c}+c) + c_0(1-c_0)(\bar{a}+a), \tag{16}$$

then we have

$$\dot{V}_2 = -\tfrac{1}{1-c_1}z_1z_2z_3 - a(1-c_0)z_1^2 - (ac_0 - c)z_2^2 - m\bar{a}^2 - n\bar{b}^2 - r\bar{c}^2. \tag{17}$$

We will cancel the first term $-\frac{1}{1-c_1}z_1z_2z_3$ in the next step. By using Eq. (16), Eq. (11) can be written in the follwing form

$$\dot{z}_2 = -\tfrac{1}{1-c_1}z_1z_3 - \tfrac{z_1}{1-c_1}[\bar{b} + c_0\bar{c} + c_0(1-c_0)\bar{a}] - (ac_0 - c)z_2 - a(1-c_1)z_1. \tag{18}$$

Step 3: The derivative of z_3 is

$$\begin{aligned} \dot{z}_3 &= \dot{x}_3 - \dot{\alpha}_2 = u - kx_3 + x_1x_2 - \dot{\alpha}_2 \\ &= u + c_0z_1^2 + (1-c_1)z_1z_2 + \bar{k}z_3 - (\bar{k}+k)z_3 - k\alpha_2 - \dot{\alpha}_2, \end{aligned} \tag{19}$$

where

$$\dot{\alpha}_2 = \frac{\partial\alpha_2}{\partial\bar{a}}\dot{\bar{a}} + \frac{\partial\alpha_2}{\partial\bar{b}}\dot{\bar{b}} + \frac{\partial\alpha_2}{\partial\bar{c}}\dot{\bar{c}} = c_0(1-c_0)\dot{\bar{a}} + \dot{\bar{b}} + c_0\dot{\bar{c}}. \tag{20}$$

Then we get the following system in the (z_1, z_2, z_3)-coordinates:

$$\begin{cases} \dot{z}_1 = a(1-c_1)z_2 - a(1-c_0)z_1, \\ \dot{z}_2 = -\frac{1}{1-c_1}z_1z_3 - \frac{z_1}{1-c_1}[\bar{b} + c_0\bar{c} + c_0(1-c_0)\bar{a}] - a(1-c_1)z_1 - (ac_0 - c)z_2, \\ \dot{z}_3 = u + c_0z_1^2 + (1-c_1)z_1z_2 + \bar{k}z_3 - (\bar{k}+k)z_3 - k\alpha_2 - c_0(1-c_0)\dot{\bar{a}} - \dot{\bar{b}} - c_0\dot{\bar{c}}. \end{cases} \tag{21}$$

In the following step, we will choose an appropriate input u to stabilize the system (21).
Considering the following Lyapunov function

$$V_3 = V_2 + \frac{1}{2(1-c_1)^2}z_3^2 + \frac{1}{2}\bar{k}^2, \tag{22}$$

we can get the derivative of V_3

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + \tfrac{1}{(1-c_1)^2}z_3\dot{z}_3 + \bar{k}\dot{\bar{k}} \\ &= \tfrac{z_3}{(1-c_1)^2}[u + c_0z_1^2 - (\bar{k}+k)z_3 - k\alpha_2 - c_0(1-c_0)\dot{\bar{a}} - \dot{\bar{b}} - c_0\dot{\bar{c}}] \quad . \\ &+ \bar{k}[\dot{\bar{k}} + \tfrac{z_3^2}{(1-c_1)^2}] - a(1-c_0)z_1^2 - (ac_0 - c)z_2^2 - m\bar{a}^2 - n\bar{b}^2 - r\bar{c}^2. \end{aligned} \tag{23}$$

We choose the following updated law

$$\dot{\bar{k}} = -\frac{1}{(1-c_1)^2} z_3^2 - h\bar{k}, \tag{24}$$

where $h > 0$. Letting

$$u = -(1-c_1)^2 c_3 z_3 - c_0 z_1^2 + (\bar{k}+k) z_3 + k\alpha_2 + c_0(1-c_0)\dot{\bar{a}} + \dot{\bar{b}} + c_0\dot{\bar{c}}, \tag{25}$$

where $c_3 > 0$, so we have

$$\dot{V}_3 = -a(1-c_0)z_1^2 - (ac_0 - c)z_2^2 - c_3 z_3^2 - m\bar{a}^2 - n\bar{b}^2 - r\bar{c}^2 - h\bar{k}^2. \tag{26}$$

Since $a > 0, m > 0, n > 0, r > 0, h > 0$ and $c_3 > 0$, we suppose that c_0 satisfies

$$\begin{cases} 1 - c_0 > 0, \\ ac_0 - c > 0. \end{cases} \tag{27}$$

Then we have

$$\frac{c}{a} < c_0 < 1. \tag{28}$$

According to Eq. (26) and Eq. (28),we obtain that $\dot{V}_3$ is negative definite. According to Eq. (25), the Eq. (19) can be written in the follwing form

$$\dot{z}_3 = -(1-c_1)^2 c_3 z_3 + (1-c_1) z_1 z_2 + \bar{k} z_3. \tag{29}$$

According to Eq. (15), Eq. (21), Eq. (25) and Eq. (29), we get the following $(z_1, z_2, z_3, \bar{a}, \bar{b}, \bar{c}, \bar{k})$-system:

$$\begin{cases} \dot{z}_1 = a(1-c_1)z_2 - a(1-c_0)z_1, \\ \dot{z}_2 = -\frac{1}{1-c_1} z_1 z_3 - \frac{z_1}{1-c_1}[\bar{b} + c_0\bar{c} + c_0(1-c_0)\bar{a}] - a(1-c_1)z_1 - (ac_0 - c)z_2, \\ \dot{z}_3 = -(1-c_1)^2 c_3 z_3 + (1-c_1)z_1 z_2 + \bar{k} z_3, \\ \dot{\bar{a}} = \frac{c_0(1-c_0)}{1-c_1} z_1 z_2 - m\bar{a}, \\ \dot{\bar{b}} = \frac{1}{1-c_1} z_1 z_2 - n\bar{b}, \\ \dot{\bar{c}} = \frac{c_0}{1-c_1} z_1 z_2 - r\bar{c}, \\ \dot{\bar{k}} = -\frac{1}{(1-c_1)^2} z_3^2 - h\bar{k}. \end{cases} \tag{30}$$

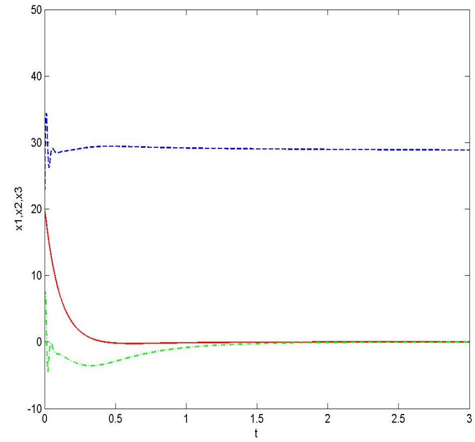
Since V_3 is a positive function and $\dot{V}_3$ is negative definite, it follows that the system (30) is globally asymptotically stabilized at the equilibrium $(z_1 = 0, z_2 = 0, z_3 = 0, \bar{a} = 0, \bar{b} = 0, \bar{c} = 0, \bar{k} = 0)$. In view of $z_1 = x_1$, $\alpha_1 = c_0 z_1 - c_1 z_2$ and $z_2 = x_2 - \alpha_1$, we know that the states x_1 and x_2 go to zero asymptotically. From $\alpha_2 = a(1-c_1)^2 + b + \bar{b} + c_0(\bar{c}+c) + c_0(1-c_0)(\bar{a}+a)$ and $z_3 = x_3 - \alpha_2$, we have $x_3 \to a(1-c_1)^2 + b + c + c_0(1-c_0)a$, as $t \to +\infty$, i.e. x_3 is bounded. From $u = -(1-c_1)^2 c_3 z_3 - c_0 z_1^2 + (\bar{k}+k)z_3 + k\alpha_2 + c_0(1-c_0)\dot{\bar{a}} + \dot{\bar{b}} + c_0\dot{\bar{c}}$, we can conclude that the control u is also bounded.

3 Numerical simulations

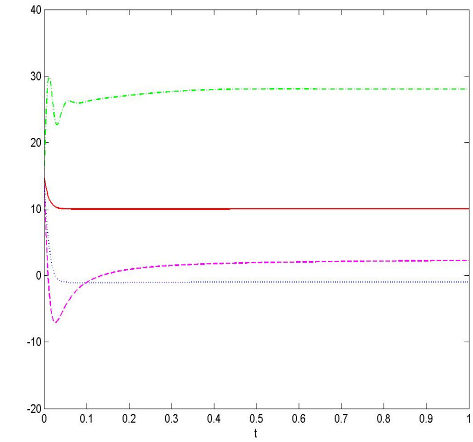
Numerical simulations show the effectiveness of the above methods. We assume that $\theta = 0$, i.e. $[a, b, c, k] = [10, 28, -1, \frac{8}{3}]$, initial conditions $c_0 = 0.05$, $c_1 = 0.90$; $\theta = 1.1$, i.e. $[a, b, c, k] = [37.5, -10.5, 30.9, \frac{9.1}{3}]$, initial conditions $c_0 = 0.85$, $c_1 = 0.90$. We select the initial conditions $[x_1(0), x_2(0), x_3(0)] = [20, 20, 20]$, $[\hat{a}, \hat{b}, \hat{c}, \hat{k}] = [15, 15, 15, 15]$, $c_3 = 10$, and $[m, n, r, h] = [100, 100, 100, 100]$ when $\theta = 1.1$. When $\theta = 0$, the initial values of the system are selected as $[x_1(0), x_2(0), x_3(0)] = [20, 1.3, 20]$, $[\hat{a}, \hat{b}, \hat{c}, \hat{k}] = [15, 15, 15, 15]$, $c_3 = 10$ and $[m, n, r, h] = [100, 100, 100, 100]$.

Fig.1 shows the effectiveness of the improved adaptive backstepping control techniques. Fig.1(a) displays the time response of the states (x_1, x_2, x_3) of the controlled Lorenz system. Fig.1(b) shows the parameters identification results of the controlled Lorenz system. Fig.1(c) is the controllers' action of the controlled Lorenz system. As can be seen from the figures, the three controllers are bounded as $t \to +\infty$. Fig.1(d) shows that the Lorenz system is controlled to bounded points.

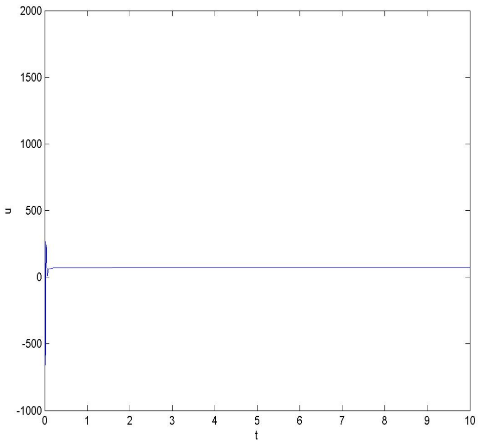
Fig.2 shows that the system can also be controlled to a bounded point with the designed controller and updated laws, when the parameter θ of the following system (1) satisfies $\theta \notin [0, 1]$ and $\theta = 1.1$. Fig.2(b) shows the system parameters identification results. Fig.2(c) shows that the controller u is bounded as $t \to +\infty$. Fig.2(d) shows that the system (1) with $\theta = 1.1$ is controlled to a bounded point.



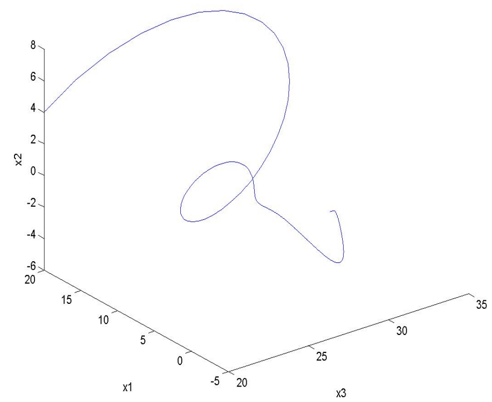
(a) System states: x_1, x_2, x_3;



(b) System parameters identification: $\hat{a}, \hat{b}, \hat{c}, \hat{k}$;



(c) Control action u;



(d) Three-dimensional view of the controlled system.

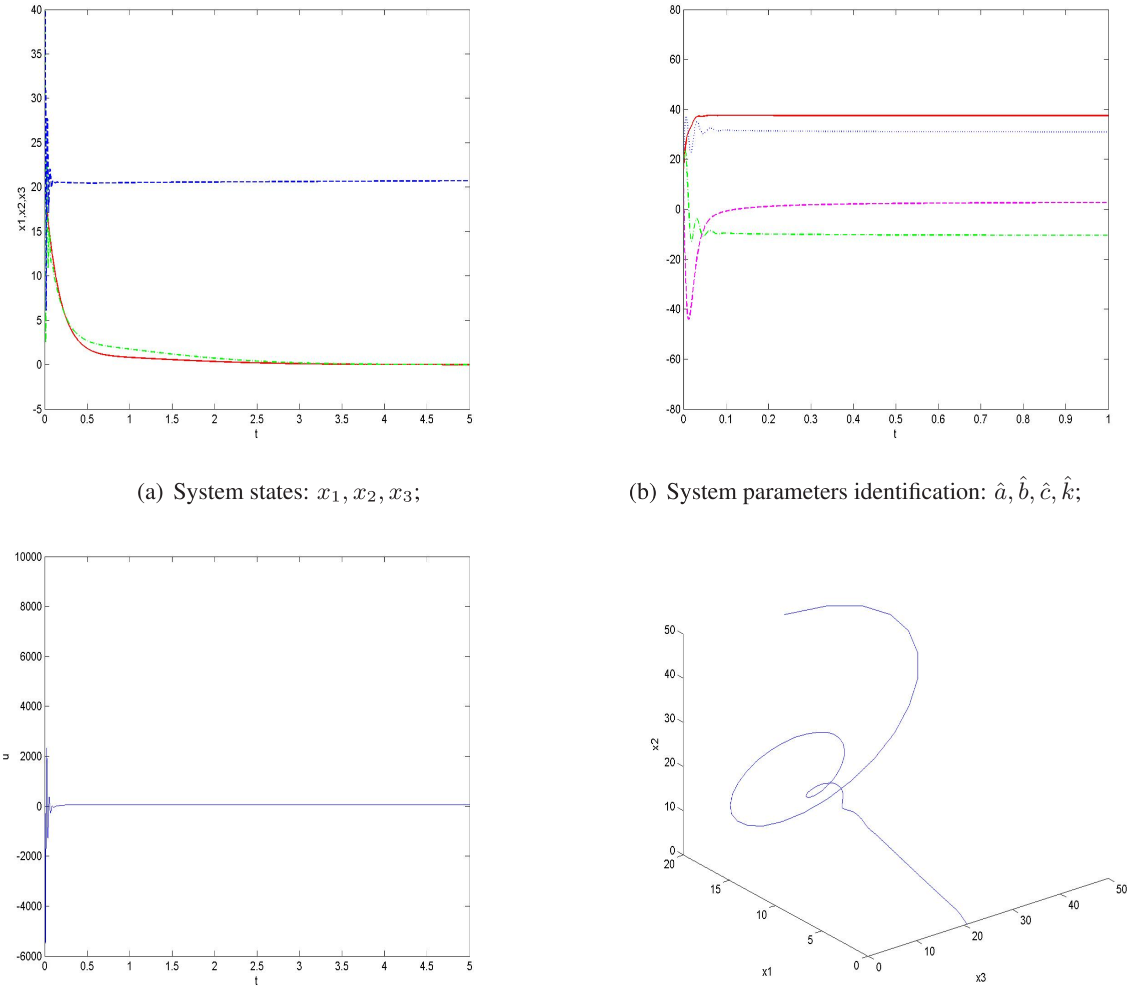
Figure 1: The control effectiveness of unified chaotic system with $\theta = 0$.

4 An unavailable adaptive backstepping design

In this paper, we suppose that the parameter c_0 satisfies $\frac{c}{a} < c_0 < 1$, i.e. the parameters a and c satisfy $c < a$. Since $a = 25\theta + 10$, $c = 29\theta - 1$ and $\theta \in [0, 1]$, we can conclude that $0 < c < a$ when $\theta \in [0, 1]$. However, since the parameter θ is unknown, i.e. the parameters a, b, c and k are unknown, it is not practical to suppose this condition. Considering the real-world application, Wu and Lu proposed an adaptive backstepping design with only one controller in studying the control of uncertain Lü system [12]. With the controller u_3 designed in [12], it is concluded that the parameters identification and control of uncertain Lü system can be achieved simultaneously in mere supposition that parameter a is positive. In the following, we will prove that the conclusions made in [12] by using the adaptive backstepping design with only one controller is not correct.

By using the controller u_3 which is described by Eq. (18) in [12], it does not guarantee that the controlled uncertain Lü system converges to a bounded point for any initial values $[x(0), y(0), z(0), \hat{a}(0), \hat{b}(0), \hat{c}(0)]$ as $t \to +\infty$, when $a > 0$, $q > 0$ and $0 < p < 1$. The Eq.(18) in [12] is described as

$$u_3 = -q\bar{z} + \hat{b}z - px^2 - \frac{c\bar{y}^2}{\bar{z}} + \frac{\partial \alpha_2}{\partial \bar{a}}\dot{\bar{a}} + \frac{\partial \alpha_2}{\partial \bar{c}}\dot{\bar{c}}. \tag{31}$$



(a) System states: x_1, x_2, x_3;

(b) System parameters identification: $\hat{a}, \hat{b}, \hat{c}, \hat{k}$;

(c) Control action u;

(d) Three-dimensional view of the controlled system.

Figure 2: The control effectiveness of unified chaotic system with $\theta = 1.1$.

The Eq. (20) in Ref. [12] is described as

$$\begin{cases} \dot{x} = a\bar{y} - a(1-p)x, \\ \dot{\bar{y}} = -x\bar{z} - x\alpha_2 + (c - ap)\bar{y} + \bar{c}px + \bar{a}p(1-p)x - (\bar{c} - c)px - (\bar{a} - a)p(1-p)x, \\ \dot{\bar{z}} = xy - bz - \frac{\partial \alpha_2}{\partial \bar{a}}\dot{\bar{a}} - \frac{\partial \alpha_2}{\partial \bar{c}}\dot{\bar{c}} + u_3, \\ \dot{\bar{a}} = (1 + p - p^2)x\bar{y} - \bar{a}, \\ \dot{\bar{b}} = -z\bar{z} - \bar{b}, \\ \dot{\bar{c}} = px\bar{y} - \bar{c}. \end{cases} \tag{32}$$

According to Eq. (31) and Eq. (32), we get the following $(x, \bar{y}, \bar{z}, \bar{a}, \bar{b}, \bar{c})$-system:

$$\begin{cases} \dot{x} = a\bar{y} - a(1-p)x, \\ \dot{\bar{y}} = -x\bar{z} - x\alpha_2 + (c - ap)\bar{y} + \bar{c}px + \bar{a}p(1-p)x - (\bar{c} - c)px - (\bar{a} - a)p(1-p)x, \\ \dot{\bar{z}} = xy - px^2 - q\bar{z} + \bar{b}z - \frac{c\bar{y}^2}{\bar{z}}, \\ \dot{\bar{a}} = (1 + p - p^2)x\bar{y} - \bar{a}, \\ \dot{\bar{b}} = -z\bar{z} - \bar{b}, \\ \dot{\bar{c}} = px\bar{y} - \bar{c}, \end{cases} \tag{33}$$

where $a > 0$, $0 < p < 1$, $q > 0$, $\alpha_2 = \bar{a} + a + (\bar{c} + c)p + (\bar{a} + a)p(1-p)$, $y = \bar{y} + px$ and $z = \bar{z} + \alpha_2$.

According to the conclusion made in [12], system (33) is global asymptotically stabilized at the origin point. From the third equation $\dot{\bar{z}} = xy - px^2 - q\bar{z} + \bar{b}z - \frac{c\bar{y}^2}{\bar{z}}$, we have that the state $\bar{z}$ satisfies $\bar{z} \neq 0$. Then we conclude that the origin point is not the equilibrium of system (33). So we can not conclude that the system (33) is global asymptotically stabilized by using Lyapunov stability theory, i.e. we can not conclude that $x \to 0, \bar{y} \to 0, \bar{z} \to 0, \bar{a} \to 0, \bar{b} \to 0, \bar{c} \to 0$ for some initial conditions $[x(0), \bar{y}(0), \bar{z}(0), \bar{a}(0), \bar{b}(0), \bar{c}(0)]$ as $t \to +\infty$. It further follows that, the controlled uncertain Lü system can not be guaranteed to converge to a bounded point for some initial values $[x(0), y(0), z(0)]$ as $t \to +\infty$, when $a > 0q > 0$ and $0 < p < 1$. We also can not conclude that $\hat{a} \to a, \hat{b} \to b, \hat{c} \to c$ for some initial conditions $[\hat{a}(0), \hat{b}(0), \hat{c}(0)]$ as $t \to +\infty$. Consequently, it fails to identify the unknown parameters of the uncertain Lü system.

For system (1), according to the adaptive backstepping design with only one controller presented in [12], we obtain the following $(z_1, z_2, z_3, \bar{a}, \bar{b}, \bar{c}, \bar{k})$ -system:

$$\begin{cases} \dot{z}_1 = a(1-c_1)z_2 - a(1-c_0)z_1, \\ \dot{z}_2 = -\frac{1}{1-c_1}z_1z_3 - \frac{z_1}{1-c_1}[\bar{b} + c_0\bar{c} + c_0(1-c_0)\bar{a}] - a(1-c_1)z_1 - (ac_0 - c)z_2, \\ \dot{z}_3 = -(1-c_1)^2c_3z_3 + (1-c_1)z_1z_2 + \bar{k}z_3 - \frac{c(1-c_1)^2}{z_3}z_2^2, \\ \dot{\bar{a}} = \frac{c_0(1-c_0)}{1-c_1}z_1z_2 - m\bar{a}, \\ \dot{\bar{b}} = \frac{1}{1-c_1}z_1z_2 - n\bar{b}, \\ \dot{\bar{c}} = \frac{c_0}{1-c_1}z_1z_2 - r\bar{c}, \\ \dot{\bar{k}} = -\frac{1}{(1-c_1)^2}z_3^2 - h\bar{k}, \end{cases} \tag{34}$$

where $a > 0, m > 0, n > 0, r > 0, h > 0, c_3 > 0, 0 < c_0 < 1, c_1 \in R$ and $c_1 \neq 1$. The variables in system (34) are defined as the same as system (29). The controller u is defined as

$$u = -(1-c_1)^2c_3z_3 - c_0z_1^2 - \frac{c(1-c_1)^2}{z_3}z_2^2 + (\bar{k} + k)z_3 + k\alpha_2 + c_0(1-c_0)\dot{\bar{a}} + \dot{\bar{b}} + c_0\dot{\bar{c}} \tag{35}$$

When we choose the initial condition $[x_1(0), x_2(0), x_3(0)] = [100, 100, 100]$, without changing other initial conditions, the numerical simulations can not be completed by using Matlab program. However, for these initial conditions, we can get the results of numerical simulation, the parameter identification and control of uncertain unified chaotic system are both achieved. Hence, we can conclude that the controlled chaotic system (2) still exist sensitive dependence in the initial conditions by using the controller u which is described by Eq. (35). As a result, it fails to control the chaotic system (1). It further follows that the adaptive backstepping design with only one controller proposed in [12] is unavailable.

When we choose the initial condition $c_0 = 0.70$, without changing other initial conditions, the numerical simulations can not be completed yet by using Matlab program. It demonstrates that the controlled uncertain unified chaotic system can not be guaranteed to converge to a bounded point for some initial conditions $c_0 \in (0, 1)$. Then we get a conclusion that the adaptive backstepping design with only one controller proposed in [12] is unavailable.

5 Conclusion

The control problem of the unified chaotic system is investigated. With the improved adaptive backstepping techniques, parameter identification and control can be achieved simultaneously with only one controller. The paper has also proved that the adaptive backstepping design with only one controller proposed in [12] is unavailable. Numerical simulations show the effectiveness and feasibility of the developed design method discussed in Section 3 and the unavailability of the method with only one controller proposed in [12].

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