Natural Convection in a Lid-driven Square Cavity Filled with Darcy-Forchheimer Porous Medium in the Presence of Thermal Radiation

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Abstract: The influence of thermal radiation on natural-convection flow in a lid-driven square cavity filled with porous medium has been studied numerically by using finite-difference technique using staggered grid formulation. The side vertical walls are considered adiabatic while both the top and bottom horizontal walls are kept at constant but with different temperatures. The upper horizontal wall moves on its own plane from left to right at constant speed keeping all other walls fixed. The hydrodynamic field in the porous medium is modelled according to the general model involving Brinkman and Forchheimer terms. The simulation has been carried out for wide range of Rayleigh number, Darcy number, thermal radiation parameter, heat generating parameter, Prandtl number and porosity in order to obtain the solutions in terms of streamlines, isotherms, velocity profiles, temperature profiles, local Nusselt numbers and the average Nusselt number by considering both Darcian and non-Darcian models. Heat transfer rates at the heated walls are presented in terms of local Nusselt number.

Keywords: natural convection; square lid-driven cavity; darcy-Forchheimer porous medium; thermal radiation; heat generation

1 Introduction

The problem of natural convection flow in a lid-driven cavity filled with a porous medium has received considerable attention in recent years because of its relevance to the thermal performance of many industrial and technological applications. Some of these include cooling of electronic devices, solar collectors, nuclear reactors, oil extraction and crystal growth. Rafique et al. [1] presented numerical simulations of natural convection heat transfer along a vertical cylinder, for four different geometries. In the past several decades, a number of experimental and numerical studies have been performed to analyze the flow field and heat transfer characteristics of lid-driven cavity flow. In this context non-Darcy effects on natural convection in porous media has received a great deal of attention in the recent times due to a large number of technological and industrial applications associated with it such as fluid flow in geothermal reservoirs, separation processes in chemical industries, storage of nuclear waste, solidification of casting, thermal insulation, crude oil production, separation process in chemical industries and so on. Mixed convection with lid-driven cavities filled with porous media having important application in various field of engineering and geophysical system such as solar ponds, thermal-hydraulics of nuclear reactors, solar power collectors, packed-bed catalytic reactors and so on [2,3]. Cheng [4] provides a comprehensive review of the literature on free convection in fluid-saturated porous medium focusing on geothermal system. Medeiros et al. [5] studied numerically heat transfer by natural convection in a porous cavity under a non-Darcian approach for uniform porosity using Darcy-Brinkman-Forchheimer model. They compared the numerical results with the works that considered uniform porosity and found to have excellent accuracy. Most recently, Basak et al. [6] investigated a natural-convection flow in a square cavity filled with a porous medium for both uniform and non-uniform heating from below by using the Darcy-Forchheimer model. Prasad and Koseff [7] performed an experimental investigation of mixed convection flow in a lid driven cavity. They considered heated moving bottom wall with high Reynolds number and high Grashof number. Cheng [8] examined the flow and heat transfer in 2D square cavity where the flow is induced by a shear force resulting from the motion of the upper lid combined with buoyancy force due to bottom heating. Kanafer

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and Chamkha [9] studied the unsteady mixed convection flow in a lid-driven enclosure filled with Darcian fluid-saturated uniform porous medium with internal heat generation. They reported that Darcy number is most important parameter in natural convection in a porous medium. Moreover, the presence of the internal heat-generation effect in the model was found to have significant influence on the isotherms and less significant effects on the streamlines for small values of the Richardson number. Nithiarasu et al. [10] examined the effect of applied heat transfer on the cold wall of the cavity flow and heat transfer inside the porous medium. Al-Amiri [11] investigated the momentum and energy transfer in a lid-driven cavity filled with a porous medium. In this study, the general formulation of the momentum equation was used by incorporating both the inertial and viscous effects whereas the forced convection was induced by sliding top constant-temperature wall. Khanafer and Vafai [12] analyzed the double-diffusive mixed convection in a lid-driven porous enclosure with horizontal walls kept at constant and different temperatures and concentrations. Oztop [13] investigated, numerically, combined convection heat transfer and fluid flow in a partially heated porous lid-driven cavity. Kandaswamy et al. [14] studied the mixed convection in lid-driven square cavity filled with porous medium numerically. Oztop and Varol [15] investigated the flow field, temperature distribution and heat transfer in a lid-driven cavity filled with porous medium with non-uniformly heated bottom wall. Oztop et al. [16] performed numerical simulations of the conduction-combined forced and natural convection (mixed convection) heat transfer fluid flow for 2-D lid-driven square enclosure divided with a finite thickness and finite conductivity. They found that heat transfer is a decreasing function of increasing thermal conductivity ratio for all Richardson number. Very recently, Basak et al. [17] analyzed mixed convection in a lid driven porous square cavity with linearly heated side wall(s) and MuthtamilSelvan et al. [18] studied the characteristics of a lid-driven flow in a two dimensional square cavity filled with heat generating porous medium. When technology processes take place at high temperatures thermal radiation heat transfer become very important and its effects cannot be neglected. Recent developments in hypersonic flights, missile reentry rocket combustion chambers and gas cooled nuclear reactors, have focused attention of researchers on thermal radiation as a mode of energy transfer and emphasize the need for inclusion of radiative transfer in these processes. For these many studies have appeared concerning the interaction of radiative flux with thermal convection flows. Recently, Aldawody and Elbashbeshy [19] investigated the effects of thermal radiation and magnetic field on flow and heat transfer over an unsteady stretching surface in a micropolar fluid. Turkylmazoglu [20] analyzed the combined influences of viscous dissipation, Joule heating, temperature-dependent viscosity, on the time-dependent MHD permeable flow having variable viscosity. Thus the motivation of this study is to examine the influence of thermal radiation and internal heat generation on natural-convection flow field and temperature distribution in a lid-driven square cavity filled with fluid-saturated porous medium. The side vertical walls are considered adiabatic while both the top and bottom horizontal walls are kept at constant but different temperatures. The upper horizontal wall is moving on its own plane from left to right at constant speed while all other walls are fixed. The numerical results for streamlines, isotherms, velocity and temperature profiles at mid horizontal plane and the heat transfer rate at the heated walls in terms of local Nusselt number and average Nusselt number are presented graphically.

2 Governing equations and boundary conditions

The physical model of the problem is shown in Fig.1 which is a square porous cavity of side length $L$ and conductivity $k$. The side vertical walls are considered adiabatic while both the top and bottom horizontal walls are kept at constant but different temperatures. The upper horizontal wall is moving on its own plane from left to right at constant speed while all other walls are fixed. The Rosseland approximation [21] is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the $x$-direction is considered negligible in comparison to the $y$-direction. The study domain is two dimensional cavity filled with non-Darcian porous medium. The fluid inside the enclosure is considered as Newtonian, incompressible and constant properties. The thermophysical properties of the fluid at a reference temperature are taken as constant except the buoyancy term in $V$– momentum equation. It means that Boussineq approximation is performed.

In the Cartesian coordinate system, the fundamental equations are as follows:

The continuity equation:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0,$$
(1)

The $U$ momentum equation:

$$\frac{\partial U}{\partial t} + \frac{U}{\varepsilon} \frac{\partial U}{\partial x} + \frac{V}{\varepsilon} \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - \frac{\nu e}{K} U - \frac{1.75 \left| \frac{V}{\sqrt{150 \varepsilon}} \right| U}{\sqrt{150 \varepsilon}},$$
(2)
Figure 1: Schematic diagram of the physical model.

The $V$ momentum equation:

$$\frac{\partial V}{\partial t} + \frac{U}{\epsilon} \frac{\partial V}{\partial X} + \frac{V}{\epsilon} \frac{\partial V}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial Y} + \nu \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{V}{K} V - \frac{1.75}{\sqrt{150K\epsilon}} |V| - g\beta(T - T_c), \tag{3}$$

The energy equation:

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \alpha \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial Y} + \frac{Q}{\rho C_p} (T - T_c). \tag{4}$$

Now defining the following non-dimensional variables:

$$x = \frac{X}{L}, \quad y = \frac{Y}{L}, \quad u = \frac{UL}{\alpha}, \quad v = \frac{VL}{\alpha}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \alpha = \frac{k}{\rho C_p}, \tag{5}$$

$$t = \frac{\alpha^4 t^4}{L^2}, \quad p = \frac{PL^2}{\rho \alpha^2}, \quad P_r = \frac{\nu}{\alpha^4} \quad D_a = \frac{K}{L^2}, \tag{6}$$

$$R_a = \frac{g\beta(T - T_c)}{\nu^2} L^3 P_r, \quad N_R = \frac{kk^*}{4\sigma T^4}, \quad H\epsilon = \frac{QL^2}{k}. \tag{7}$$

Here the radiation heat flux $q_r$ is consider according to Rosseland approximation [21] such that

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial Y}, \tag{8}$$

where $\sigma$ and $k^*$ are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. As done by Raptis [22], the fluid-phase temperature differences within the flow are assumed to be sufficiently small so that $T^4$ may be expressed as a linear function of temperature. This is done by expanding $T^4$ in a Taylor series about the free stream temperature $T_\infty$ and neglecting higher order terms to yield,

$$T^4 = 4T^3_\infty T - 3T^4_\infty. \tag{9}$$
Using the parameters given by equations (5)-(9), we obtain the following group of dimensionless equations:

\[
\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\epsilon} \left( \frac{\partial u^2}{\partial x} \right) - \frac{1}{\epsilon} \left( \frac{\partial uv}{\partial y} \right) - \frac{P_r \epsilon}{Da} u - \frac{1.75 \sqrt{u^2 + v^2}}{\sqrt{150 Da \epsilon}} u,
\]

(10)

\[
\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{1}{\epsilon} \left( \frac{\partial v^2}{\partial x} \right) - \frac{1}{\epsilon} \left( \frac{\partial uv}{\partial y} \right) - \frac{P_r \epsilon}{Da} v - \frac{1.75 \sqrt{u^2 + v^2}}{\sqrt{150 Da \epsilon}} v + Pr R_a \epsilon \theta,
\]

(11)

\[
\frac{\partial \theta}{\partial t} = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \left( \frac{\partial u \theta}{\partial x} \right) + \frac{4}{3 N_R} \frac{\partial^2 \theta}{\partial y^2} + H \epsilon \theta.
\]

(12)

Equations (10)-(13) are solved using the following dimensionless boundary conditions:

\[
u = v = 0 \quad \text{and} \quad \theta = 1 \quad \text{at} \quad y = 0,
\]

(14)

\[
u = 1, \quad v = 0 \quad \text{and} \quad \theta = 0 \quad \text{at} \quad y = 1,
\]

(15)

\[
u = v = 0 \quad \text{and} \quad \frac{\partial \theta}{\partial x} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = 1.
\]

(16)

3 Method of solution

3.1 Finite-difference method

Control-volume based finite-difference discretization of the above equations (10-13) is carried out in the present work with staggered grid. This method is known as MAC cell method. In this method Pressure-Poisson equation is derived from discretized momentum and continuity equations. The pressure-Poisson equation takes the final form as

\[
2(a + b) p^n_{i,j} - a p^n_{i+1,j} - a p^n_{i-1,j} - b p^n_{i,j+1} - b p^n_{i,j-1} = -\frac{D^n_{ij}}{\Delta t} + \frac{1}{\Delta x} \{(U D P C)^n_{i,j} - (U D P C)^n_{i-1,j}\} + \frac{1}{\Delta y} \{(V D P C)^n_{i,j} - (V D P C)^n_{i,j-1}\},
\]

(17)

where \(D^n_{ij}\) is called the divergence of the the velocity field at the cell \((i,j)\) and \(n\)th time level, which is to be zero for the convergence of the flow in successive iterations of the method, where

\[
a = \frac{1}{\Delta x^2}, \quad b = \frac{1}{\Delta y^2}, \quad D^n_{i,j} = \left( \frac{u^n_{i,j} - u^n_{i-1,j}}{\Delta x} + \frac{v^n_{i,j} - v^n_{i,j-1}}{\Delta y} \right).
\]

(18)

In the equation (19) \(D i f f u^n_{i,j}\) and \(C o n u^n_{i,j}\) are diffusive and convective terms, respectively of the \(u\)-momentum equation at the \(n\)th time level at \((i, j)\)th cell and in equation (17), \((U D P C)^n_{i,j}\) is given by

\[
(U D P C)^n_{i,j} = Pr \text{ Diff } u^n_{i,j} - \frac{1}{\epsilon} \text{ Con } u^n_{i,j} - \frac{P_r \epsilon}{Da} u^n_{i,j} - \frac{1.75 \sqrt{(u^n_{i,j})^2 + (v^n_{i,j})^2}}{\sqrt{150 Da \epsilon}} u^n_{i,j},
\]

(19)

In the equation (19) \(D i f f v^n_{i,j}\) and \(C o n v^n_{i,j}\) are diffusive and convective terms of the \(v\)-momentum equation at the \(n\)th time level at \((i, j)\)th cell.

The pressure boundary condition is not needed for a MAC cell at the boundaries where the normal velocities are specified. Thus, for this type of boundary the pressure Poisson equations become as follows:

\[
(a + 2b) p^n_{i,j} - a p^n_{i+1,j} - b p^n_{i,j+1} - b p^n_{i,j-1} = -\left[ \frac{D^n_{ij}}{\Delta t} + \frac{1}{\Delta x} \{(U D P C)^n_{i,j} + \frac{1}{\Delta y} \{(V D P C)^n_{i,j} - (V D P C)^n_{i,j-1}\} \right],
\]

(20)
for the cells adjacent to the left boundary, since $p_{n-1,j} = p_{n,j} - \delta x (UDPC)_{n-1,j}$, at the left boundary wall, and
\[
(2a + b)p_n^{n,j} - ap_{n+1,j} - ap_{n-1,j} - bp_{n,j+1} = -\left[ \frac{D_p^{n,j}}{\delta t} + \frac{1}{\delta x} ((UDPC)_{n,j}^{n} - (UDPC)_{n-1,j}^{n} ) + \frac{1}{\delta y} (VDPCC)_{n,j}^{n} \right],
\]
for the cells adjacent to the lower boundary, since $p_{n,j-1} = p_{n,j} - \delta y (VDPCC)_{n,j-1}^{n}$, at the bottom boundary wall. Similarly, the pressure Poisson equations at the upper and at the lower boundaries can be specified.

We now describe the iteration process to obtain the solutions of the basic equations with appropriate boundary conditions. In the derivation of pressure Poisson equation, the divergence term at $n$-th time level $(D_{ij}^n)$ is retained and evaluated in the pressure-Poisson iteration. It is done because the discretized form of divergence of velocity field, i.e, $D_{ij}$ is not guaranteed to be zero.

The solution procedure starts with the initializing the velocity field. This is done either from the result of previous cycle or from the prescribed initial and boundary conditions. Using this velocity field, pressure-Poisson equation is solved using Bi-CG-Stab method. Knowing pressure field, $u,v,\theta$ from the prescribed initial and boundary conditions. Using this velocity field, pressure-Poisson equation is solved using Bi-CG-Stab method. Knowing pressure field, $u,v,\theta$ are updated to get the values at $(n+1)$th time level. Using the values of $u$ and $v$ at $(n+1)$th time level, the value of the divergence of velocity field is checked for its limit. If its absolute value is less than $0.5 \times 10^{-5}$ and steady state reaches then iteration process stops, otherwise again pressure-Poisson equation is solved for pressure.

### 3.2 Numerical Stability Criteria

Linear stability gives $\delta t_1 \leq \text{Min} \left[ \frac{\delta x}{|u|}, \frac{\delta y}{|v|} \right]$ which is related to convection of fluid, i.e, fluid should not move more than one cell width per time step (Courant, Friedrichs and Lewy condition).

Also, from the Hirt’s stability analysis, $\delta t_2 \leq \text{Min} \left[ \frac{1}{2M_r} \frac{\delta x^2 \delta y}{\delta x \delta y \delta x + \delta x \delta y} \right]$. This condition roughly states that momentum cannot diffuse more than one cell width per time step. The time step in the computations is determined from
\[
\delta t = FCT \times \left[ \text{Min}(\delta t_1, \delta t_2) \right],
\]
where the factor FCT varies from 0.2 to 0.4. The upwinding parameter $\beta$ is governed by the inequality condition $1 \geq \beta \geq \text{Max} \left[ \frac{|u|}{\delta x}, \frac{|v|}{\delta y} \right]$. As a rule of thumb, $\beta$ is taken to be approximately 1.2 times larger than the above inequality condition.

### 3.3 Stream function and Nusselt Number

The fluid motion is displayed using the stream function $\psi$ is obtained from velocity components $u$ and $v$. The relationships between stream function, $\psi$ and velocity components for two-dimensional flows are
\[
u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}.
\]

Positive sign of $\psi$ denotes anti-clockwise circulation and the clockwise is represented by the negative sign of $\psi$. The heat transfer coefficient in terms of the local Nusselt number ($Nu$) is defined by
\[
Nu = -\frac{\partial \theta}{\partial n},
\]
where $n$ denotes the normal direction to a plane. The local Nusselt number at the bottom wall ($Nu_b$) is defined as
\[
Nu_b = -\frac{\partial \theta}{\partial y} \bigg|_{y=0}.
\]
The average Nusselt number at the hot wall is given by,
\[
\overline{Nu_H} = \int_0^1 Nu_b \, dx.
\]
4 Results and Discussion

In this study, numerical results have been obtained for natural convection flow in a unit square cavity for different Rayleigh numbers $R_a (10^3 \leq R_a \leq 10^6)$, Darcy numbers $Da (10^{-4} \leq Da \leq 10^{-2})$, thermal radiation parameters $N_R (0.5 \leq N_R \leq 2.0)$, heat generating parameters $He (0.5 \leq He \leq 5.0)$, Prandtl numbers $Pr (0.7 \leq Pr \leq 7.0)$ and the porosity of the porous medium $\epsilon (0.2 \leq \epsilon \leq 0.6)$ which are illustrated through a Table and several Figures. All the calculations are done using $80 \times 80$ grid size. Table 1 presents the computed values of the average Nusselt number ($\bar{Nu}_{y=0}$) of the hot wall ($y = 0$) for various values of $Da$, $Pr$, $N_R$, $He$ and $R_a$ with uniform porosity ($0.2 \leq \epsilon \leq 0.6$). From this table, it is seen that when $Pr$ increases from 0.7 to 7.0 the average Nusselt number increases. Similar trend is observed when the values of $\epsilon$, $N_R$, $Da$ and $R_a$ is increased from 0.2 to 0.6, 0.5 to 2.0, $10^{-4}$ to $10^{-2}$ and $10^4$ to $10^6$. It is also observed from this table that the average Nusselt number decreases with increasing the $He$ from 1.0 to 5.0.

Table 1: Computed values of $\bar{Nu}_{y=0}$ for various values of $Ra$, $N_R$, $He$, $Pr$, and $\epsilon$.

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<th>$Ra$</th>
<th>$Da$</th>
<th>$He$</th>
<th>$N_R$</th>
<th>$\epsilon$</th>
<th>$Pr$</th>
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Figures 2 & 3 show the variation of vertical velocity and temperature profiles, respectively, at the mid-horizontal plane for $Pr = 0.7$, $R_a = 10^6$ and $He = 1.0$ for two values of $Da=10^{-4}$ and $10^{-3}$. It is seen from Fig. 2 that when $Da=10^{-4}$, vertical velocity increases with increase in $N_R$ near the left vertical wall whereas reverse effect is observed at the right vertical wall but opposite trend is observed when $Da = 10^{-3}$. Further, it is observed that the vertical velocity profile increases with increase in the value of $Da$ whereas reverse effect is observed at the right vertical wall as seen in Fig. 2. From Fig. 3 it is observed that in the left half of the cavity, temperature at a point on the mid horizontal plane increases with increase in $Da$ but decreases with increase in $N_R$. The opposite trend is found in the right half of the cavity.

Figures 4 & 5 depict the variation of vertical velocity and temperature at the mid-horizontal plane of the cavity by considering uniform porosity for various values of $Da$ and $He$. It is observed from the Fig. 4 that the effect of increasing the value of $He$ is to increase the vertical velocity near the left vertical wall whereas reverse effect is observed at the right vertical wall for $Da = 10^{-4}$. But when $Da = 10^{-3}$ the vertical velocity decreases with increase $He$ whereas at middle area of the cavity, opposite effect is observed and the profile again takes the same pattern near the right vertical wall as that observed near the left vertical wall. Fig. 5 shows the effects of $He$ on temperature at the mid- horizontal plane of the cavity for $Pr = 0.7$, $R_a = 10^6$, $N_R = 1.0$, $\epsilon = 0.4$ and $Da = 10^{-4}$ and $10^{-3}$. It is seen that temperature increases when $He$ increases at the left wall for both $Da = 10^{-4}$, $10^{-3}$. It is well known that the heat generation causes the fluid temperature to increase which has a tendency to increase the thermal buoyancy effects. These effects are clearly seen from this figure.
Figure 2: Velocity profiles at mid-horizontal plane for different values of $N_R$ and $Da$ when $Ra = 10^6$, $Pr = 0.7$ and $He = 1.0$.

Figure 3: Temperature profiles at mid-horizontal plane for different values of $N_R$ and $Da$ when $Ra = 10^6$, $Pr = 0.7$ and $He = 1.0$.

Figure 4: Velocity profiles at mid-horizontal plane for different values of $He$ and $Da$ when $Ra = 10^6$, $Pr = 0.7$ and $N_R = 1.0$.

Figure 5: Temperature profiles at mid-horizontal plane for different values of $He$ and $Da$ when $Ra = 10^6$, $Pr = 0.7$ and $N_R = 1.0$.
The effects of Darcy number $Da$ and porosity of the porous medium $\epsilon$ on velocity and temperature profiles are shown in the Figs. 6 & 7 when $Ra = 10^6$, $Pr = 0.7$, $N_R = 1.0$ and $He = 1.0$. From Fig. 6, we observe that the velocity profiles at mid-horizontal plane in the presence of porous medium increases with increase in $Da$ and $\epsilon$ near the left vertical wall of the square cavity, by keeping other parameters fixed, whereas reverse effect is observed near the right vertical wall of the cavity for both the parameters $Da$ and $\epsilon$. Fig. 7 shows that the temperature profiles at the mid-horizontal plane increases near the left wall with increase in porosity of the porous medium $\epsilon$ but the opposite trend is observed near the right wall of the cavity for $Da = 10^{-4}$, whereas the effects of $\epsilon$ on temperature profiles are opposite for $Da = 10^{-3}$.

Figures 8 & 9 depict the effect of $Da$ on vertical velocity and temperature at mid-horizontal plane when $Ra = 10^4$, $Pr = 1.0$, $N_R = 1.0$, $He = 1.0$ and $\epsilon = 0.4$. It is observed from Fig. 8 that the vertical velocity profiles at the mid-horizontal plane decreases with decrease in the values of $Da$ in the left half of the cavity keeping other parameters fixed, whereas reverse effect is observed at the right half of the cavity. Fig. 9 shows that the temperature profiles at mid-horizontal plane decreases when $Da$ decreases in the left half of the cavity but the opposite trend is observed near the right half of the cavity. Further, it is observed from this figure that the temperature coincides at the mid point of the cavity for all value of the Darcy number.

The effects of the $Da$ and $Ra$ on streamlines and isotherms are illustrated in the Figs. 10 – 14 for different values of $Da = 10^{-4}$ to $10^{-2}$ and $Ra = 10^4$ to $10^6$ with uniform heating of bottom wall. Due to heating of the bottom wall the fluid rises up along the sides of the hot wall and flows towards the cold wall, forming a roll with clockwise rotation inside the cavity filled with non-Darcian porous medium. Fig.10 demonstrates that the natural convection flow dominates the lid-driven flow for $Da = 10^{-4}$. The streamlines are all most symmetric with respect to the center of the cavity. Comparing Figs. 10(a), 11(a) and 12(a), it is observed that as Da increases the expansion of streamlines take place. It is also seen from the above comparison that the flow pattern in nearly same but with the change in the flow intensity. Further, it is observed that when the Darcy number is increased to $10^{-2}$, the streamlines indicate an elongation of the recirculation region of the flow(Fig.12a). It is seen from the comparison of Figs.10(b), 11(b), and 12(b) that at a smaller value of $Da(= 10^{-4})$ the isotherms are similar, but at higher value of $Da (= 10^{-3}$ or $10^{-2}$) they are not similar. Fluid circulation is strongly dependent on Darcy number as can be seen from Figs. 10 – 12. As the $Da$ is increased, the intensity

\[ \text{value of } 10 \]
within the cavity increases due to reduced resistance from the porous medium which shows that the higher permeability of the porous medium is significantly more important for the heat transfer rate. Thus, when the \(Da\) is increased from \(10^{-4}\) to \(10^{-2}\), the isotherm plot indicates that the thermal boundary layers become denser due to uniform heating. Thus, Darcy number helps in minimizing the conduction heat transfer and increasing the convection heat transfer due to the reason that permeability increases with increase in the \(Da\). Figs. 11, 13 and 14 show the effect of \(Ra\) on streamlines and isotherms. Fig. 13 depicts that the flow pattern is nearly same as Fig. 11 but with change in flow intensity i.e, the value of the stream function at the center of the cavity increased from \(-16.44\) to \(-2.44\). Figure 14 is the plot of the streamlines and isotherms for \(Da = 10^{-3}\) and \(Ra = 10^{4}\). It is observed that the streamlines concentrated at the top horizontal wall due to stronger circulation, which results in lower heat transfer rate due to convection. It is also observed that isotherms are almost parallel to the horizontal wall for all values of the buoyancy ratios which indicates that most of the heat transfer is carried out by heat conduction due to an increase in the thermal boundary-layer thickness.

The effects of \(Da\) \((10^{-4} \leq Da \leq 10^{-2})\) and \(NR\) \((1.0 \leq NR \leq 5.0)\) on the local Nusselt numbers at the bottom wall are displayed in Figure 15 when \(Ra = 10^6\), \(Pr = 0.7\), \(He = 1.0\) and \(\epsilon = 0.4\). It is observed from this figure that the heat transfer rate increases when radiation parameter increases for both \(Da = 10^{-4}\), \(10^{-3}\). In the case of uniform heating, the heat transfer rate at the bottom wall or \(Nu_b\) is very high at the right edge of vertical wall, due to the presence of discontinuity in the temperature boundary conditions at this edge. Further, the local heat transfer rate at the left vertical wall has its minimum value. The effects of \(Da\), \(He\) and porosity of the porous medium on the heat transfer rates are displayed in Fig. 16−18. From Fig. 16 it is seen that \(Nu_b\) decreases when \(He\) increases. Further, \(Nu_b\) is very high at the right edge of vertical wall and it has its minimum value at the left vertical wall. Figure 17 shows that the heat transfer rate increases with decreasing \(Da\) in the left half of the cavity but in the right half of the cavity the opposite trend is observed. The value of \(Nu_b\) is same at this point for any value of \(Da\) when \(Ra = 10^4\), \(Pr = 0.7\), \(He = 1.0\), \(NR = 1.0\) and \(\epsilon = 0.4\). Figure 18 presents the plot of variation of \(Nu_b\) with \(x\) for various values of \(Da\) and \(\epsilon\). It is observed that \(Nu_b\) increases when \(\epsilon\) increases for both \(Da = 10^{-4}\) and \(10^{-3}\).
Figure 10: (a) Streamlines, (b) Isotherms for $Da = 10^{-4}$, $Ra = 10^6$, $Pr = 1.0$, $N_R = 1.0$ and $He = 1.0$.

Figure 11: (a) Streamlines, (b) Isotherms for $Da = 10^{-3}$, $Ra = 10^6$, $Pr = 1.0$, $N_R = 1.0$ and $He = 1.0$. 

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Figure 12: (a) Streamlines, (b) Isotherms for $Da = 10^{-2}, Ra = 10^6, Pr = 1.0, Nr = 1.0$ and $He = 1.0$.

Figure 13: (a) Streamlines, (b) Isotherms for $Da = 10^{-3}, Ra = 10^5, Pr = 1.0, Nr = 1.0$ and $He = 1.0$. 

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Figure 14: (a) Streamlines, (b) Isotherms for $Da = 10^{-3}$, $Ra = 10^4$, $Pr = 1.0$, $NR = 1.0$ and $He = 1.0$.

Figure 15: Local Nusselt number at the bottom wall for different values of $NR$ and $Da$ when $Ra = 10^6$, $Pr = 0.7$, $He = 1.0$.

Figure 16: Local Nusselt number at the bottom wall for different values of $He$ and $Da$ when $Ra = 10^6$, $Pr = 0.7$, $NR = 1.0$.
5 Conclusions

Effects of thermal radiation and heat generation on natural convection in a lid-driven square cavity filled with Darcy-Forchheimer porous medium is studied in the present paper. Numerical results for the velocity and temperature at the mid-horizontal plane of the cavity, local Nusselt number and average Nusselt number are obtained for representative governing physical parameters. Streamlines and isotherms for various values of Darcy number and Rayleigh number are shown graphically. As a summary, we conclude the following:

1) The effect of increasing the thermal radiation parameter is to enhance the vertical velocity.
2) The vertical velocity profiles decreases with increasing the value of heat generating parameter on the left vertical wall whereas reverse effect is observed on the right vertical wall.
3) The temperature decreases with increase in the value of Rayleigh number up to certain value of $x$ and beyond that distance the opposite trend is observed.
4) Average Nusselt number increases with increase in the thermal radiation parameter whereas reverse effect is observed in the case of heat generating parameter increase.

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