ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.12(2011) No.1,pp.16-24

Existence and Uniqueness Result for Couple Stress Bio-Fluid Flow Model via Adomian Decomposition Method

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(Received 14 October 2011, accepted 24 March 2011)

Abstract: In this paper, we examine the effect of viscosity on blood flow. In some previous papers, blood viscosity has been assumed as constants. However, some experiments suggest that blood viscosity varies under certain conditions. Here we assume that the source of variable viscosity is the hematocrit parameter (β). We assume that β could be positive, negative or zero. We proved the existence and uniqueness results for the non-trivial solution of the nonlinear initial and boundary value problem by using Adomian decomposition method (ADM).

Keywords: couple stress fluid, Variable viscosity, stenosis, porous medium, ADM

1 Introduction

In a recent paper [1], the effect of couple stresses on the hydromagnetic non-Newtonian fluid using Eyring-Powel was studied by assuming constant viscosity. However, for some reasons blood viscosity can vary [2]-[4]. We assume that the cause of variations in viscosity is due to stenosis [5], and hematocrit variations [6]. It is known that magnetic field has great influence on biological fluids [7]-[10] In the present study, the work in [1] is extended to include the effects of hematocrit variations and channel porosity on the Hydromagnetic blood flow and we prove existence and uniqueness of approximate solution to the non-linear model by using Adomian decomposition method [11]. An excellent application of the method can be found in [12]-[21].

2 Fluid model and mathematical formulation

Consider an incompressible flow of non-Newtonian fluid through a porous artery, we select rectangular coordinate in such a way that the x-axis lies along the centre line and the y-axis normal to it, the Bousinesq approximation for the flow is
Continuity Equation:

$$\frac{\partial u}{\partial x} = 0 \tag{1}$$

Momentum Equation:

$$\rho(\frac{\partial u}{\partial t} + \nu_0 \frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \sigma B_0^2 u - \mu \frac{\partial^4 u}{\partial y^4} - \frac{\mu}{K} u \tag{2}$$

$$\frac{\partial P}{\partial y} = 0 \tag{3}$$

here τ_{xy} represents the stress tensor in the case of the non-polar theory of fluids. The stress tensor in the Eyring-Powell model for non-Newtonian fluids takes the form [1], [22].

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IJNS.2011.08.15/505

$$\tau_{xy} = \mu\frac{\partial u}{\partial y} + \frac{1}{\beta}\sinh^{-1}(\frac{1}{c}\frac{\partial u}{\partial y}) \tag{4}$$

Substituting (4) into (2) we obtain

$$\rho(\frac{\partial u}{\partial t} + \nu_0\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y} + \frac{1}{\beta}\sinh^{-1}(\frac{1}{c}\frac{\partial u}{\partial y})) - \sigma B_0^2 u - \mu\frac{\partial^4 u}{\partial y^4} - \frac{\mu}{K}u \tag{5}$$

by assuming a small electric field, (5) can be written as

$$\rho(\frac{\partial u}{\partial t} + \nu_0\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}((\mu\frac{\partial u}{\partial y} + \frac{1}{\beta}\sinh^{-1}(\frac{1}{c}\frac{\partial u}{\partial y}))\frac{\partial u}{\partial y}) - \sigma B_0^2 u - \mu\frac{\partial^4 u}{\partial y^4} - \frac{\mu}{K}u \tag{6}$$

Following [6], we assume

$$\mu = \mu_0 e^{\beta(1-y^2)} \tag{7}$$

where
μ_0 is the viscosity of blood plasma
β is the hematocrit variation parameter
the expansion of the elastic part of red blood cell is given to be

$$\sinh -1(\frac{1}{c}\frac{\partial u}{\partial y}) \cong \frac{1}{c}\frac{\partial u}{\partial y} + \frac{1}{6}(\frac{1}{c}\frac{\partial u}{\partial y})^3; \qquad |\frac{1}{c}\frac{\partial u}{\partial y}| < 1 \tag{8}$$

introducing the following dimensionless parameters and variables

$$u' = \frac{u}{\nu_0},\ \ x' = \frac{x}{h},\ \ y' = \frac{y}{h},\ \ t' = \frac{\nu_0 t}{h},\ \ p' = \frac{P}{\rho\nu_0^2},\ \ Ha = \frac{\sigma B_0^2 h^2}{\mu_0}$$

$$Re = \frac{\nu_0 h}{\nu}, Da = \frac{K}{h^2}, a = \frac{1}{\nu\rho\alpha c h}, s^2 = \frac{1}{Da}, \epsilon = \frac{\nu_0^2}{2\nu\alpha c^3\rho h^2}, b = \frac{\mu}{\rho h^2\nu} \tag{9}$$

we obtain

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial u}{\partial y}) + \frac{1}{Re}(a + \epsilon(\frac{\partial u}{\partial y})^2)\frac{\partial^2 u}{\partial y^2}$$
$$-\frac{1}{Re}(Ha^2 + s^2 e^{\beta(1-y^2)})u - \frac{b}{Re}\frac{\partial^4 u}{\partial y^4} \tag{10}$$

together with the initial condition

$$u(0,y) = \sin(\pi y) \qquad 0 \le y \le 1 \tag{11}$$

and the boundary conditions

$$\begin{aligned} u(t,0) &= 0 = u''(t,0) \\ u(t,1) &= 0 = u''(t,1) \end{aligned} \tag{12}$$

where $t \ge 0$; u - flow velocity, P - pressure, t - time, x-horizontal coordinate, y - vertical coordinate, ν_0 -vertical constant velocity, h- distance between the two plates, Re-Reynolds number, Da-Darcy number, Ha- Hartmann number, H_0 - magnetic field intensity, B_0= -electromagnetic induction, ρ - fluid density, μ -dynamic viscosity, σ -conductivity of fluid, μ_e - magnetic permeability, β -hematocrit parameter, a, e - Eyring-Powel parameters η -couple stress parameter, Re - is the Reynolds's number of plasma

Although much work has been done on the convergence of the Adomian decomposition method [23]-[26]; now we want to establish a convergence of the Adomian decomposition model applied to the non-linear model.

Theorem 1 *(Existence of Solution) Let*

$$\begin{aligned}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y} &= -\frac{\partial P}{\partial x}+\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial u}{\partial y})+\frac{1}{Re}(a+\epsilon(\frac{\partial u}{\partial y})^2)\frac{\partial^2 u}{\partial y^2}\\ &\quad -\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})u-\frac{b}{Re}\frac{\partial^4 u}{\partial y^4}\end{aligned}$$

the initial condition

$$u(0,y)=\sin(\pi y) \qquad 0\le y\le 1$$

and the boundary conditions

$$\begin{aligned}u(t,0)=0=u''(t,0)\\ u(t,1)=0=u''(t,1)\end{aligned}$$

for $t\ge 0$, then the problem has an approximate solution.

Proof. For pulsatile flow
Let

$$\frac{-dP}{dx}=P_1+P_0e^{i\omega t} \qquad t\ge 0 \tag{13}$$

So that by substituting (13) in (10) we get

$$\begin{aligned}\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y} &= P_1+P_0e^{i\omega t}+\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial u}{\partial y})+\frac{1}{Re}(a+\epsilon(\frac{\partial u}{\partial y})^2)\frac{\partial^2 u}{\partial y^2}\\ &\quad -\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})u-\frac{b}{Re}\frac{\partial^4 u}{\partial y^4}\end{aligned} \tag{14}$$

the nonlinear term in (14) is identified as

$$B_n=(\frac{\partial u}{\partial y})^2 \tag{15}$$

substituting (15) in (14) and integrating with respect to t gives

$$\begin{aligned}u(t,y)=u(0,y)+\int_0^t(P_1+P_0e^{i\omega t})dt\\ \int_0^t(\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial u}{\partial y})=\frac{1}{Re}(a+\epsilon B_n)\frac{\partial^2 u}{\partial y^2}\\ -\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})u-\frac{b}{Re}\frac{\partial^4 u}{\partial y^4})dt\end{aligned} \tag{16}$$

from (16) we have the iterative scheme

$$\begin{aligned}u(t,y)=\sin(\pi y)+\int_0^t(P_1+P_0e^{i\omega t})dt\\ u_{n+1}=\int_0^t(\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial u}{\partial y})+\frac{1}{Re}(a+\epsilon B_n)\frac{\partial^2 u}{\partial y^2}\\ -\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})u-\frac{b}{Re}\frac{\partial^4 u}{\partial y^4})dt\end{aligned} \tag{17}$$

by Taylor's expansion we have

$$e^{\beta(1-y^2)} = 1 + \beta(1-y^2) + \frac{\beta^2}{2}(1-y^2)^2 + \frac{\beta^3}{3!}(1-y^2)^3 + ... + \frac{\beta^n}{n}(1-y^2)^n \tag{18}$$

obviously (18) is an infinite series therefore let $\beta << 3$, we have

$$e^{\beta(1-y^2)} = 1 + \beta(1-y^2) + \frac{\beta^2}{2}(1-y^2)^2 + O(\beta^3) \tag{19}$$

now substituting (19) in (17) yields obviously (18) is an infinite series therefore let $\beta << 3$, we have

$$\begin{aligned} u_0(t,y) &= \sin(\pi y) + \int_0^t (P_1 + P_0 e^{i\omega t})dt \\ u_{n+1} &= \int_0^t (-\frac{\partial u_n}{\partial y} + \frac{1}{Re}\frac{\partial}{\partial y}((1+\beta(1-y^2) + \frac{\beta^2}{2}(1-y^2)^2)\frac{\partial u_n}{\partial y}) + \frac{1}{Re}(a+\epsilon B_n)\frac{\partial^2 u_n}{\partial y^2})dt \\ &- \int_0^t (\frac{1}{Re}(Ha^2 + s^2(1+\beta(1-y^2) + \frac{\beta^2}{2}(1-y^2)^2))u_n + \frac{b}{Re}\frac{\partial^4 u_n}{\partial y^4})dt \end{aligned} \tag{20}$$

using the algorithm proposed by Wazwaz, [27], the Adomian polynomial can be decomposed by substituting the series

$$u = \sum_{n=0}^{\infty} u_n \tag{21}$$

in equation (15), giving

$$B_n = (\frac{\partial}{\partial y}(u_0 + u_1 + u_2 + u_3 + ...))^2 \tag{22}$$

now let

$$B_n = (\frac{\partial}{\partial y}(u_0(1+\phi)))^2 \tag{23}$$

where

$$\phi = \frac{1}{u_0}(u_1 + u_2 + u_3 + ...) \tag{24}$$

by binomial expansion we have

$$B_n = (\frac{du_0}{dy})^2 + \frac{du_0}{dy}(2\frac{du_1}{dy} + 2\frac{du_2}{dy} + 2\frac{du_3}{dy} + ...) + (\frac{du_1}{dy})^2 + 2\frac{du_1}{dy}\frac{du_2}{dy} + ...(\frac{du_2}{dy})^2 + ... \tag{25}$$

by Adomian the polynomial is given

$$\begin{aligned} B_0 &= (\frac{du_0}{dy})^2 \\ B_1 &= 2\frac{du_0}{dy}\frac{du_1}{dy} \\ B_2 &= 2\frac{du_0}{dy}\frac{du_2}{dy} + (\frac{du_1}{dy})^2 \\ &... \end{aligned} \tag{26}$$

with each term of (26) known, other terms of the series can be obtained by using Wazwaz, [28] to reduce the computational load, that is,

$$\begin{aligned} &u_0(t,y)=\int_0^t (P_1+P_0e^{i\omega t})dt \\ &u_1(t,y)=\sin(\pi y)+\int_0^t (\frac{1}{Re}\frac{\partial}{\partial y}((1+\beta(1-y^2)+\frac{\beta^2}{2}(1-y^2)^2)\frac{\partial u_0}{\partial y}) \\ &+\frac{1}{Re}(a+\epsilon B_n)\frac{\partial^2 u_0}{\partial y^2})dt-\int_0^t (\frac{1}{Re}(Ha^2+s^2(1+\beta(1-y^2)+\frac{\beta^2}{2}(1-y^2)^2))u_0 \\ &+\frac{b}{Re}\frac{\partial^4 u_0}{\partial y^4})dt \end{aligned} \tag{27}$$

for $n>1$ we have

$$\begin{aligned} Un+1=&\int_0^t (\frac{1}{Re}\frac{\partial}{\partial y}((1+\beta(1-y^2)+\frac{\beta^2}{2}(1-y^2)^2)\frac{\partial u_n}{\partial y})+\frac{1}{Re}(a+\epsilon B_n)\frac{\partial^2 u_n}{\partial y^2})dt \\ &-\int_0^t (\frac{1}{Re}(Ha^2+s^2(1+\beta(1-y^2)+\frac{\beta^2}{2}(1-y^2)^2))u_n+\frac{b}{Re}\frac{\partial^4 u_n}{\partial y^4})dt \end{aligned} \tag{28}$$

by Adomian the k^{th} approximant can be obtained as

$$u(t,y)=\sum_{n=0}^{k} u_n(t,y) \tag{29}$$

therefore the problem has an approximate solution ■

Theorem 2 *(Uniqueness Theorem)*
If $u(t,y)$ is a continuously differentiable function and satisfies the differential equation

$$\begin{aligned} \frac{\partial u}{\partial t}+\frac{\partial u}{\partial y} &= -\frac{\partial P}{\partial x}+\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial u}{\partial y})+\frac{1}{Re}(a+\epsilon(\frac{\partial u}{\partial y})^2)\frac{\partial^2 u}{\partial y^2} \\ &-\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})u-\frac{b}{Re}\frac{\partial^4 u}{\partial y^4} \end{aligned} \tag{30}$$

the initial condition

$$u(0,y)=\sin(\pi y) \qquad 0\le y\le 1$$

and the boundary conditions

$$\begin{aligned} u(t,0)=0=u''(t,0) \\ u(t,1)=0=u''(t,1) \end{aligned}$$

for $t\ge 0$, *then it is unique.*

Proof. Suppose there are two solutions of $u_1(t,y)-u_2(t,y)$
then let the initial condition

$$\nu(0,y)=u_1(t,y)-u_2(t,y) \tag{31}$$

then (30) must satisfy

$$\begin{aligned} \frac{\partial \nu}{\partial t}+\frac{\partial \nu}{\partial y} &= -\frac{\partial P}{\partial x}+\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial \nu}{\partial y})+\frac{1}{Re}(a+\epsilon(\frac{\partial \nu}{\partial y})^2)\frac{\partial^2 \nu}{\partial y^2} \\ &-\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})\nu-\frac{b}{Re}\frac{\partial^4 \nu}{\partial y^4} \end{aligned}$$

together with the initial condotion

$$\begin{aligned}\nu(0,y) &= u_1(0,y)-u_2(0,y)\\ &= \sin(\pi y)-\sin(\pi y)\\ &= 0\end{aligned} \tag{32}$$

for $0 \le y \le 1$, and the boundary condition

$$\begin{aligned}\nu(t,0) = u_1(t,0)-u_2(t,0) = u_1''(t,0)-u_2''(t,0) = 0\\ \nu(t,1) = u_1(t,1)-u_2(t,1) = u_1''(t,1)-u_2''(t,1) = 0\end{aligned} \tag{33}$$

for $t \ge 0$, substituting (13) then we must have

$$\begin{aligned}\frac{\partial\nu}{\partial t}+\frac{\partial\nu}{\partial y} &= P_1+P_0e^{i\omega t}+\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial\nu}{\partial y})+\frac{1}{Re}(a+\epsilon(\frac{\partial\nu}{\partial y})^2)\frac{\partial^2\nu}{\partial y^2}\\ &\quad -\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})\nu-\frac{b}{Re}\frac{\partial^4\nu}{\partial y^4}\end{aligned} \tag{34}$$

satisfying the initial condition

$$\nu(0,y) = 0 \qquad 0 \le y \le 1 \tag{35}$$

and the boundary condition

$$\begin{aligned}\nu(t,0) = \nu_1''(t,0) = 0\\ \nu(t,1) = \nu_1''(t,1) = 0\end{aligned} \tag{36}$$

for $t \ge 0$, consider the function Myint-U and Debnatn, [29]

$$J'(t) = \int_0^t \nu^2 dy \tag{37}$$

differentiating with respect to t, we obtain

$$J'(t) = \int_0^t \nu\nu_t dy \tag{38}$$

substituting for ν_t we have

$$\begin{aligned}J'(t) &= \int_0^1 \nu(P_1+P_0e^{i\omega t}-\frac{\partial\nu}{\partial y}+\frac{1}{Re}\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial\nu}{\partial y})+\frac{1}{Re}(a+\epsilon(\frac{\partial\nu}{\partial y})^2)\frac{\partial^2\nu}{\partial y^2}\\ &\quad -\frac{1}{Re}(Ha^2+s^2e^{\beta(1-y^2)})\nu-\frac{b}{Re}\frac{\partial^4\nu}{\partial y^4})dy\end{aligned} \tag{39}$$

then

$$\begin{aligned}&J'(t) = (P_1+P_0e^{i\omega t})\int_0^1 \nu dy-\int_0^1 \nu\frac{\partial\nu}{\partial y}dy+\frac{1}{Re}\int_0^1 \nu\frac{\partial}{\partial y}(e^{\beta(1-y^2)}\frac{\partial\nu}{\partial y})dy+\\ &\frac{\epsilon}{Re}\int_0^1 \nu(\frac{\partial\nu}{\partial y})^2)\frac{\partial^2\nu}{\partial y^2}dy+\frac{a}{Re}\int_0^1 \nu\frac{\partial^2\nu}{\partial y^2}dy-\int_0^1 \nu^2(Ha^2+s^2e^{\beta(1-y^2)})dy\\ &-\frac{b}{Re}\int_0^1 \nu\frac{\partial^4\nu}{\partial y^4}dy\end{aligned} \tag{40}$$

assuming $\beta << 1$, we have $e^{\beta(1-y^2)} = 1 + \beta(1-y^2)$ so that

$$J'(t) = (P_1 + P_0 e^{i\omega t})\int_0^1 \nu dy - \int_0^1 \nu\frac{\partial \nu}{\partial y}dy + \frac{1}{Re}\int_0^1 \nu\frac{\partial}{\partial y}(1+\beta(1-y^2))\frac{\partial \nu}{\partial y})dy +$$
$$\frac{\epsilon}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2\frac{\partial^2 \nu}{\partial y^2}dy + \frac{a}{Re}\int_0^1 \nu\frac{\partial^2 \nu}{\partial y^2}dy - \int_0^1 \nu^2(Ha^2 + s^2(1+\beta(1-y^2)))dy$$
$$-\frac{\beta}{Re} - \frac{b}{Re}\int_0^1 \nu\frac{\partial^4 \nu}{\partial y^4}dy \quad (41)$$

expanding we obtain

$$J'(t) = (P_1 + P_0 e^{i\omega t})\int_0^1 \nu dy + \frac{(1+a+\beta)}{Re}\int_0^1 \nu\frac{\partial^2 \nu}{\partial y^2}dy + \frac{\epsilon}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2\frac{\partial^2 \nu}{\partial y^2}dy - \int_0^1 \nu\frac{\partial \nu}{\partial y}dy$$
$$-\frac{\beta}{Re}\int_0^1 y^2\nu\frac{\partial^2 \nu}{\partial y^2}dy - \frac{2\beta}{Re}\int_0^1 y\nu\frac{\partial \nu}{\partial y}dy - \int_0^1 \nu^2(Ha^2 + s^2(1+\beta(1-y^2)))dy$$
$$-\frac{b}{Re}\int_0^1 \nu\frac{\partial^4 \nu}{\partial y^4}dy \quad (42)$$

integrating by parts, we have

$$J'(t) = (P_1 + P_0 e^{i\omega t})\nu|_0^1 + \frac{(1+a+\beta)}{Re}[\nu\nu_y]|_0^1 - \frac{(1+a+\beta)}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2 dy$$
$$+\frac{\epsilon}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2\frac{\partial^2 \nu}{\partial y^2}dy - \int_0^1 \nu\frac{\partial \nu}{\partial y}dy - \frac{\beta}{Re}\int_0^1 y^2\nu\frac{\partial^2 \nu}{\partial y^2}dy - \frac{2\beta}{Re}\int_0^1 y\nu\frac{\partial \nu}{\partial y}dy$$
$$-\int_0^1 \nu^2(Ha^2 + s^2(1+\beta(1-y^2)))dy - \frac{b}{Re}\int_0^1 \nu\frac{\partial^4 \nu}{\partial y^4}dy \quad (43)$$

since

$$\nu(t,0) = 0 = \nu(t,1) \quad (44)$$

then (43) reduces to

$$J'(t) = -\frac{(1+a+\beta)}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2 dy + \frac{\epsilon}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2\frac{\partial^2 \nu}{\partial y^2}dy$$
$$-\int_0^1 \nu\frac{\partial \nu}{\partial y}dy - \frac{\beta}{Re}\int_0^1 y^2\nu\frac{\partial^2 \nu}{\partial y^2}dy - \frac{2\beta}{Re}\int_0^1 y\nu\frac{\partial \nu}{\partial y}dy$$
$$-\int_0^1 \nu^2(Ha^2 + s^2(1+\beta(1-y^2)))dy - \frac{b}{Re}\int_0^1 \nu\frac{\partial^4 \nu}{\partial y^4}dy \quad (45)$$

thus

$$J'(t) = -\frac{(1+a+\beta)}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2 dy - \frac{\epsilon}{Re}\int_0^1 \nu(\frac{\partial \nu}{\partial y})^2\frac{\partial^2 \nu}{\partial y^2}dy$$
$$-\int_0^1 \nu\frac{\partial \nu}{\partial y}dy - \frac{\beta}{Re}\int_0^1 y^2\nu\frac{\partial^2 \nu}{\partial y^2}dy - \frac{2\beta}{Re}\int_0^1 y\nu\frac{\partial \nu}{\partial y}dy$$
$$-\int_0^1 \nu^2(Ha^2 + s^2(1+\beta(1-y^2)))dy - \frac{b}{Re}\int_0^1 \nu\frac{\partial^4 \nu}{\partial y^4}dy \quad (46)$$

the implication of (45) is that

$$J'(t) \le 0 \qquad J(t) \le 0 \quad (47)$$

however from the intial condition

$$\nu(0,y)=0 \tag{48}$$

we have

$$J(0)=0 \tag{49}$$

by definition, the functional

$$J(t)=\frac{1}{2}\int_0^1 \nu^2 dy \tag{50}$$

we should have $J(t) \geq 0$ so that it is an increasing function of time but (50) showed a contradiction, that is, a non increasing function of time thus $J(t)=0$ and this implies that $u_1(t,y)=u_2(t,y)$ in $0 \leq y \leq 1, t \geq 0$ thus the solution converges to $u(t,y)$,in conclusion the solution is unique. ■

Acknowledgment

The first author is full of thanks to Prof. A.M. Wazwaz of the Department of Mathematics and Computer Science, Saint Xavier University Chicago, IL60655, USA. For all the encouragements and support on the Adomian decomposition method.

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