The Controller Designing of the Type-2 Fuzzy Impulsive Nonlinear Systems

Ruoyu Zhu *, Yuanyuan Sun, Dianchen Lu
Faculty of Science, Jiangsu University, Zhenjiang, 212013,China
(Received 23 February 2011, accepted 12 May 2011)

Abstract: In this paper a novel impulsive control approach based on interval Type-2 T-S fuzzy model has been presented for nonlinear systems. With the help of the controller, the resulting closed-loop system is globally stable in the sense that all signals involved are uniformly bounded. Finally, the simulation of the inverted pendulum model demonstrates the validity and superiority of the proposed method by easily determining the membership functions and choosing minimum number of fuzzy rules and show that the IT2FIC achieves the best tracking performance in comparison with the T1FLC.

Keywords: Type-2 fuzzy set; impulsive control; interval type-2 fuzzy control; stability

1 Introduction

Type-2 fuzzy sets (T2 FSS), originally introduced by Zadeh [1], provide additional design degrees of Mamdani and TSK fuzzy logic systems (FLSS), which can be very useful when such systems are used in situations where lots of uncertainties are present [2]. The resulting type-2 fuzzy logic systems (T2 FLS) have the potential to provide better performance than a type-1 (T1) FLS [3]. To-date, because of the computational complexity of using a general T2 FS, most people only use interval T2FSs in a T2 FLS, the result being an interval T2 FLS (IT2 FLS) [4].

But less research and development on type-2 fuzzy impulsive system has been done before. Due to the advantages of impulsive control, Type-2 fuzzy system and TS fuzzy dynamic model, a stable fuzzy impulsive controller based on interval type-2 T-S fuzzy model is proposed in this paper. It can show a great potential in handling various modeling as well as control applications.

This paper is organized as follows. Indirect adaptive fuzzy impulsive controller design using interval type-2 FNN is constructed in Section 2. In Section 3, the proposed IT2FIC is described and the Lyapunov function is utilized to testify the stability of the proposed method. The conclusions are drawn in Section 4.

2 Interval type-2 TS fuzzy impulsive

Based on the IT2FLC mentioned in the previous subsection, the design procedures for IT2FIC will be described in this subsection.

2.1 Problem formulations

Consider the following nth-order nonlinear system

\[
\begin{cases}
\dot{X}_i = X_{i+1}, i = 2, ..., n - 1, \\
\dot{X}_n = f(X) + g(X)u + d(t), \\
y = X_1,
\end{cases}
\]  

(1)

where \(X\) is the state vector, \(u\) is the control input of the plan, \(y\) is the output of the system. \(f\) is the unknown continuous function, \(g\) is the unknown function control gain \(d(t)\) denotes the unknown external disturbance. The tracking problem of (1) is to find a suitable control law to let the states follow a desired trajectory \(y_d\).

*Corresponding author. E-mail address: zhuzhu860616@126.com

Copyright © World Academic Press, World Academic Union

IJNS.2011.06.15/483
The tracking error is defined as:

\[ e = [y_d - y, \dot{y}_d - \dot{y}, \ldots, y_d^{(n-1)} - y^{(n-1)}]^T. \] (2)

Then we design an adaptive fuzzy controller \( u \) and an impulsive controller \( U_k \) such that the resulting closed-loop system is globally stable in the sense that all signals involved are uniformly bounded and the tracking error between the output of the system \( y \) and the given bounded reference signal \( y_d \) is convergent to zero asymptotically, i.e., \( e_1 \to 0 \) as \( t \to +\infty \).

The impulsive controller \( U_k \) is to be designed later and the impulsive time instants \( t_k \) satisfy:

\[ 0 < t_1 < t_2 < \ldots < t_{k-1} < t_k < \ldots, \lim_{k \to \infty} t_k = \infty. \]

we obtain the following error dynamical system

\[
\begin{cases}
\dot{e}_i = e_{i+1}, i = 1, \ldots, n - 1, \\
\dot{e}_n = y_d^{(n)} - f(X) - g(X)u - d(t), t \not= t_k \\
\Delta e(t_k) = -U_k(e(t_k^-)), k = 1, 2, \ldots,
\end{cases}
\] (3)

In order to design stable adaptive fuzzy and impulsive control, we make the following assumptions, which have been used in many papers, for example [5].

Based on approximation capability of fuzzy systems, a novel adaptive type-2 fuzzy impulsive control is developed. Let \( k = [k_0, k_{n-1}, \ldots, k_1]^T \in \mathbb{R}^n \) be such that all roots of polynomial \( h(s) = s^n + k_1 s^{n-1} + \ldots + k_n = 0 \) are in the open left-half complex plane. The (3) can be transformed into

\[
\begin{align*}
\dot{e} &= \Lambda_c e + b_c[y_d^{(n)} - f(X) - g(X)u + k^T e], t \not= t_k \\
\Delta e(t_k) &= -U_k(e(t_k^-)), k = 1, 2, \ldots,
\end{align*}
\] (4)

\[
\Lambda_c = \begin{pmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & 1 \\
-k_n & -k_{n-1} & \cdots & -k_1
\end{pmatrix}_{n \times n},
\quad b_c = \begin{pmatrix}
0 \\
0 \\
\vdots \\
1
\end{pmatrix}_{n \times 1}
\]

Because \( \Lambda_c \) is stable, there always exists a positive definite matrix \( P = P^T \in \mathbb{R}^{n \times n} \) satisfying the Lyapunov equation

\[ \Lambda_c^T P + P \Lambda_c + Q = 0, \]

where \( Q \in \mathbb{R}^{n \times n} \) is an arbitrary positive definite matrix specified by the designer.

### 2.2 Controller designing

If both functions \( f(X) \) and \( g(X) \) are known, and there is no external disturbance and impulsive control, then the control law

\[ u = \frac{1}{g(X|\theta_g)} \left[ \hat{f}(X|\theta_f) + y_d^{(n)} + k^T e \right] \]

applied to system (1) can result in \( e_1^{(n)} + k_1 e^{(n-1)} + \ldots + k_n e_1 = 0 \), which implies that \( \lim_{t \to \infty} e_1(t) = 0 \).

Define: \( \Omega_{\theta_f} = \{ \theta_f \| \| \theta_f \| \leq M_{\theta_f} \} \quad \Omega_{\theta_g} = \{ \theta_g \| \| \theta_g \| \leq M_{\theta_g} \} \),

we adopt the following adaptive fuzzy control law:

\[ u = u_c + u_\omega \] (5)

In which:

\[ u_c = \frac{1}{g(X|\theta_g)} \left[ \hat{f}(X|\theta_f) + y_m^{(n)} + K^T e \right], u_\omega = \frac{1}{g_0} \text{sgn}(e^T P b_c) (\dot{\epsilon}_\omega + D), t \not= t_k. \]

LINS homepage: http://www.nonlinearscience.org.uk/
where $u_c$ is called equivalent controller, $u_\omega$ is an additional robustifying control term. The minimum approximation error is defined as:

$$
\omega = \hat f (X | \theta_f^\ast) - f (X) + (\hat g (X | \theta_g^\ast) - g (X)) u_c .
$$

(6)

Let $\varepsilon_\omega = \max_{X \in \Omega_X, \theta_f \in \Omega_{\theta_f}, \theta_g \in \Omega_{\theta_g}} [\hat f (X | \theta_f^\ast) - f (X) + (\hat g (X | \theta_g^\ast) - g (X)) u_c]$.

Then $\varepsilon_\omega$ is an unknown bounded constant. Let $\tilde \varepsilon_\omega$ be an estimator of $\varepsilon_\omega$ at time $t$.

It adopts the following impulsive control law:

$$
U_k(e(t_k^-)) = G_k(e(t_k^-)), k = 1, 2, ..., (7)
$$

where $G_k \in \mathbb{R}^{n \times n}$ are constant matrices to be designed later.

Then

$$
\dot{e} = \Lambda e + b_\omega \hat g(X | \theta_g) u_c + b_c [(f(X | \theta_f)) + (\hat g(X | \theta_g)) u_c + \omega - d(t)], t \neq t_k .
$$

3 Stability analysis

**Theorem 1** Consider the nonlinear system (1) with control laws defined by (5) and (7). If there exist two positive constants $\lambda_1, \lambda_2$ such that $\lambda_1 = \inf_k \{t_k - t_{k-1}\} > 0$,

$$
\lambda_2 = \sup_k \{t_k - t_{k-1}\} < \infty,
$$

and the impulsive matrices $G_k$ satisfy the following inequalities

$$
G_k^T P G_k - G_k^T P - P G_k \leq 0, k = 1, 2, ..., (8)
$$

Then

(1) If $\hat \theta_f(0) \in \Omega_{\theta_f}, \hat \theta_g(0) \in \Omega_{\theta_g}$ then $\left\| \dot{\hat \theta}_f(t) \right\| \leq M_{\theta_f}, \left\| \dot{\hat \theta}_g(t) \right\| \leq M_{\theta_g}, \forall t \geq 0$.

(2) If $M_X = 2 \sqrt{\lambda_{\min}(P)} V + M_d$, then the state vector $X \in \Omega_x = \{X \mid X \leq M_x\}$. Thus the overall closed-loop fuzzy control system is globally stable in the sense that all of the closed-loop signals are bounded.

**Proof.** Define the Lyapunov function candidate

$$
V = \frac{1}{2} e^T P e + \frac{1}{2} \hat \theta_f^T \hat \theta_f + \frac{1}{2} \hat \theta_g^T \hat \theta_g + \frac{1}{2} \varepsilon_\omega^2 .
$$

Consider the Lyapunov function candidate $V_1$. Taking the Dini derivative of $V(t)$ for $t \in [t_k-1, t_k)$, then we obtain:

$$
D^+ V_1 = \frac{1}{2} e^T P e + \frac{1}{2} \hat \theta_f^T \hat \theta_f + \frac{1}{2} \hat \theta_g^T \hat \theta_g + \frac{1}{2} \varepsilon_\omega^2 .
$$

On the other hand, when $t \neq t_k$

$$
\Delta V_1 = \frac{1}{2} (e - G_k e)^T P (e - G_k e) - \frac{1}{2} e^T P e = \frac{1}{2} e^T (G_k^T P G_k - G_k^T P - P G_k) e \leq 0 .
$$

4 Simulation results

In this section, we will provide a biological simulation example with chaotic dynamics performance to illustrate the feasibility of the control scheme proposed in this paper.

Consider this following system:

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= 3x_1^2 - 0.1x_2 - 2x_1^3 - x_1 + 0.01 \cos t + u \\
y &= x_1
\end{align*}
$$

IENS email for contribution: editor@nonlinearscience.org.uk
where \( d = 0.01 \cos t, f = 3x_1^2 - 0.1x_2 - 2x_1^3 - x_1, g = 1, \) \( x_1 \) is indicated the blood velocity in the aneurysm. \( x_2 \) represents the change rate of blood velocity in the aneurysm. The objective is to generate an appropriate actuator force \( u \) to control the blood velocity in aneurysm.

In the following simulation, the design parameters of controller (3) and parameter update laws are chosen. The initial conditions are chosen that \( X_1 = 0.2, X_2 = 0.2, \gamma = 1. \) This indicates the adaptive fuzzy impulsive method is effective to the biomathematical model from theory we can see the chaotic state of the vessel is controlled.

![Figure 1: The trajectories of \( x_1(t) \) and \( x_2(t) \), for the Type-2 fuzzy impulsive controller.](image)

The above simulation results show that the interval type-2 fuzzy impulsive systems can handle unpredicted disturbance and data uncertainties very well. This method proves that the interval type-2 fuzzy impulsive system and the impulsive controller have better robustness.

## 5 Conclusions

Combining interval type-2 T-S fuzzy model with impulsive control, a fuzzy impulsive controller for nonlinear systems based on the Type-1 T-S fuzzy model is proposed in this paper.

From the simulation results, it is obvious to see that the output performance obtained from the interval type-2 fuzzy impulsive controller is better than the one obtained from the type-1 fuzzy impulsive controller. Moreover, the interval type-2 fuzzy impulsive system can handle unpredicted internal disturbance and data uncertainties well. Finally, the effectiveness and feasibility of the developed methods have been shown by some numerical simulations.
References


