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Nonlinear Schrödinger Equations and N=2 Superconformal Algebra

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Abstract: We obtain new coupled super Nonlinear Schrödinger equations by using AKNS scheme and soliton connection taking values in N=2 superconformal algebra .

Keywords: AKNS scheme; soliton connection; coupled super nonlinear Schrödinger equation; N=2 superconformal algebra

1 Introduction

Coupled Nonlinear Schrödinger (NLS) equations can be obtained using Ablowitz, Kaub, Newell, Segur (AKNS) scheme [1–3]. Extensions of coupled NLS equations have been obtained using a simple Lie algebra[4], a Kac-Moody algebra [5], a Lie superalgebra [5–7], a Virasoro algebra[8] and a N=1 superconformal algebra [9]in the literature.

The N=2 superconformal algebras were discovered in the seventies independently by Ademollo et al.[10] and by Kac [11]. The first authors derived the algebras for physical purposes, in order to define supersymmetric strings, whereas Kac derived them for mathematical purposes along with his classification of Lie superalgebras.

The N=2 superconformal algebras provide the symmetries underlying the N=2 strings[12–20]. These seem to be related to M-theory since many of the basic objects of M-theory are realized in the heterotic (2,1) N=2 strings[21]. In addition, the topological version of the N=2 superconformal algebra is realized in the world-sheet of the bosonic string[22, 23], as well as in the world-sheet of the superstrings [24].

In literature super-extensions of Nonlinear Schröedinger equations involve finite number of bosonic and finite number of fermionic fields [5–7, 25–27]. In literature super-extensions of Korteweg-de Vries equation with finite number of fields is also obtained using Hamiltonian reduction formalism (see[28]). The super-extensions of Nonlinear Schroedinger equations with infinite number of bosonic fields and infinite number of fermionic fields has been studied in ref.[9] and in this paper.Mathematically this paper contains extension of the work done in ref.[9]. The super-extensions of Korteweg-de Vries equation with infinite number of bosonic fields and infinite number of fermionic fields is an open problem.

In this paper we will obtain super - extensions of coupled NLS equations using N=2 superconformal algebra with Neveu-Schwarz and Ramond types. In sec.2 we will discuss the sl(2,1) superalgebra valued soliton connection and we will obtain coupled super NLS equations. Sec.3 and sec.4 concern the soliton connection for the N=2 superconformal algebra with Neveu-Schwarz type and Ramond type, respectively and we will obtain in these sections two different types of super-extensions of coupled NLS equations.

2 AKNS Scheme with sl(2,1) Superalgebra

In AKNS scheme in 1+1 dimension the connection is defined as

$$\Omega = \Omega_b + \Omega_f \tag{1}$$

where

$$\Omega_b = \Big(i\lambda J_1 + i\lambda H_1 + Q^{+1}E_{+1} + Q^{-1}E_{-1}\Big)dx + \Big(-A_1 J_1 - A_2 H_1 + B^{+1}E_{+1} + B^{-1}E_{-1}\Big)dt \tag{2}$$

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$$\Omega_f = \Big(P_1^{+\frac{1}{2}}F_{+\frac{1}{2}} + P_1^{-\frac{1}{2}}F_{-\frac{1}{2}} + P_2^{+\frac{1}{2}}\bar{F}_{+\frac{1}{2}} + P_2^{-\frac{1}{2}}\bar{F}_{-\frac{1}{2}}\Big)dx + \Big(C_1^{+\frac{1}{2}}F_{+\frac{1}{2}} + C_1^{-\frac{1}{2}}F_{-\frac{1}{2}} + C_2^{+\frac{1}{2}}\bar{F}_{+\frac{1}{2}} + C_2^{-\frac{1}{2}}\bar{F}_{-\frac{1}{2}}\Big)dt \tag{3}$$

where J_1, H_1, $E_{\pm1}$ are bosonic generators and $F_{\pm\frac{1}{2}}\bar{F}_{\pm\frac{1}{2}}$ are fermionic generators of $sl(1,2) \cong sl2,1)$ superalgebra which is the (N=2)extended supersymmetric version of sl(2) algebra. These generators have matrix representations as

$$H_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}; J_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}; E_{+1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; E_{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$F_{+\frac{1}{2}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}; F_{-\frac{1}{2}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \bar{F}_{+\frac{1}{2}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \bar{F}_{-\frac{1}{2}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

also, these generators satisfy the following commutation and anticommutation relations

$$\begin{array}{ccc} [H_1, E_{\pm1}] = \pm E_{\pm1}; & [H_1, F_{\pm1}] = \pm\frac{1}{2}F_{\pm1}; & [H_1, \bar{F}_{\pm1}] = \pm\frac{1}{2}\bar{F}_{\pm1} \\ [J_1, H_{\pm1}] = [J_1, E_{\pm1}] = 0; & [J_1, F_{\pm1}] = \frac{1}{2}F_{\pm1}; & [J_1, \bar{F}_{\pm1}] = -\frac{1}{2}\bar{F}_{\pm1} \\ \Big[E_{\pm1}, F_{\mp\frac{1}{2}}\Big] = \Big[E_{\pm1}, \bar{F}_{\mp\frac{1}{2}}\Big] = 0; & \Big[E_{\pm1}, F_{\mp\frac{1}{2}}\Big] = -F_{\pm\frac{1}{2}}; & \Big[E_{\pm1}, \bar{F}_{\mp\frac{1}{2}}\Big] = \bar{F}_{\pm\frac{1}{2}} \\ \Big\{F_{\pm1}, F_{\pm\frac{1}{2}}\Big\} = \Big\{\bar{F}_{\pm1}, \bar{F}_{\pm\frac{1}{2}}\Big\} = 0; & \Big\{F_{\pm1}, F_{\mp\frac{1}{2}}\Big\} = \Big\{\bar{F}_{\pm1}, \bar{F}_{\mp\frac{1}{2}}\Big\} = 0; & \Big\{F_{\pm1}, \bar{F}_{\mp\frac{1}{2}}\Big\} = E_{\pm1} \\ [E_{+1}, E_{-1}] = H_1; & \Big\{F_{\pm\frac{1}{2}}, \bar{F}_{\pm\frac{1}{2}}\Big\} = J_1 \mp H_1. & \end{array} \tag{5}$$

In Eq.(1) λ is the spectral parameter, $Q^{\pm1}$, $P_1^{\pm\frac{1}{2}}$ and $P_2^{\pm\frac{1}{2}}$ are fields depending on space and time, namely x and t, and functions A_1, A_2, $B^{\pm1}$, $C_1^{\pm\frac{1}{2}}$ and $C_2^{\pm\frac{1}{2}}$ are x, t and λ dependent. The integrability condition is given by

$$d\,\Omega \;+\; \Omega \;\wedge\; \Omega = 0. \tag{6}$$

By using Eqs.(1) and (6) one can obtain following equations:

$$A_{1x} = P_1^{-\frac{1}{2}}C_2^{+\frac{1}{2}} + P_1^{+\frac{1}{2}}C_2^{-\frac{1}{2}} + P_2^{-\frac{1}{2}}C_1^{+\frac{1}{2}} + P_2^{+\frac{1}{2}}C_1^{-\frac{1}{2}} \tag{7}$$

$$A_{2x} = -2Q^{-1}B^{+1} + 2Q^{+1}B^{-1} + P_1^{-\frac{1}{2}}C_2^{+\frac{1}{2}} - P_1^{+\frac{1}{2}}C_2^{-\frac{1}{2}} - P_2^{-\frac{1}{2}}C_1^{+\frac{1}{2}} + P_2^{+\frac{1}{2}}C_1^{-\frac{1}{2}} \tag{8}$$

$$Q^{+1}{}_t = \; B^{+1}{}_x + i\lambda B^{+1} + Q^{+1}A_2 + P_1^{+\frac{1}{2}}C_2^{+\frac{1}{2}} + P_2^{+\frac{1}{2}}C_1^{+\frac{1}{2}} \tag{9}$$

$$Q^{-1}{}_t = \; B^{-1}{}_x - i\lambda B^{-1} - Q^{-1}A_2 + P_1^{-\frac{1}{2}}C_2^{-\frac{1}{2}} + P_2^{-\frac{1}{2}}C_1^{-\frac{1}{2}} \tag{10}$$

$$P_1^{+\frac{1}{2}}{}_t = \; C_1^{+\frac{1}{2}}{}_x + i\lambda C_1^{+\frac{1}{2}} + P_1^{-\frac{1}{2}}B^{+1} + \tfrac{1}{2}P_1^{+\frac{1}{2}}A_1 + \tfrac{1}{2}P_1^{+\frac{1}{2}}A_2 - Q^{+1}C_1^{-\frac{1}{2}} \tag{11}$$

$$P_1^{-\frac{1}{2}}{}_t = \; C_1^{-\frac{1}{2}}{}_x + P_1^{+\frac{1}{2}}B^{-1} + \tfrac{1}{2}P_1^{-\frac{1}{2}}A_1 - \tfrac{1}{2}P_1^{-\frac{1}{2}}A_2 - Q^{-1}C_1^{+\frac{1}{2}} \tag{12}$$

$$P_2^{+\frac{1}{2}}{}_t = \; C_2^{+\frac{1}{2}}{}_x - P_2^{-\frac{1}{2}}B^{+1} - \tfrac{1}{2}P_2^{+\frac{1}{2}}A_1 + \tfrac{1}{2}P_2^{+\frac{1}{2}}A_2 + Q^{+1}C_2^{-\frac{1}{2}} \tag{13}$$

$$P_2^{-\frac{1}{2}}{}_t = \; C_2^{-\frac{1}{2}}{}_x - i\lambda C_2^{-\frac{1}{2}} - P_2^{+\frac{1}{2}}B^{-1} - \tfrac{1}{2}P_2^{-\frac{1}{2}}A_1 - \tfrac{1}{2}P_2^{-\frac{1}{2}}A_2 + Q^{-1}C_2^{+\frac{1}{2}}. \tag{14}$$

In AKNS scheme we expand A_1, A_2, $B^{\pm1}$, $C_1^{\pm\frac{1}{2}}$ and $C_2^{\pm\frac{1}{2}}$ in terms of positive powers of λ as

$$A_1 = \sum_{n=0}^{2}\lambda^n a_{1n}; \;\; A_2 = \sum_{n=0}^{2}\lambda^n a_{2n}; \;\; B^{\pm1} = \sum_{n=0}^{2}\lambda^n b_n^{\pm1}; \;\; C_1^{\pm\frac{1}{2}} = \sum_{n=0}^{2}\lambda^n c_{1n}^{\pm\frac{1}{2}}; \;\; C_2^{\pm\frac{1}{2}} = \sum_{n=0}^{2}\lambda^n c_{2n}^{\pm\frac{1}{2}}. \tag{15}$$

Inserting Eq.(15) into Eqs.(7-14)gives 24 relations in terms of a_{1n},a_{2n}, $b_n^{\pm1}$, $c_{1n}^{\pm\frac{1}{2}}$ and $c_{2n}^{\pm\frac{1}{2}}$. By solving these relations we get

$$a_{10} = -iP_1^{-\frac{1}{2}}P_2^{+\frac{1}{2}} - iP_1^{+\frac{1}{2}}P_2^{-\frac{1}{2}}; \;\; a_{11} = 0; \;\; a_{12} = -i;$$
$$a_{20} = -2iQ^{+1}Q^{-1} - iP_1^{-\frac{1}{2}}P_2^{+\frac{1}{2}} + iP_1^{+\frac{1}{2}}P_2^{-\frac{1}{2}}; \;\; a_{21} = 0; \;\; a_{22} = -i;$$
$$b_0^{\pm1} = \pm iQ^{\pm1}{}_x + iP_2^{\pm\frac{1}{2}}P_1^{\pm\frac{1}{2}}; \; b_1^{\pm1} = Q^{\pm1}; \; b_2^{\pm1} = 0 \tag{16}$$

$$c_{10}^{\pm\frac{1}{2}} = iP_1^{\mp\frac{1}{2}}Q^{\pm 1} + iP^{\pm}_{1\ x};\ c_{20}^{\pm\frac{1}{2}} = iP_2^{\mp\frac{1}{2}}Q^{\pm 1} - iP^{\pm}_{2\ x}$$
$$c_{11}^{+\frac{1}{2}} = P^{+\frac{1}{2}};\ c_{21}^{-\frac{1}{2}} = P^{-\frac{1}{2}};\ c_{11}^{-\frac{1}{2}} = c_{21}^{+\frac{1}{2}} = 0;\ c_{12}^{\pm\frac{1}{2}} = c_{22}^{\pm\frac{1}{2}} = 0.$$

By using the relations given by Eq.(16) from Eqs.(9-14) we obtain the coupled super NLS equations as

$$\begin{aligned}
-iQ^{+1}{}_t &= Q^{+1}{}_{xx} - 2(Q^{+1})^2Q^{-1} + 2Q^{+1}P_2^{+\frac{1}{2}}P_1^{-\frac{1}{2}} - 2Q^{+1}P_2^{-\frac{1}{2}}P_1^{+\frac{1}{2}} + 2(P_2^{+\frac{1}{2}}P_1^{+\frac{1}{2}})_x \\
iQ^{-1}{}_t &= Q^{+1}{}_{xx} - 2(Q^{-1})^2Q^{+1} - 2Q^{-1}P_2^{-\frac{1}{2}}P_1^{+\frac{1}{2}} + 2Q^{-1}P_2^{+\frac{1}{2}}P_1^{-\frac{1}{2}} - 2(P_2^{-\frac{1}{2}}P_1^{-\frac{1}{2}})_x \\
-iP_1^{+\frac{1}{2}}{}_t &= P_1^{+\frac{1}{2}}{}_{xx} - 2P_1^{+\frac{1}{2}}Q^{+1}Q^{-1} + 2P_1^{-\frac{1}{2}}Q^{+1}{}_x - 2P_2^{+\frac{1}{2}}P_1^{+\frac{1}{2}}P_1^{-\frac{1}{2}} \\
-iP_1^{-\frac{1}{2}}{}_t &= P_1^{-\frac{1}{2}}{}_{xx} - 2P_1^{-\frac{1}{2}}Q^{+1}Q^{-1} - 2P_2^{-\frac{1}{2}}P_1^{-\frac{1}{2}}P_1^{+\frac{1}{2}} \\
iP_2^{+\frac{1}{2}}{}_t &= P_2^{+\frac{1}{2}}{}_{xx} + 2P_2^{+\frac{1}{2}}Q^{+1}Q^{-1} + 2P_2^{+\frac{1}{2}}P_2^{-\frac{1}{2}}P_1^{+\frac{1}{2}} \\
iP_2^{-\frac{1}{2}}{}_t &= P_2^{-\frac{1}{2}}{}_{xx} - 2P_2^{-\frac{1}{2}}Q^{+1}Q^{-1} - 2P_2^{+\frac{1}{2}}Q^{-1}{}_x + 2P_2^{+\frac{1}{2}}P_2^{-\frac{1}{2}}P_1^{-\frac{1}{2}}.
\end{aligned} \tag{17}$$

3 AKNS Scheme with N=2 Superconformal Algebra (Neveu-Schwarz Type)

We generalize the connection given by Eq.(1) as

$$\Omega = \Omega_b + \Omega_f \tag{18}$$

where

$$\Omega_b = \begin{array}{l} \Big(i\lambda J_0 + i\lambda L_0 + Q_1^{+m}J_{+m} + Q_1^{-m}J_{-m} + Q_2^{+m}L_{+m} + Q_2^{-m}L_{-m}\Big)dx + \\ \Big(-A_1J_0 - A_2L_0 + B_1^{+m}J_{+m} + B_1^{-m}J_{-m} + B_2^{+m}L_{+m} + B_2^{-m}L_{-m}\Big)dt \end{array} \tag{19}$$

$$\Omega_f = \begin{array}{l} \Big(+P_1^{+\frac{m}{2}}G^1_{+\frac{m}{2}} + P_1^{-\frac{m}{2}}G^1_{-\frac{m}{2}} + P_2^{+\frac{m}{2}}G^2_{+\frac{m}{2}} + P_2^{-\frac{m}{2}}G^2_{-\frac{m}{2}}\Big)dx + \\ \Big(+C_1^{+\frac{m}{2}}G^1_{+\frac{m}{2}} + C_1^{-\frac{m}{2}}G^1_{-\frac{m}{2}} + C_2^{+\frac{m}{2}}G^2_{+\frac{m}{2}} + C_2^{-\frac{m}{2}}G^2_{-\frac{m}{2}}\Big)dt \end{array} \tag{20}$$

where L_0, $J_{\pm m}$, $L_{\pm m}$ are bosonic generators and $G^1_{\pm\frac{m}{2}}$, $G^2_{\pm\frac{m}{2}}$ are fermionic generators of centerless N=2 superconformal algebra of Neveu-Schwarz type. In modern notation namely this algebra satisfy the following commutation and anticommutation relations

$$\begin{array}{lcl}
[L_r, L_s] &=& (r-s)\, L_{r+s} \\
[J_r, J_s] &=& 0 \\
[L_r, J_s] &=& -s\, J_{r+s} \\
\{G^1_r, G^2_s\} &=& 2\, L_{r+s} + (r-s)\, J_{r+s} \\
\left[J_r, G^{1,2}_s\right] &=& \pm G^{1,2}_{r+s} \\
\left[L_r, G^{1,2}_s\right] &=& (\frac{r}{2} - s)\, G^{1,2}_{r+s} \\
\{G^{1,2}_r, G^{1,2}_s\} &=& 0
\end{array} \tag{21}$$

here, $J_{\pm m}$,$L_{\pm m}$ are generators with positive(negative) integer indices and $G^{1,2}_{\pm\frac{m}{2}}$ are with positive(negative) half integer indices. In Eq.(18) we assume summation over the repeated indices. The fields $Q^{\pm m}_{1,2}$ and $P^{\pm\frac{m}{2}}_{1,2}$ are x,t dependent and also functions $A_{1,2}$, $B^{\pm m}_{1,2}$ and $C^{\pm\frac{m}{2}}_{1,2}$ are x, t and λ dependent.

In N=2 superconformal algebra if we restrict $J_{\pm m}$ to have only J_0 components,$L_{\pm m}$ to have only L_0 , $L_{\pm 1}$ components, and $G^{1,2}_{\pm\frac{m}{2}}$ to have only $G^{1,2}_{\pm\frac{1}{2}}$ components we obtain sl(1,2) algebra given by Eq.(5) with the following definitions:

$$\begin{array}{l}
J_1 = \frac{1}{2}J_0;\ \ H_1 = -L_0;\ \ E_{+1} = L_{+1};\ \ E_{-1} = -L_{-1} \\
F_{+\frac{1}{2}} = G^1_{+\frac{1}{2}};\ \ F_{-} = -\frac{1}{2}G^1_{-\frac{1}{2}}; \bar{F}_{+\frac{1}{2}} = \frac{1}{2}G^2_{+\frac{1}{2}};\ \ \bar{F}_{\frac{1}{2}} = -\frac{1}{2}G^2_{-\frac{1}{2}}.
\end{array} \tag{22}$$

From the integrability condition given by Eq.(6) we obtain

$$A_{1x} = \sum_{r=1}^{\infty} r(B_1^{-r}Q_2^{+r} - B_1^{+r}Q_2^{-r} + B_2^{-r}Q_1^{+r} - B_2^{+r}Q_1^{-r} - P_1^{-\frac{r}{2}}C_2^{+\frac{r}{2}} + P_1^{+\frac{r}{2}}C_2^{-\frac{r}{2}} + P_2^{-\frac{r}{2}}C_1^{+\frac{r}{2}} - P_2^{+\frac{r}{2}}C_1^{-\frac{r}{2}}) \tag{23}$$

$$A_{2x} = 2\sum_{r=1}^{\infty}(rB_2^{-r}Q_2^{+r} - rB_2^{+r}Q_2^{-r} + P_1^{-\frac{r}{2}}C_2^{+\frac{r}{2}} + P_1^{+\frac{r}{2}}C_2^{-\frac{r}{2}} + P_2^{-\frac{r}{2}}C_1^{+\frac{r}{2}} + P_2^{+\frac{r}{2}}C_1^{-\frac{r}{2}}) \tag{24}$$

$$Q_1^{\pm m}{}_t = B_1^{\pm m}{}_x \mp im\lambda B_1^{\pm m} \mp mA_2 Q_1^{\pm m} + \delta_J^{\pm m} \tag{25}$$

$$Q_2^{\pm m}{}_t = B_2^{\pm m}{}_x \mp im\lambda B_2^{\pm m} \mp mA_2 Q_2^{\pm m} + \delta_L^{\pm m} \tag{26}$$

$$P_1^{\pm\frac{m}{2}}{}_t = C_1^{\pm\frac{m}{2}}{}_x \mp \tfrac{i}{2}(m \mp 2)\lambda C_1^{\pm\frac{m}{2}} \mp \tfrac{m}{2} P_1^{\pm\frac{m}{2}} A_2 + P_1^{\pm\frac{m}{2}} A_1 + \delta_{G^1}^{\pm\frac{m}{2}} \tag{27}$$

$$P_2^{\pm\frac{m}{2}}{}_t = C_2^{\pm\frac{m}{2}}{}_x \mp \tfrac{i}{2}(m \pm 2)\lambda C_2^{\pm\frac{m}{2}} \mp \tfrac{m}{2} P_2^{\pm\frac{m}{2}} A_2 - P_2^{\pm\frac{m}{2}} A_1 + \delta_{G^2}^{\pm\frac{m}{2}} \tag{28}$$

where

$$\begin{aligned}
\delta_J^{+m} = &\sum_{r,s=1}^{\infty} \Big[(rB_2^{+s}Q_1^{+r} - sB_1^{+s}Q_2^{+r})\delta_{r+s,m} + (r-s)(P_1^{+\frac{r}{2}}C_2^{+\frac{s}{2}} - P_2^{+\frac{r}{2}}C_1^{+\frac{s}{2}})\delta_{r+s,2m}\Big] \\
&+ \sum_{\substack{r,s=1\\ r>s}}^{\infty} \Big[(rB_2^{-s}Q_1^{+r} + sB_1^{-s}Q_2^{+r})\delta_{r-s,m} + \frac{(r+s)}{2}(P_1^{+\frac{r}{2}}C_2^{-\frac{s}{2}} - P_2^{+\frac{r}{2}}C_1^{-\frac{s}{2}})\delta_{r-s,2m}\Big] \\
&- \sum_{\substack{r,s=1\\ r<s}}^{\infty} \Big[(rB_2^{+s}Q_1^{-r} + sB_1^{+s}Q_2^{-r})\delta_{-r+s,m} + \frac{(r+s)}{2}(P_1^{-\frac{r}{2}}C_2^{+\frac{s}{2}} - P_2^{-\frac{r}{2}}C_1^{+\frac{s}{2}})\delta_{-r+s,2m}\Big]
\end{aligned} \tag{29}$$

$$\begin{aligned}
\delta_L^{+m} = &\sum_{r,s=1}^{\infty} \Big[(r-s)B_2^{+s}Q_2^{+r}\delta_{r+s,m} + 2(P_1^{+\frac{r}{2}}C_2^{+\frac{s}{2}} + P_2^{+\frac{r}{2}}C_1^{+\frac{s}{2}})\delta_{r+s,2m}\Big] \\
&+ \sum_{\substack{r,s=1\\ r>s}}^{\infty} \Big[(r+s)B_2^{-s}Q_2^{+r}\delta_{r-s,m} + 2(P_1^{+\frac{r}{2}}C_2^{-\frac{s}{2}} + P_2^{+\frac{r}{2}}C_1^{-\frac{s}{2}})\delta_{r-s,2m}\Big] \\
&- \sum_{\substack{r,s=1\\ r<s}}^{\infty} \Big[(r+s)B_2^{+s}Q_2^{-r}\delta_{-r+s,m} - 2(P_1^{-\frac{r}{2}}C_2^{+\frac{s}{2}} + P_2^{-\frac{r}{2}}C_1^{+\frac{s}{2}})\delta_{-r+s,2m}\Big]
\end{aligned} \tag{30}$$

$$\begin{aligned}
\delta_{G^1}^{+\frac{m}{2}} = &-\frac{1}{2}\sum_{r,s=1}^{\infty} \Big[2B_1^{+s} - (r-s)B_2^{+s})P_1^{+\frac{r}{2}}\delta_{r+2s,m} - 2(Q_1^{+r} + (r-s)Q_2^{+r})C_1^{+\frac{s}{2}}\delta_{2r+s,m}\Big] \\
&+\frac{1}{2}\sum_{\substack{r,s=1\\ 2r>s}}^{\infty} \Big[2Q_1^{+r} + (r+s)Q_2^{+r})C_1^{-\frac{s}{2}}\delta_{2r-s,m}\Big] - \frac{1}{2}\sum_{\substack{r,s=1\\ r>2s}}^{\infty} \Big[2(B_1^{-s} - (r+s)B_2^{-s})P_1^{+\frac{r}{2}}\delta_{r-2s,m}\Big] \\
&-\frac{1}{2}\sum_{\substack{r,s=1\\ r<2s}}^{\infty} \Big[2B_1^{+s} + (r+s)B_2^{+s})P_1^{-\frac{r}{2}}\delta_{-r+2s,m}\Big] + \frac{1}{2}\sum_{\substack{r,s=1\\ 2r<s}}^{\infty} \Big[2(Q_1^{-r} - (r+s)Q_2^{-r})C_1^{+\frac{s}{2}}\delta_{-2r+s,m}\Big]
\end{aligned} \tag{31}$$

$$\delta_{G^2}^{+\frac{m}{2}} = \delta_{G^1}^{+\frac{m}{2}} \begin{pmatrix} P_1 \to -P_2 \\ C_1 \to -C_2 \end{pmatrix} \tag{32}$$

and

$$\delta_J^{-m}, \delta_L^{-m}, \delta_{G^1}^{-\frac{m}{2}}, \delta_{G^2}^{-\frac{m}{2}} = \delta_J^{+m}, \delta_L^{+m}, \delta_{G^1}^{+\frac{m}{2}}, \delta_{G^2}^{+\frac{m}{2}} \begin{pmatrix} +m \to -m \\ +r \to -r \\ +s \to -s \end{pmatrix}. \tag{33}$$

In AKNS scheme we expand A_1, A_2, $B^{\pm 1}$, $C_1^{\pm\frac{1}{2}}$ and $C_2^{\pm\frac{1}{2}}$ in terms of the positive powers of λ as

$$A_1 = \sum_{n=0}^{2} \lambda^n a_{1n}; \;\; A_2 = \sum_{n=0}^{2} \lambda^n a_{2n}; \;\; B_1^{\pm m} = \sum_{n=0}^{2} \lambda^n b_{1n}^{\pm m}; \;\; B_2^{\pm m} = \sum_{n=0}^{2} \lambda^n b_{2n}^{\pm m}; \tag{34}$$

$$C_1^{\pm\frac{m}{2}} = \sum_{n=0}^{2} \lambda^n c_{1n}^{\pm\frac{m}{2}}; \;\; C_2^{\pm\frac{m}{2}} = \sum_{n=0}^{2} \lambda^n c_{2n}^{\pm\frac{m}{2}}. \tag{35}$$

Inserting Eq.(34-35) into Eqs.(23-28)gives 30 relations in terms of a_{1n}, a_{2n}, $b_{1n}^{\pm m}$, $b_{2n}^{\pm m}$, $c_{1n}^{\pm\frac{m}{2}}$ and $c_{2n}^{\pm\frac{m}{2}}$ (n=0,1,2). By solving these relations we get

$$a_{10} = i\sum_{r=1}^{\infty}(Q_2^{+r}Q_1^{-r} + Q_2^{-r}Q_1^{+r}) - i\sum_{r=1}^{\infty}\left[\frac{2}{r-2}\right]P_2^{-\frac{r}{2}}P_1^{+r} - i\sum_{r=1}^{\infty}\left[\frac{2}{r+2}\right]P_2^{+r}P_1^{-r};\ a_{11} = a_{12} = 0;$$

$$a_{20} = 2i\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r} - i\sum_{r=1}^{\infty}\left[\frac{4}{r-2}\right]P_2^{-r}P_1^{+r} + i\sum_{r=1}^{\infty}\left[\frac{4}{r+2}\right]P_2^{+r}P_1^{-r};\ a_{21} = a_{22} = 0;$$

$$b_{10}^{\pm m} = \mp\frac{i}{m}Q_1^{\pm m}{}_x;\ b_{20}^{\pm m} = \mp\frac{i}{m}Q_2^{\pm m}{}_x;\ b_{11}^{\pm m} = Q_1^{\pm m};\ b_{21}^{\pm m} = Q_2^{\pm m};\ b_{12}^{\pm m} = 0;\ b_{22}^{\pm m} = 0 \tag{36}$$

$$c_{10}^{\pm\frac{m}{2}} = \mp\frac{2i}{m-2}P_1^{\pm\frac{m}{2}}{}_x;\ c_{20}^{\pm\frac{m}{2}} = \pm\frac{2i}{m+2}P_2^{\mp\frac{m}{2}}{}_x;\ c_{11}^{\pm\frac{m}{2}} = P_1^{\pm\frac{m}{2}};\ c_{21}^{\pm\frac{m}{2}} = P_2^{\pm\frac{m}{2}};\ c_{12}^{\pm\frac{m}{2}} = c_{22}^{\pm\frac{m}{2}} = 0.$$

By using the relations given by Eq.(36) from Eqs.(25-28) we obtain the coupled super NLS equations as

$$-iQ_1^{\pm m}{}_t = \mp\frac{1}{m}Q_1^{\pm m}{}_{xx} - 2Q_1^{\pm m}\left(\sum_{r=1}^{\infty}Q_2^{-r}Q_2^{+r}\right) \pm 4mQ_1^{\pm m}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r-2}\right]P_2^{-\frac{r}{2}}P_1^{+\frac{r}{2}}\right)$$
$$\mp 4mQ_1^{\pm m}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r+2}\right]P_2^{+\frac{r}{2}}P_1^{-\frac{r}{2}}\right) + \delta_J^{\pm m} \tag{37}$$

$$-iQ_2^{\pm m}{}_t = \mp\frac{1}{m}Q_2^{\pm m}{}_{xx} - 2Q_2^{\pm m}\left(\sum_{r=1}^{\infty}Q_2^{-r}Q_2^{+r}\right) \pm 4mQ_2^{\pm m}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r-2}\right]P_2^{-\frac{r}{2}}P_1^{+\frac{r}{2}}\right)$$
$$\mp 4mQ_2^{\pm m}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r+2}\right]P_2^{+\frac{r}{2}}P_1^{-\frac{r}{2}}\right) + \delta_L^{\pm m} \tag{38}$$

$$-iP_1^{\pm\frac{m}{2}}{}_t = \mp\frac{2}{m\mp 2}P_1^{\pm\frac{m}{2}}{}_{xx} \pm 2(m\mp 1)P_1^{\pm\frac{m}{2}}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r-2}\right]P_2^{-\frac{r}{2}}P_1^{+\frac{r}{2}}\right)$$
$$\mp 2(m\pm 1)P_1^{\pm\frac{m}{2}}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r+2}\right]P_2^{+\frac{r}{2}}P_1^{-\frac{r}{2}}\right) + P_1^{\pm\frac{m}{2}}\sum_{r=1}^{\infty}Q_1^{+r}Q_2^{-r} \tag{39}$$
$$+P_1^{\pm\frac{m}{2}}\sum_{r=1}^{\infty}Q_1^{-r}Q_2^{+r} \mp mP_1^{\pm\frac{m}{2}}\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r} + \delta_{G^1}^{\pm\frac{m}{2}}$$

and

$$-P_2^{\pm\frac{m}{2}}{}_t = \mp\frac{2}{m\pm 2}P_2^{\pm\frac{m}{2}}{}_{xx} \pm 2(m\pm 1)P_2^{\pm\frac{m}{2}}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r-2}\right]P_2^{-\frac{r}{2}}P_1^{+\frac{r}{2}}\right)$$
$$\mp 2(m\mp 1)P_2^{\pm\frac{m}{2}}\left(\sum_{r=1}^{\infty}\left[\frac{1}{r+2}\right]P_2^{+\frac{r}{2}}P_1^{-\frac{r}{2}}\right) - P_2^{\pm\frac{m}{2}}\sum_{r=1}^{\infty}Q_1^{+r}Q_2^{-r} \tag{40}$$
$$-P_2^{\pm\frac{m}{2}}\sum_{r=1}^{\infty}Q_1^{-r}Q_2^{+r} \mp mP_1^{\pm\frac{m}{2}}\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r} + \delta_{G^2}^{\pm\frac{m}{2}}$$

where the terms $\delta_J^{\pm m}, \delta_L^{\pm m}, \delta_{G^1}^{\pm\frac{m}{2}}, \delta_{G^2}^{\pm\frac{m}{2}}$ are given blow:

$$-i\delta_J^{+m} = \sum_{\substack{r=1\\ r<m}}^{\infty} Q_2^{+r}Q_{1x}^{+(m-r)} - \sum_{\substack{r=1\\ r<m}}^{\infty}\left[\frac{r}{m-r}\right]Q_1^{+r}Q_{2x}^{+(m-r)} + 2\sum_{\substack{r=1\\ r<2m}}^{\infty}\left[\frac{r-m}{2m-r-2}\right]P_2^{+\frac{r}{2}}P_{1x}^{+(m-\frac{r}{2})}$$
$$-2\sum_{\substack{r=1\\ r<2m}}^{\infty}\left[\frac{r-m}{2m-r+2}\right]P_1^{+\frac{r}{2}}P_{2x}^{+(m-\frac{r}{2})} + \sum_{\substack{r=1\\ r>m}}^{\infty}Q_2^{+r}Q_{1x}^{-(r-m)} + \sum_{\substack{r=1\\ r>m}}^{\infty}\left[\frac{r}{r-m}\right]Q_1^{+r}Q_{2x}^{-(r-m)}$$

$$+2\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{r-m}{r-2m-2}\Big]P_1^{+\frac{r}{2}}P_{2x}^{-(\frac{r}{2}-m)}-2\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{r-m}{r-2m+2}\Big]P_2^{+\frac{r}{2}}P_{1x}^{-(\frac{r}{2}-m)}+\sum_{\substack{r=1\\ r>m}}^{\infty}Q_2^{-(r-m)}Q_{1x}^{+r} \quad (41)$$

$$+\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{(r-m)}{r}\Big]Q_1^{-(r-m)}Q_{2x}^{+s}+2\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{r-m}{r+2}\Big]P_1^{-(\frac{r}{2}-m)}P_{2x}^{+\frac{s}{2}}-2\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{r-m}{r-2}\Big]P_2^{-(\frac{r}{2}-m)}P_{1x}^{+\frac{r}{2}}$$

$$-i\delta_L^{+m}=-\sum_{\substack{r=1\\ r<m}}^{\infty}\Big[\frac{2r-m}{m-r}\Big]Q_2^{+r}Q_{2x}^{+(m-r)}-4\sum_{\substack{r=1\\ r<2m}}^{\infty}\Big[\frac{1}{2m-r+2}\Big]P_1^{+\frac{r}{2}}P_{2x}^{+(m-\frac{r}{2})}-4\sum_{\substack{r=1\\ r<2m}}^{\infty}\Big[\frac{1}{2m-r-2}\Big]P_2^{+\frac{r}{2}}P_{1x}^{+(m-\frac{r}{2})}$$

$$+\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{2r-m}{r-m}\Big]Q_2^{+r}Q_{2x}^{-(r-m)}+4\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{1}{r-2m+2}\Big]P_1^{+\frac{r}{2}}P_{2x}^{+(\frac{r}{2}-m)}+4\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{1}{r-2m+2}\Big]P_2^{+\frac{r}{2}}P_{1x}^{-(\frac{r}{2}-m)} \quad (42)$$

$$+\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{2r+m}{r}\Big]Q_2^{-(r-m)}Q_{2x}^{+r}-4\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{1}{r+2}\Big]P_1^{-(\frac{r}{2}-m)}P_{2x}^{+\frac{r}{2}}-4\sum_{\substack{r=1\\ r>2m}}^{\infty}\Big[\frac{1}{r-2}\Big]P_2^{-(\frac{r}{2}-m)}P_{1x}^{+\frac{r}{2}}$$

$$-i\delta_{G^1}^{+\frac{m}{2}}=\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{1}{r}\Big]P_1^{+(\frac{m}{2}-r)}Q_{1x}^{+r}-\frac{1}{2}\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{m-3r}{r}\Big]P_1^{+(\frac{m}{2}-r)}Q_{2x}^{+r}-2\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{1}{m-2r-2}\Big]P_{1x}^{+(\frac{m}{2}-r)}Q_1^{+r}$$

$$-2\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{3r-m}{m-2r-2}\Big]P_{1x}^{+(\frac{m}{2}-r)}Q_2^{+r}+2\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{1}{2r-m+2}\Big]P_{1x}^{-(r-\frac{m}{2})}Q_1^{+r}+2\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{3r-m}{2r-m+2}\Big]P_{1x}^{-(r-\frac{m}{2})}Q_2^{+r}$$

$$-2\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{1}{r-m}\Big]P_1^{+\frac{r}{2}}Q_{1x}^{-(\frac{r}{2}-\frac{m}{2})}+\frac{1}{2}\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{3r-m}{r-m}\Big]P_1^{+\frac{r}{2}}Q_{2x}^{-(\frac{r}{2}-\frac{m}{2})}+\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{1}{r}\Big]P_1^{-(r-\frac{m}{2})}Q_{1x}^{+r} \quad (43)$$

$$+\frac{1}{2}\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{3r-m}{r}\Big]P_1^{-(r-\frac{m}{2})}Q_{2x}^{+r}-2\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{1}{r-2}\Big]P_{1x}^{+\frac{r}{2}}Q_1^{-(\frac{r}{2}-\frac{m}{2})}-\frac{1}{2}\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{3r-m}{r-2}\Big]P_{1x}^{+\frac{r}{2}}Q_2^{-(\frac{r}{2}-\frac{m}{2})}$$

$$-i\delta_{G^2}^{+\frac{m}{2}}=-\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{1}{r}\Big]P_2^{+(\frac{m}{2}-r)}Q_{1x}^{+r}-\frac{1}{2}\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{m-3r}{r}\Big]P_2^{+(\frac{m}{2}-r)}Q_{2x}^{+r}-2\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{1}{m-2r+2}\Big]P_{2x}^{+(\frac{m}{2}-r)}Q_1^{+r}$$

$$-2\sum_{\substack{r=1\\ 2r<m}}^{\infty}\Big[\frac{3r-m}{m-2r+2}\Big]P_{2x}^{+(\frac{m}{2}-r)}Q_2^{+r}+2\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{1}{2r-m-2}\Big]P_{2x}^{-(r-\frac{m}{2})}Q_1^{+r}+2\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{3r-m}{2r-m-2}\Big]P_{2x}^{-(r-\frac{m}{2})}Q_2^{+r}$$

$$+2\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{1}{r-m}\Big]P_2^{+\frac{r}{2}}Q_{1x}^{-(\frac{r}{2}-\frac{m}{2})}+\frac{1}{2}\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{3r-m}{r-m}\Big]P_2^{+\frac{r}{2}}Q_{2x}^{-(\frac{r}{2}-\frac{m}{2})}-\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{1}{r}\Big]P_2^{-(r-\frac{m}{2})}Q_{1x}^{+r} \quad (44)$$

$$+\frac{1}{2}\sum_{\substack{r=1\\ 2r>m}}^{\infty}\Big[\frac{3r-m}{r}\Big]P_2^{-(r-\frac{m}{2})}Q_{2x}^{+r}+2\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{1}{r+2}\Big]P_{2x}^{+\frac{r}{2}}Q_1^{-(\frac{r}{2}-\frac{m}{2})}+\frac{1}{2}\sum_{\substack{r=1\\ r>m}}^{\infty}\Big[\frac{3r-m}{r-2}\Big]P_{2x}^{+\frac{r}{2}}Q_2^{-(\frac{r}{2}-\frac{m}{2})}$$

and

$$\delta_J^{-m},\delta_L^{-m},\delta_{G^1}^{-\frac{m}{2}},\delta_{G^2}^{-\frac{m}{2}}=\delta_J^{+m},\delta_L^{+m},\delta_{G^1}^{+\frac{m}{2}},\delta_{G^2}^{+\frac{m}{2}},\begin{pmatrix}+m\to-m\\ +r\to-r\end{pmatrix}. \quad (45)$$

The Eqs.(37-45) are the most general super extensions of NLS equations related to N=2 superconformal algebra with Neveu-Schwarz type involving infinite number of bosonic and fermionic fields

4 AKNS Scheme with N=2 Superconformal Algebra (Ramond Type)

We take the soliton connection as

$$\Omega = \Omega_b + \Omega_f \tag{46}$$

where

$$\Omega_b = \begin{array}{l} \Big(\quad i\lambda L_0 + Q_1^{+m} J_{+m} + Q_1^{-m} J_{-m} + Q_2^{+m} L_{+m} + Q_2^{-m} L_{-m} \Big) dx + \\ \Big(\quad -A_1 J_0 - A_2 L_0 + B_1^{+m} J_{+m} + B_1^{-m} J_{-m} + B_2^{+m} L_{+m} + B_2^{-m} L_{-m} \Big) dt \end{array} \tag{47}$$

$$\Omega_f = \begin{array}{l} \Big(\quad +P_1^{+m} G^1_{+m} + P_1^{-m} G^1_{-m} + P_2^{+m} G^2_{+m} + P_2^{-m} G^2_{-m} \Big) dx + \\ \Big(\quad -A_3 G^1_0 - A_4 G^2_0 + C_1^{+m} G^1_{+m} + C_1^{-m} G^1_{-m} + C_2^{+m} G^2_{+m} + C_2^{-m} G^2_{-m} \Big) dt \end{array} \tag{48}$$

where L_0 , $J_{\pm m}$, $L_{\pm m}$ are bosonic generators and G^1_0,G^2_0, $G^1_{\pm m}$,$G^2_{\pm m}$ are fermionic generators of centerless N=2 superconformal algebra of Ramond type , namely they satisfy the following commutation and anticommutation relations

$$\begin{array}{lcl} [L_r, L_s] & = & (r-s)\, L_{r+s} \\ [J_r, J_s] & = & 0 \\ [L_r, J_s] & = & -s\, J_{r+s} \\ \{G^1_r, G^2_s\} & = & 2\, L_{r+s} + (r-s)\, J_{r+s} \\ [J_r, G^{1,2}_s] & = & \pm G^{1,2}_{r+s} \\ [L_r, G^{1,2}_s] & = & (\frac{r}{2} - s)\, G^{1,2}_{r+s} \\ \{G^{1,2}_r, G^{1,2}_s\} & = & 0 \end{array} \tag{49}$$

here, $J_{\pm m}$,$L_{\pm m}$,$G^1_{\pm m}$ and $G^2_{\pm m}$ are generators with positive(negative) integer indices . In Eq.(46) we assume summation over the repeated indices. The fields $Q^{\pm m}_{1,2}$ and $P^{\pm m}_{1,2}$are x,t dependent and also functions $A_{1,2}$,$A_{3,4}$, $B^{\pm m}_{1,2}$ and $C^{\pm m}_{1,2}$ are x,t and λ dependent.

From the integrability condition given by Eq.(6) we obtain

$$A_{1x} = \sum_{r=1}^{\infty} r(B_1^{+r} Q_2^{-r} - B_1^{-r} Q_2^{+r} + B_2^{+r} Q_1^{-r} - B_2^{-r} Q_1^{+r} - 2(P_1^{-r} C_2^{+r} - P_1^{+r} C_2^{-r} - P_2^{-r} C_1^{+r} + P_2^{+r} C_1^{-r})) \tag{50}$$

$$A_{2x} = \sum_{r=1}^{\infty} (2r(B_2^{-r} Q_2^{+r} - B_2^{+r} Q_2^{-r}) + P_1^{-r} C_2^{+r} + P_1^{+r} C_2^{-r} + P_2^{-r} C_1^{+r} + P_2^{+r} C_1^{-r}) \tag{51}$$

$$A_{3x} = -\frac{3}{2} \sum_{r=1}^{\infty} r(B_2^{+r} P_1^{-r} - B_2^{-r} P_1^{+r} + C_1^{+r} Q_2^{-r} - C_1^{-r} Q_2^{+r}) - (B_1^{+r} P_1^{-r} + B_1^{-r} P_1^{-r} - C_1^{+r} Q_1^{-r} - C_1^{-r} Q_1^{+r})) \tag{52}$$

$$A_{4x} = -\frac{3}{2} \sum_{r=1}^{\infty} r(B_2^{+r} P_2^{-r} - B_2^{-r} P_2^{+r} + C_2^{+r} Q_2^{-r} - C_2^{-r} Q_2^{+r}) + (B_1^{+r} P_2^{-r} + B_1^{-r} P_2^{-r} - C_2^{+r} Q_1^{-r} - C_2^{-r} Q_1^{+r})) \tag{53}$$

$$Q_1^{\pm m}{}_t = B_1^{\pm m}{}_x \mp im\lambda B_1^{\pm m} \mp mP_1^{\pm m} A_4 \mp mP_2^{\pm m} A_3 \mp mA_2 Q_1^{\pm m} + \delta_J^{\pm m} \tag{54}$$

$$Q_2^{\pm m}{}_t = B_2^{\pm m}{}_x \mp im\lambda B_2^{\pm m} - 2P_1^{\pm m} A_4 - 2P_2^{\pm m} A_3 \mp mA_2 Q_2^{\pm m} + \delta_L^{\pm m} \tag{55}$$

$$P_1^{\pm m}{}_t = C_1^{\pm m}{}_x \mp im\lambda C_1^{\pm m} + P_1^{\pm m} A_1 \mp P_1^{\pm m} A_2 - A_3 Q_1^{\pm m} \mp mA_3 Q_2^{\pm m} + \delta_{G_1}^{\pm m} \tag{56}$$

$$P_2^{\pm m}{}_t = C_2^{\pm m}{}_x \mp im\lambda C_2^{\pm m} - P_1^{\pm m} A_1 \mp P_2^{\pm m} A_2 + A_4 Q_1^{\pm m} \mp mA_4 Q_2^{\pm m} + \delta_{G_2}^{\pm m} \tag{57}$$

where

$$\delta_J^{+m} = \sum_{r,s=1}^{\infty} \left((r-s) P_1^{+r} C_2^{+s} - (r-s) P_2^{+r} C_1^{+s} + rQ_1^{+r} B_2^{+s} - sQ_2^{+r} B_1^{+s}\right) \delta_{r+s,m}$$

$$+ \sum_{\substack{r,s=1 \\ r>s}}^{\infty} \left((r+s) P_1^{+r} C_2^{-s} - (r+s) P_2^{+r} C_1^{-s} + rQ_1^{+r} B_2^{-s} + sQ_2^{+r} B_1^{-s}\right) \delta_{r-s,m} \tag{58}$$

$$+\sum_{\substack{r,s=1\\r<s}}^{\infty}\left((r+s)P_1^{-r}C_2^{+s}-(r+s)P_2^{-r}C_1^{+s}+rQ_1^{-r}B_2^{+s}+sQ_2^{-r}B_1^{+s}\right)\delta_{-r+s,m}$$

$$\begin{aligned}\delta_L^{+m}=&\sum_{r,s=1}^{\infty}\left(2P_1^{+r}C_2^{+s}+2P_2^{+r}C_1^{+s}+(r-s)Q_2^{+r}B_2^{+s}\right)\delta_{r+s,m}\\&+\sum_{\substack{r,s=1\\r>s}}^{\infty}\left(2P_1^{+r}C_2^{-s}+2P_2^{+r}C_1^{-s}+(r+s)Q_2^{+r}B_2^{-s}\right)\delta_{r-s,m}\\&+\sum_{\substack{r,s=1\\r<s}}^{\infty}\left(2P_1^{-r}C_2^{+s}+2P_2^{-r}C_1^{+s}-(r+s)Q_2^{-r}B_2^{+s}\right)\delta_{-r+s,m}\end{aligned}\quad(59)$$

$$\begin{aligned}\delta_{G^1}^{+m}=&-\sum_{r,s=1}^{\infty}\left(B_1^{+s}P_1^{+r}-\frac{1}{2}(2r-s)B_2^{+s}P_1^{+r}-C_1^{+s}Q_1^{+r}-\frac{1}{2}(r-2s)C_1^{+s}Q_2^{+r}\right)\delta_{r+s,m}\\&-\sum_{\substack{r,s=1\\r>s}}^{\infty}\left(B_1^{-s}P_1^{+r}-\frac{1}{2}(2r+s)B_2^{-s}P_1^{+r}-C_1^{-s}Q_1^{+r}-\frac{1}{2}(r+2s)C_1^{-s}Q_2^{+r}\right)\delta_{r-s,m}\\&-\sum_{\substack{r,s=1\\r<s}}^{\infty}\left(B_1^{+s}P_1^{-r}+\frac{1}{2}(2r+s)B_2^{+s}P_1^{-r}-C_1^{+s}Q_1^{-r}+\frac{1}{2}(r+2s)C_1^{+s}Q_2^{-r}\right)\delta_{-r+s,m}\end{aligned}\quad(60)$$

$$\begin{aligned}\delta_{G^2}^{+m}=&+\sum_{r,s=1}^{\infty}\left(B_1^{+s}P_2^{+r}-\frac{1}{2}(2r-s)B_2^{+s}P_2^{+r}+C_2^{+s}Q_1^{+r}-\frac{1}{2}(r-2s)C_2^{+s}Q_2^{+r}\right)\delta_{r+s,m}\\&+\sum_{\substack{r,s=1\\r>s}}^{\infty}\left(B_1^{-s}P_2^{+r}-\frac{1}{2}(2r+s)B_2^{-s}P_2^{+r}+C_2^{-s}Q_1^{+r}-\frac{1}{2}(r+2s)C_2^{-s}Q_2^{+r}\right)\delta_{r-s,m}\\&-\sum_{\substack{r,s=1\\r<s}}^{\infty}\left(B_1^{+s}P_2^{-r}+\frac{1}{2}(2r+s)B_2^{+s}P_2^{-r}+C_2^{+s}Q_1^{-r}+\frac{1}{2}(r+2s)C_2^{+s}Q_2^{-r}\right)\delta_{-r+s,m}\end{aligned}\quad(61)$$

and

$$\delta_J^{-m},\delta_L^{-m},\delta_{G^1}^{-m},\delta_{G^2}^{-m}=\delta_J^{+m},\delta_L^{+m},\delta_{G^1}^{+m},\delta_{G^2}^{+m}\begin{pmatrix}+m\to-m\\+r\to-r\\+s\to-s\end{pmatrix}\quad(62)$$

In AKNS scheme we expand A_1,A_2, A_3, A_4, $B_1^{\pm m}$, $B_2^{\pm m}$, $C_1^{\pm m}$ and $C_2^{\pm m}$ in terms of the positive powers of λ as

$$A_1=\sum_{n=0}^{2}\lambda^n a_{1n};\;\; A_2=\sum_{n=0}^{2}\lambda^n a_{2n};\;\; A_3=\sum_{n=0}^{2}\lambda^n a_{3n};\;\; A_4=\sum_{n=0}^{2}\lambda^n a_{4n};\quad(63)$$

$$B_1^{\pm m}=\sum_{n=0}^{2}\lambda^n b_{1n}^{\pm m};\;\; B_2^{\pm m}=\sum_{n=0}^{2}\lambda^n b_{2n}^{\pm m};\;\; C_1^{\pm m}=\sum_{n=0}^{2}\lambda^n c_{1n}^{\pm m};\;\; C_2^{\pm m}=\sum_{n=0}^{2}\lambda^n c_{2n}^{\pm m}\quad(64)$$

inserting Eq.(63-64) into Eqs.(50-57)gives 36 relations in terms of $a_{1n}, a_{2n}, a_{3n}, a_{4n}, b_{1n}^{\pm m}$, $b_{2n}^{\pm m}$, $c_{1n}^{\pm m}$ and $c_{2n}^{\pm m}$ (n=0,1,2). By solving these relations we get

$$a_{10}=i\sum_{r=1}^{\infty}(Q_1^{+r}Q_2^{-r}+Q_1^{-r}Q_2^{+r})-2i\sum_{r=1}^{\infty}(P_2^{-r}P_1^{+r}+P_2^{+r}P_1^{-r});\; a_{11}=a_{12}=0;$$

$$a_{20}=2i\sum_{r=1}^{\infty}(Q_2^{-r}Q_2^{+r})-2i\sum_{r=1}^{\infty}(\Big[\frac{1}{r}\Big]P_2^{-r}P_1^{+r}-\Big[\frac{1}{r}\Big]P_2^{+r}P_1^{-r});\; a_{21}=a_{22}=-i;$$

$$a_{30}=\frac{3}{2}i\sum_{r=1}^{\infty}(P_1^{+r}Q_2^{-r}+P_1^{-r}Q_2^{+r})-i\sum_{r=1}^{\infty}\left[\frac{1}{r}\right]P_1^{+r}Q_1^{-r}+i\sum_{r=1}^{\infty}\left[\frac{1}{r}\right]P_1^{-r}Q_1^{+r};\ a_{31}=a_{32}=0; \tag{65}$$

$$a_{40}=\frac{3}{2}i\sum_{r=1}^{\infty}(P_2^{+r}Q_2^{-r}+P_2^{-r}Q_2^{+r})+i\sum_{r=1}^{\infty}\left[\frac{1}{r}\right]P_2^{+r}Q_1^{-r}-i\sum_{r=1}^{\infty}\left[\frac{1}{r}\right]P_2^{-r}Q_1^{+r};\ a_{41}=a_{42}=0;$$

$$b_{10}^{\pm m}=\mp\frac{i}{m}Q_1^{\pm m}{}_x;\ b_{11}^{\pm m}=Q_1^{\pm m};\ b_{12}^{\pm m}=0;\ b_{20}^{\pm m}=\mp\frac{i}{m}Q_2^{\pm m}{}_x;\ b_{21}^{\pm m}=Q_2^{\pm m};\ b_{22}^{\pm m}=0$$

$$c_{10}^{\pm m}=\mp\frac{i}{m}P_1^{\pm m}{}_x;\ c_{20}^{\pm m}=\pm\frac{i}{m}P_2^{\mp m}{}_x;\ c_{11}^{\pm m}=P_1^{\pm m};\ c_{21}^{\pm m}=P_2^{\pm m};\ c_{12}^{\pm m}=c_{22}^{\pm m}=0.$$

By using the relations given by Eq.(65) from Eqs.(54-57) we obtain the coupled super NLS equations as

$$\begin{aligned}
-iQ_1^{\pm m}{}_t &= \pm\frac{1}{m}Q_1^{\pm m}{}_{xx}\pm\left(\sum_{r=1}^{\infty}P_2^{+r}Q_1^{-r}\right)P_1^{\pm m}\\
&\pm P_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{+r}Q_1^{-r}\right)\pm\left(\sum_{r=1}^{\infty}P_2^{-r}Q_1^{-r}\right)P_1^{\pm m}\mp P_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{-r}Q_1^{+r}\right)\\
&\mp 2\left(\sum_{r=1}^{\infty}P_2^{-r}P_1^{+r}\right)Q_1^{\pm m}\pm 2\left(\sum_{r=1}^{\infty}P_2^{+r}P_1^{-r}\right)Q_1^{\pm m}\pm 2m\left(\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r}\right)Q_1^{\pm m}\\
&\mp\frac{3}{2}m\left(\sum_{r=1}^{\infty}P_2^{+r}Q_2^{-r}\right)P_1^{\pm m}\pm\frac{3}{2}mP_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{+r}Q_2^{-r}\right)\\
&\mp\frac{3}{2}m\left(\sum_{r=1}^{\infty}P_2^{-r}Q_2^{+r}\right)P_1^{\pm m}\pm\frac{3}{2}mP_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{-r}Q_2^{+r}\right)+\delta_J^{\pm m}
\end{aligned} \tag{66}$$

$$\begin{aligned}
-iQ_2^{\pm m}{}_t &= \pm\frac{1}{m}Q_2^{\pm m}{}_{xx}-\frac{2}{m}\left(\sum_{r=1}^{\infty}P_2^{+r}Q_1^{-r}\right)P_1^{\pm m}\\
&-\frac{2}{m}P_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{+r}Q_1^{-r}\right)+\frac{2}{m}\left(\sum_{r=1}^{\infty}P_2^{-r}Q_1^{+r}\right)P_1^{\pm m}+\frac{2}{m}P_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{-r}Q_1^{+r}\right)\\
&-3\left(\sum_{r=1}^{\infty}P_2^{+r}Q_1^{-r}\right)P_1^{\pm m}+3P_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{+r}Q_2^{-r}\right)\\
&-3\left(\sum_{r=1}^{\infty}P_2^{-r}Q_2^{+r}\right)P_1^{\pm m}+3P_2^{\pm m}\left(\sum_{r=1}^{\infty}P_1^{-r}Q_2^{+r}\right)\mp 2\left(\sum_{r=1}^{\infty}P_2^{-r}P_1^{+r}\right)Q_2^{\pm m}\\
&\pm 2\left(\sum_{r=1}^{\infty}P_2^{+r}P_1^{-r}\right)Q_2^{\pm m}\pm 2m\left(\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r}\right)Q_2^{\pm m}+\delta_L^{\pm m}
\end{aligned} \tag{67}$$

$$\begin{aligned}
-iP_1^{\pm m}{}_t &= \pm\frac{1}{m}P_1^{\pm m}{}_{xx}+4\left(\sum_{r=1}^{\infty}P_2^{+r}P_1^{+r}\right)P_1^{\pm m}\\
&-\left(\sum_{r=1}^{\infty}Q_2^{-r}Q_1^{+r}\right)P_1^{\pm m}-\left(\sum_{r=1}^{\infty}Q_2^{+r}Q_1^{-r}\right)P_1^{\pm m}\pm 2m\left(\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r}\right)P_1^{\pm m}\\
&-\frac{1}{m}\left(\sum_{r=1}^{\infty}Q_1^{-r}P_1^{+r}\right)Q_1^{\pm m}+\frac{1}{m}\left(\sum_{r=1}^{\infty}Q_1^{+r}P_1^{-r}\right)Q_1^{\pm m}\\
&+\frac{3}{2}\left(\sum_{r=1}^{\infty}Q_2^{-r}P_1^{+r}\right)Q_1^{\pm m}+\frac{3}{2}\left(\sum_{r=1}^{\infty}Q_2^{+r}P_1^{-r}\right)Q_1^{\pm m}\mp\frac{1}{2}\left(\sum_{r=1}^{\infty}Q_1^{-r}P_1^{+r}\right)Q_2^{\pm m}
\end{aligned} \tag{68}$$

$$\pm\frac{1}{2}\left(\sum_{r=1}^{\infty}Q_1^{+r}P_1^{-r}\right)Q_2^{\pm m}\pm\frac{3}{4}m\left(\sum_{r=1}^{\infty}Q_2^{-r}P_1^{+r}\right)Q_2^{\pm m}\pm\frac{3}{4}m\left(\sum_{r=1}^{\infty}Q_2^{+r}P_1^{-r}\right)Q_2^{\pm m}+\delta_{G^1}^{\pm m}$$

$$-iP_2^{\pm m}{}_t=\pm\frac{1}{m}P_2^{\pm m}{}_{xx}-4\left(\sum_{r=1}^{\infty}P_2^{-r}P_1^{+r}\right)P_2^{\pm m}$$
$$+\left(\sum_{r=1}^{\infty}Q_2^{-r}Q_1^{+r}\right)P_2^{\pm m}+\left(\sum_{r=1}^{\infty}Q_2^{+r}Q_1^{-r}\right)P_2^{\pm m}\pm 2m\left(\sum_{r=1}^{\infty}Q_2^{+r}Q_2^{-r}\right)P_2^{\pm m}$$
$$-\frac{1}{m}\left(\sum_{r=1}^{\infty}Q_1^{-r}P_2^{+r}\right)Q_1^{\pm m}+\frac{1}{m}\left(\sum_{r=1}^{\infty}Q_1^{+r}P_2^{-r}\right)Q_1^{\pm m} \tag{69}$$
$$-\frac{3}{2}\left(\sum_{r=1}^{\infty}Q_2^{-r}P_2^{+r}\right)Q_1^{\pm m}-\frac{3}{2}\left(\sum_{r=1}^{\infty}Q_2^{+r}P_2^{-r}\right)Q_1^{\pm m}\mp\frac{1}{2}\left(\sum_{r=1}^{\infty}Q_1^{-r}P_2^{+r}\right)Q_2^{\pm m}$$
$$\pm\frac{1}{2}\left(\sum_{r=1}^{\infty}Q_1^{+r}P_2^{-r}\right)Q_2^{\pm m}\pm\frac{3}{4}m\left(\sum_{r=1}^{\infty}Q_2^{-r}P_2^{+r}\right)Q_2^{\pm m}\pm\frac{3}{4}m\left(\sum_{r=1}^{\infty}Q_2^{+r}P_2^{-r}\right)Q_2^{\pm m}+\delta_{G^2}^{\pm m}$$

where the terms $\delta_J^{\pm m},\delta_L^{\pm m},\delta_{G^1}^{\pm m},\delta_{G^2}^{\pm m}$ are given blow:

$$-i\delta_J^{+m}=\sum_{r=1}^{\infty}\left[\frac{2r-m}{m-r}\right]P_1^{+r}P_{2x}^{+(m-r)}-\sum_{r=1}^{\infty}\left[\frac{r}{m-r}\right]P_2^{+r}P_{1x}^{+(m-r)}-\sum_{r=1}^{\infty}Q_2^{+r}Q_{1x}^{+(m-r)}$$
$$+\sum_{r=1}^{\infty}\left[\frac{r}{m-r}\right]Q_1^{+r}Q_{2x}^{+(m-r)}-\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{2r-m}{r-m}\right]P_1^{+r}P_{2x}^{-(r-m)}+\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{2r-m}{m-r}\right]P_2^{+r}P_{1x}^{-(r-m)}$$
$$-\sum_{\substack{r=1\\r>m}}^{\infty}Q_2^{+r}Q_{1x}^{-(r-m)}+\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{r}{r-m}\right]Q_1^{+r}Q_{2x}^{-(r-m)}-\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{2r-m}{r}\right]P_1^{-(r-m)}P_{2x}^{+r} \tag{70}$$
$$+\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{2r-m}{r}\right]P_2^{-(r-m)}P_{1x}^{+r}-\sum_{\substack{r=1\\r<m}}^{\infty}Q_2^{-(r-m)}Q_{1x}^{+r}-\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{r-m}{r}\right]Q_1^{+(r-m)}Q_{2x}^{+r}$$

$$-i\delta_L^{+m}=\sum_{r=1}^{\infty}\left[\frac{2}{m-r}\right]P_1^{+r}P_{2x}^{+(m-r)}+\sum_{r=1}^{\infty}\left[\frac{2}{m-r}\right]P_2^{+r}P_{1x}^{+(m-r)}-\sum_{r=1}^{\infty}\left[\frac{2r-m}{m-r}\right]Q_2^{+r}Q_{1x}^{+(m-r)}$$
$$+\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{2}{r}\right]P_1^{+r}P_{2x}^{-(r-m)}+\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{2}{r-m}\right]P_2^{+r}P_{1x}^{-(r-m)}-\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{2r-m}{r-m}\right]Q_2^{+r}Q_{1x}^{-(r-m)} \tag{71}$$
$$+\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{2}{r}\right]P_1^{-(r-m)}P_{2x}^{+r}+\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{2}{r}\right]P_2^{-(r-m)}P_{1x}^{+r}-\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{2r-m}{r}\right]Q_2^{-(r-m)}Q_{1x}^{+r}$$

$$-i\delta_{G^1}^{+m}=\sum_{r=1}^{\infty}\left[\frac{1}{m-r}\right]Q_1^{+r}P_{1x}^{+(m-r)}+\sum_{r=1}^{\infty}\left[\frac{1}{m-r}\right]P_1^{+r}Q_{1x}^{+(m-r)}+\frac{1}{2}\sum_{r=1}^{\infty}\left[\frac{3r-m}{m-r}\right]Q_2^{+r}P_{1x}^{+(m-r)}$$
$$+\frac{1}{2}\sum_{r=1}^{\infty}\left[\frac{3r-m}{m-r}\right]P_1^{+r}Q_{2x}^{+(m-r)}-\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{1}{r-m}\right]Q_1^{+r}P_{1x}^{-(r-m)}+\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{1}{r-m}\right]P_1^{+r}Q_{1x}^{-(r-m)} \tag{72}$$
$$-\frac{1}{2}\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{3r-2m}{r-m}\right]Q_2^{+r}P_{1x}^{-(r-m)}-\frac{1}{2}\sum_{\substack{r=1\\r>m}}^{\infty}\left[\frac{3r-2m}{r-m}\right]P_1^{+r}Q_{2x}^{-(r-m)}+\sum_{\substack{r=1\\r<m}}^{\infty}\left[\frac{1}{r}\right]Q_1^{-(r-m)}P_{1x}^{+r}$$

$$+\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{1}{r}\Big]P_1^{-(r-m)}Q_{1x}^{+r}-\frac{1}{2}\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{3r-m}{r}\Big]Q_2^{-(r-m)}P_{1x}^{+r}-\frac{1}{2}\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{3r-m}{r}\Big]P_1^{-(r-m)}Q_{2x}^{+r}$$

$$-i\delta_{G^2}^{+m}=-\sum_{r=1}^{\infty}\Big[\frac{1}{m-r}\Big]Q_1^{+r}P_{2x}^{+(m-r)}+\sum_{r=1}^{\infty}\Big[\frac{1}{m-r}\Big]P_1^{+r}Q_{1x}^{+(m-r)}+\frac{1}{2}\sum_{r=1}^{\infty}\Big[\frac{3r-m}{m-r}\Big]Q_2^{+r}P_{2x}^{+(m-r)}$$
$$+\frac{1}{2}\sum_{r=1}^{\infty}\Big[\frac{3r-m}{m-r}\Big]P_1^{+r}Q_{2x}^{+(m-r)}+\sum_{\substack{r=1\\r>m}}^{\infty}\Big[\frac{1}{r-m}\Big]Q_1^{+r}P_{2x}^{-(r-m)}+\sum_{\substack{r=1\\r>m}}^{\infty}\Big[\frac{1}{r-m}\Big]P_1^{+r}Q_{1x}^{-(r-m)} \quad (73)$$
$$-\frac{1}{2}\sum_{\substack{r=1\\r>m}}^{\infty}\Big[\frac{3r-2m}{r-m}\Big]Q_2^{+r}P_{2x}^{-(r-m)}-\frac{1}{2}\sum_{\substack{r=1\\r>m}}^{\infty}\Big[\frac{3r-2m}{r-m}\Big]P_1^{+r}Q_{2x}^{-(r-m)}-\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{1}{r}\Big]Q_1^{-(r-m)}P_{2x}^{+r}$$
$$+\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{1}{r}\Big]P_1^{-(r-m)}Q_{1x}^{+r}-\frac{1}{2}\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{3r-m}{r}\Big]Q_2^{-(r-m)}P_{2x}^{+r}-\frac{1}{2}\sum_{\substack{r=1\\r<m}}^{\infty}\Big[\frac{3r-m}{r}\Big]P_1^{-(r-m)}Q_{2x}^{+r}$$

and

$$\delta_J^{-m},\delta_L^{-m},\delta_{G^1}^{-m},\delta_{G^2}^{-m}=\delta_J^{+m},\delta_L^{+m},\delta_{G^1}^{+m},\delta_{G^2}^{+m},\begin{pmatrix}+m\to-m\\+r\to-r\end{pmatrix}. \quad (74)$$

The Eqs.(66-74) are the most general super extensions of NLS equations related to N=2 superconformal algebra with Ramond type involving infinite number of bosonic and fermionic fields

5 Conclusions

Using AKNS scheme and N=2 superconformal algebra of Neveu-Schwarz and Ramond types we obtain two different new super- extensions of coupled Nonlinear Schrödinger equations.These super- extensions of coupled NLS equations involve infinite number of bosonic and infinite number of fermionic fields,and mathematically my work is an extension of the work done in ref.6. The solution of these NLS equations with infinite number of bosonic and infinite number of femionic fields is an open problem.

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