ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.12(2011) No.3,pp.305-312

Approximate Solutions of System of PDEEs Arising in Physics

M A Abdou *
Faculty of education of girls, King khaled University, Physics Department, Bisha, Saudi Arabia
(Received 24 June 2010 , accepted 16 March 2011)

Abstract: Reduced differential transform method (RDTM) has been successively used to find the explicit and numerical solutions of a coupled nonlinear evolution equations. Comparing the methodology with some other known techniques shows that the present approach is the effective and powerful. In addition, Two models of special interest in physics are discussed to illustrate the effectiveness of the reduced differential transform method. It is worthwhile to mention that the RDTM is easy to be applied to other coupled nonlinear system in physics.

Keywords: reduced differential transform method;adomian decomposition method; variational iteration method; coupled Schrodinger-KdV equations.

1 Introduction

The investigation of exact solutions to nonlinear partial differential equation plays an important role in the study of nonlinear physical phenomena. In the past several decades, great progress have been made on the construction of exact solutions of NLPDEs and many significant methods have been established $[2-10]$. With the rapid development of computer algebraic system like Maple or Mathematical, many powerful algebraic methods have been presented to obtain a great many exact solutions of NLPDEs.

The numerical methods which do not require discretization of space-time variables or linearization of the nonlinear equations, among which are the Adomian decomposition method (ADM) [11] and the variational iteration method, which is suggested by Ji- Huan He [12], and the Homotopy perturbation method (HPM) [13]. The numerical methods can provide approximate solutions rather than analytic solutions of the problems. The differential transform method $[14-17]$ is a numerical method for solving differential equations or system of the differential equations. The reduced Differential transform method for solving differential equations has recently renewed interest due to many important applications $[18-21]$.

In this work, we shall extend the technique of reduced differential transform method of nonlinear evolution equations of special interest physically. The paper has been organized as follows. **In Section 2**,we begin by introducing the definition and the basic mathematical operations of reduced differential transform method . **In Section 3**, two models are chosen to illustrate the validity of the reduced differential Transform Method (RDTM). Finally, some conclusions and discussions are provided in **Section 4**.

2 Methods and its applications

In what follows Summarize the setps reports in Reduced differentional transforme method [18-21]. Based on the properties of differential transform, function $u(x,t)$ can be represented as

$$u(x,t)=\sum_{k=0}^{\infty}U_k(x)t^k, \tag{1}$$

where $U_k(x)$ is called t-dimensional spectrum function of $u(x,t)$.

The basic definitions of reduced differential transform (RDTM) method $[18-21]$ are intoduced as

* *E-mail address*: m_abdou_eg@yahoo.com

IJNS.2011.12.15/543

Definition 1 *If function $u(x,t)$ is analytic and differentiated continuously with respect to t and x in the domian of interst,then let*

$$U_k(x) = \frac{1}{k}[[\frac{d^k u(x,t)}{dt^k}]|_{t=0}, \tag{2}$$

where the t dimensional spectrum function $u_k(x)$ is the transformed function.In this paper,the lowercase $u(x,t)$ represent the original function while the uppercase $U_k(x)$ stand for the transformed function.

The differential inverse transform of $U_k(x)$ is defined as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k \tag{3}$$

Then combining Eqs.(1) and (3) we write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k}[\frac{d^k u(x,t)}{dt^k}]|_{t=0}t^k \tag{4}$$

With the aid above definitions, it can be found that the concept of the reduced differential transform is derived from the power series expansion.

For the purpose of illustration of the methodlology to the proposed method, we write nonlinear equation in the standered operator form as follows

$$L(u(x,t)) + R(u(x,t)) + N(u(x,t)) = g(x,t), \tag{5}$$

with initial condition

$$u(x,0) = f(x), \tag{6}$$

where $L = \frac{\partial}{\partial t}$,$R$ is a linear operator which has partial derivatives, $Nu(x,t)$ is a nonlinear term and $g(x,t)$ is an inhomogeneous term.

According to the RDTM and Table[1], we can constructing the following iteration formulas

$$(k+1)U_{k+1}(x) = G_k(x) - RU_k(x) - NU_k(x), \tag{7}$$

where $U_k(x)$,$RU_k(x)$,$NU_k(x)$ and $G_k(x)$ are transformation of the functions $Lu(x,t)$,$Ru(x,t)$,$Nu(x,t)$ and $g(x,t)$ respectively. From the initial condition, we write

$$U_0(x) = f(x), \tag{8}$$

Substituting Eq.(8) and (7) and by a straight forward iterative formula, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values ${U_k(x)^n}_{k=0}$ gives approximation solution as,

$$u^*(x,t) = \sum_{k=0}^{n} U_k(x)t^k \tag{9}$$

where n is order of approximation solution.Therefore,the exact solution of problem is given by

$$u(x,t) = lim_{n \longrightarrow \infty} u_n^*(x,t), i = 1,2,....,n \tag{10}$$

3 Applications

To illustrate the effectiveness and the advantages of the proposed method, two models of nonlinear evolution equations arising in mathematical physics are chosen, namely, a coupled MKdV equations and coupled Schrodinger-KdV equations.

3.1 Example[1].A coupled MKdV equations

Let us first consider a coupled MKdV equations [22] as

$$u_t = \frac{1}{2}u_{xxx} - 3u^2u_x + \frac{3}{2}v_{xx} + 3(uv)_x - 3\lambda u_x,$$

$$v_t = -v_{xxx} - 3vv_x - 3u_xv_x + 3u^2v_x + 3\lambda v_x, \tag{11}$$

with initial conditions

$$u(x,0) = \frac{b_1}{2k} + ktanh[kx],$$

$$v(x,0) = \frac{\lambda}{2}(1+\frac{b_1}{2}) + b_1tanh[kx], \tag{12}$$

where k, $b_1 \neq 0$, and λ are arbitrary constants. This equation describes interactions of two long waves with different dispersion relations.

With the aid of Table[1], the differential transform of Eqs.(11) admits to

$$(k+1)U_{k+1}(x) = \frac{1}{2}\frac{\partial^3}{\partial x^3}U_k(x) - 3A_k(x) + 3B_k(x) + \frac{3}{2}\frac{\partial^2}{\partial x^2}V_k(x) - 3\lambda\frac{\partial}{\partial x}U_k(x),$$

$$(k+1)V_{k+1}(x) = -\frac{1}{2}\frac{\partial^3}{\partial x^3}V_k(x) - 3C_k(x) - 3D_k(x) + E_k(x) + 3\lambda\frac{\partial}{\partial x}V_k(x), \tag{13}$$

where the t-dimensional spectrum function $U_k(x)$ and $V_k(x)$ are the transformed function, $A_k(x)$,$B_k(x)$,$C_k(x)$ and $D_k(x)$ are transformed form of the nonlinear terms.From the initial conditions (12), we write

$$U_0(x) = \frac{b_1}{2k} + ktanh[kx],$$

$$V_0(x) = \frac{\lambda}{2}(1+\frac{b_1}{2}) + b_1tanh[kx] \tag{14}$$

The first three component of the nonlinear term reads

$$A_0 = U_0^2U_{0x}, A_1 = 2U_0U_1U_{0x} + U_0^2U_{1x}, A_2 = 2U_0U_2U_{0x} + U_1^2U_{0x} + 2U_0U_1U_{1x} + U_{2x}U_0^2,$$

$$B_0 = (U_0V_0)_x, B_1 = (V_0V_1 + U_0V_1)_x, B_2 = (V_0U_2 + U_0V_2 + U_1V_1)_x,$$

$$C_0 = V_0V_{0x}, C_1 = (V_0V_{1x} + V_1V_0)_x, C_2 = V_2V_{0x} + V_1V_{1x} + V_0V_{2x},$$

$$D_0 = U_{0x}V_{0x}, D_1 = U_{1x}V_{0x}, D_2 = U_{2x}V_{0x} + U_{1x}V_{1x} + U_{0x}V_{2x},$$

$$E_0 = U_0^2V_{0x}, E_1 = 2U_0V_{0x}U_1 + V_0^2V_{1x}, E_2 = 2U_0V_{0x}U_2 + U_1^2V_{0x} + 2U_0U_1V_{1x} + V_{2x}U_0^2 \tag{15}$$

Using Eqs.(15) into Eqs.(13), we can directly evaluated the values of $U_k(x)$ and $V_k(x)$. Explicit form for $U_k(x)$ and $V_k(x)$ are obtained for 5 order of approximation. For simplicity should be omitte here.

The inverse transform of of the set of values $U_k(x)$ and $V_k(x)$ gives five term approximate solutions as

$$u^*(x,t) = \sum_{k=0}^{5} U_k(x)t^k, \tag{16}$$

$$v^*(x,t) = \sum_{k=0}^{5} V_k(x)t^k, \tag{17}$$

The approximate solutions of $u(x,t)$ and $v(x,t)$ are readily found to be

$$u(x,t) = lim_{n\longrightarrow\infty} u_n^*(x,t), i = 1,2,....,n \tag{18}$$

$$v(x,t) = lim_{n\longrightarrow\infty} v_n^*(x,t), i = 1,2,....,n \tag{19}$$

The numerical behaviour of the approximate solutions of $u(x,t)$ and $v(x,t)$ obtained by RDTM with the exact solutions by Adomian decomposition method [22] are shown graphically in Figs.[(1.(a-f)] for a different values of time t with a fixed values of $k = 0.1$,$\lambda = 1.5$ and $b_1 = 0.1$, which proofs the two solutions are quite good. It is to be noted that the exact solutions of $u(x,t)$ and $v(x,t)$ can be written as

$$u(x,t) = \frac{b_1}{2k} + ktanh[k\xi],$$

$$v(x,t) = \frac{\lambda}{2}(1 + \frac{b_1}{2}) + b_1 tanh[k\xi], \tag{20}$$

with $\xi = x + \frac{1}{4}[-4k^2 - 6\lambda + \frac{6k\lambda}{b_1} + \frac{3b_1^2}{k^2}]t$,where k,$b_1 \neq 0$,and λ is arbitrary constants.

3.2 Example (2)

The coupled Schrodinger-KdV equations

In this case we shall deal with coupled Schrodinger-KdV equations [7]

$$iu_t - (u_{xx} + uv) = 0,$$

$$v_t + 6uv_x + v_{xxx} - (|u^2|)_x) = 0, \tag{21}$$

with the initial conditions [7]

$$u(x,0) = 6\sqrt{2}k^2 e^{i\alpha x} sech^2(kx),$$

$$v(x,0) = \frac{\alpha + 16k^2}{3} - 16k^2 tanh^2(kx) \tag{22}$$

This nonlinear system is an important mathematical physica model with wide applications in quantum mechanics, nonlinear optics, plasma physics and chemical physics.

Making use of the differential transform of Eqs.(21) as follows

$$(k+1)U_{k+1}(x) = i[\frac{\partial^2}{\partial x^2}U_k(x) + A_k(x)], \tag{23}$$

$$(k+1)V_{k+1}(x) = -6B_k(x) - \frac{\partial^3}{\partial x^3}V_k(x) + 2C_k(x), \tag{24}$$

where the t-dimensional spectrum function $U_k(x)$ and $V_k(x)$ are the transformed function, $A_k(x)$,$B_k(x)$ and $C_k(x)$ are transformed form of the nonlinear terms. From the initial conditions, we write

$$U_0(x) = 6\sqrt{2}k^2 e^{i\alpha x} sech^2(kx),$$

$$V_0(x) = \frac{\alpha + 16k^2}{3} - 16k^2 tanh^2(kx) \tag{25}$$

The first three component of nonlinear term can be written as

$$C_0 = U_0U_{0x}, C_1 = U_1U_{0x} + U_0U_{1x}, C_2 = U_2U_{0x} + U_1U_{1x} + U_0U_{2x},$$

$$C_3 = U_3U_{0x} + U_2U_{1x} + U_1U_{2x} + U_0U_{3x}, B_0 = U_0V_{0x}, B_1 = U_1V_{0x} + U_0U_{1x},$$

$$B_2 = U_2V_{0x} + U_1V_{1x} + U_0V_{2x}, B_3 = U_3V_{0x} + U_2V_{1x} + U_1V_{2x} + U_0V_{3x}, A_0 = U_0V_0,$$

$$A_1 = U_1V_0 + V_1U_0, A_2 = U_1V_1 + V_0U_2 + U_0V_2, A_3 = U_3U_{0x} + U_2U_{1x} + U_1U_{2x} + U_0U_{3x},$$

$$B_0 = U_0V_{0x}, B_1 = U_1V_{0x} + U_0U_{1x} \tag{26}$$

Inserting Eqs.(25) and (26) into Eqs.(23) and (24), the values of $U_k(x)$ and $V_k(x)$ can be diretly evaluated. Explicit form for $U_k(x)$ and $V_k(x)$ obtained by RDTM are obtained for 5 order of approximation.

Therefore the inverse transform of the values of $U_k(x)$ and $V_k(x)$ gives five term approximate solutions as follows

$$u^*(x,t) = \sum_{k=0}^{5} U_k(x)t^k, \tag{27}$$

$$v^*(x,t) = \sum_{k=0}^{5} V_k(x)t^k \tag{28}$$

Then approximate solutions in this case are given by

$$u(x,t) = lim_{n\longrightarrow\infty} u_n^*(x,t), i = 1,2,....,n \tag{29}$$

$$v(x,t) = lim_{n\longrightarrow\infty} v_n^*(x,t), i = 1,2,....,n \tag{30}$$

It is to be noted that,the exact solutions of $u(x,t)$ and $v(x,t)$ can be written as [7]

$$u(x,t) = 6\sqrt{2}k^2 e^{i\theta} sech^2(k\xi), \tag{31}$$

$$v(x,t) = \frac{\alpha + 16k^2}{3} - 16k^2 tanh^2(k\xi), \tag{32}$$

where k is constants and $\theta = [\frac{\alpha t}{2} + \alpha^2 t - \frac{10k^2 t}{3} + \alpha x]$.Figs.[2.(a-d)] shows the comparison of the RDTM approximation solution of order six and exact solutions obtained with a fixed values $k = \alpha = 0.05$ for a different values of t by variational iteration method [7]. From the figures, it seen that the RDTM approximation and the exact solutions are nearly the same.

4 Summary and Discussion

In the present work, the RDTM is used for constructing the approximate solutions of two nonlinear coupled systems arising in physics, namely, coupled MKdV system and coupled schrodinger KdV equations.

It is worth noting that the solutions obtained by the proposed method confirm the correctness of those obtained by other methods. The method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in mathematical physics. This is our task in future work.

Table 1: Reduced Differentional Transformation

$FunctionalFrom$	$TransformedForm$
$u(x,t)$	$U_k(x) = \frac{1}{k\vert}[\frac{\partial u(x,t)}{dt^k}]_{t=0}$
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \alpha u(x,t)$	$W_k(x) = \alpha U_k(x)$,α is a constant
$w(x,t) = x^m t^n$	$W_k(x) = x^m \delta(k-n)$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U(k-n)$
$w(x,t) = u(x,t)v(x,t)$	$W_k(x) = \sum_{r=0}^{k} U_r(x)V_{k-r}(x)$
$w(x,t) = \frac{\partial^r}{\partial t^r}u(x,t)$	$W_k(x) = (k+1)....(k+r)U_{k+1}(x) = \frac{(k+r)!}{k!}U_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x}u(x,t)$	$W_k(x) = \frac{\partial}{\partial x}U_k(x)$

Acknowledgments

The author would like to express sincerely thanks to the referees for useful comments and discussions.

References

[1] L.Debtnath,Nonlinear partial differential equations for scientist and Eng.,Birkhauser,Boston,1997

[2] S.A.El-Wakil,M.A.Abdou.Modified extended tanh function method for solving nonlinear partial differential equations.*Chaos,Solitons and Fractals*, 31(2007):1256-1264.

[3] S A El-Wakil and M A Abdou,New applications of Adomian decomposition method.*Chaos Solitons & Fractals*,33(2007):513-522.

[4] S.A.El-Wakil,M.A.Abdou,A.Elhanbaly.New solitons and periodic wave solutions for nonlinear evolution equations.*Phys.Lett.A*, 353(2006):40-47.

[5] M.A.Abdou and A.Elhanbaly.Construction of periodic and solitary wave solutions by the extended Jacobi elliptic function expansion method.*Commun.in Non.Sci. and Numer.Simu.*,12 (2007):1229-1241.

[6] M A Abdou.On the variational iteration method.*Phys.Lett.A*,366(2007):61-68.

[7] M A abdou,A A Soliman,New applications of Variational iteration method,Physica D 211(2005)1-8

[8] S.A.El-Wakil,M.A.Abdou.New exact travelling wave solutions using Modified extended tanh function method. *Chaos,Solitons and Fractals*, 31(2007):840-848.

[9] M.A.Abdou.The extended F-expansion method and its application for nonlinear evolution equations.*Chaos Solitons and Fractals*, 31(2007):95-104.

[10] M A Abdou,S Zhang,New periodic wave solutions via extended mapping method.*Communcation in Nonlinear Science and Numerical Simulation*,14(2009):2-11.

[11] G.Adomian,Solving Frontier Problems of Physics: The Decomposition Method,Kluwer,1994.

[12] J.H.He,Variational iteration method-a kind of non-linear analytical technique: Some examples. *International Journal of Non-Linear Mechanics*, 34(1999):699-708.

[13] He J.H.,Homotopy perturbation method: A new nonlinear technique. *Applied Mathematics and Computation*, 135(2003):73-79.

[14] G.E.Pukhov.Differential transformations and mathematical modelling of physical processes,Kiev,1986.

[15] J.K.Zhou,Differential transform and its Applications for Electrical Circuits,Huazhong University Press, Wuhan, China,1986.

[16] G.E.Pukhov,Differential transformations and mathematical modelling of physical processes,Kiev,1986.

[17] J.K.Zhou,Differential transform and its Applications for Electrical Circuits,Huazhong University Press, Wuhan, China,1986.

[18] Y.Keskin, Ph.D.Thesis,University of Selcuk(to appear).

[19] Keskin Y.,Oturanc G.,Reduced Differential Transform Method for Partial Differential Equations.*International Journal of Nonlinear Sciences and Numerical Simulation*,10(2009):741-749.

[20] Keskin Y.,Oturanc G.,Numerical simulations of systems of PDEs by reduced differential transform method. Communications in Nonlinear Science and Numerical Simulations(In Press).

[21] Keskin Y.,Oturanc G.,Reduced differential transform method for fractional parial differential equations.*Non.Sci.Lett.A 1*,(2010):61-72.

[22] D.Kaya,I.E.Tnan,Appl.Math. Comput.151(2004):775-787.

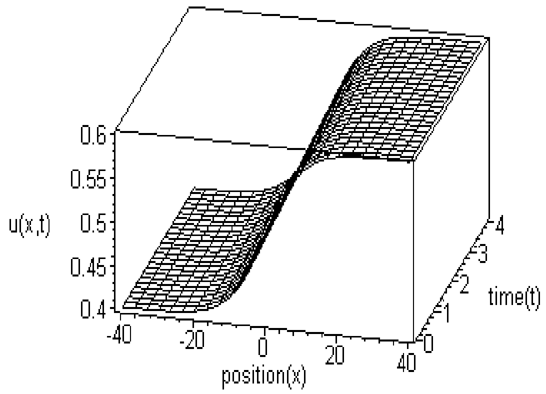

Fig.[1.a].Numerical solution of u(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=0.1 and λ=1.5 for a different values of t

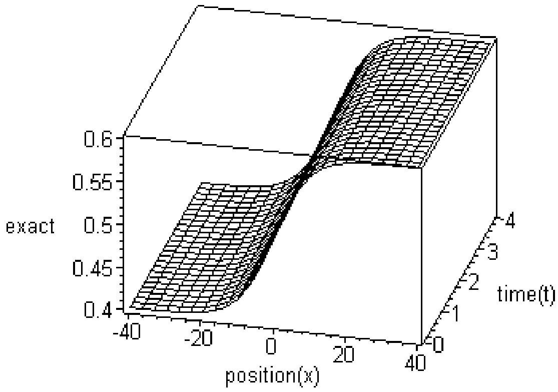

Fig.[1.b].the exact solution of u(x,t) Eq.(20) obtained by **Variational iteration method** with a fixed values of k=0.1 and λ=1.5 for a different values of t

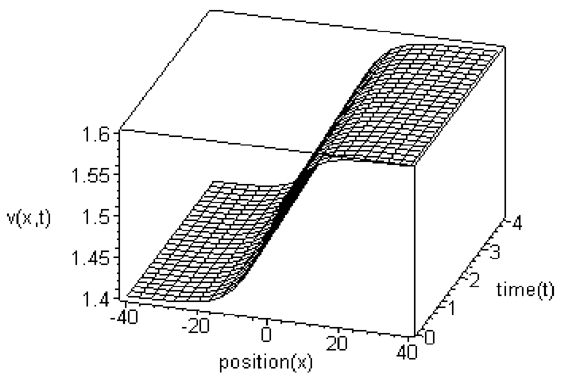

Fig.[1.c].Numerical solution of v(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=0.1 and λ=1.5 for a different values of t

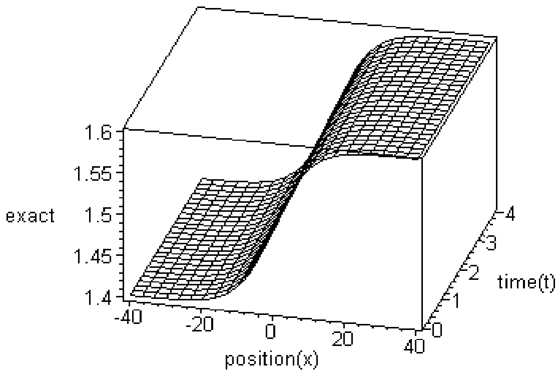

Fig.[1.d].the exact solution of v(x,t) Eq.(20) obtained by **Adomian decomposition method** with a fixed values of k=0.1 and λ=1.5 for a different values of t

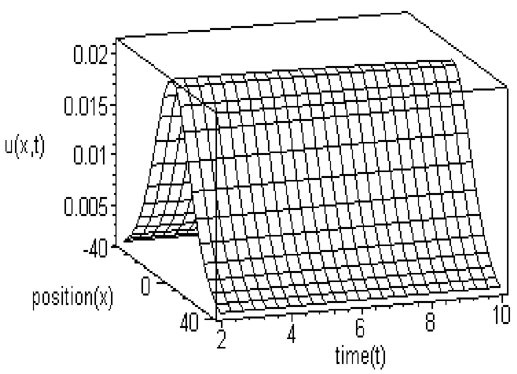

Fig.[2.a].Numerical solution of u(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=0051 and c=.05 for a different values of t

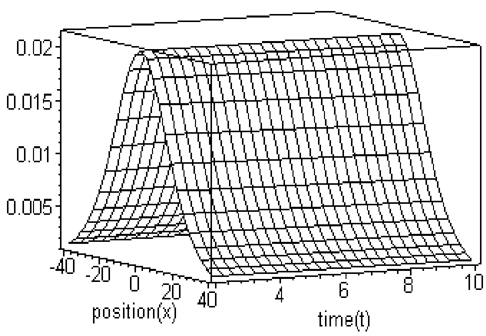

Fig.[2.b].the exact solution of u(x,t) Eq.(31) obtained by **Adomian decomposition method** with a fixed values of k=0.05 and c=0.05 for a different values of t

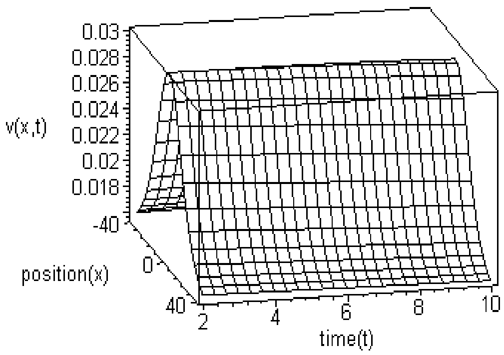

Fig.[2.c]Numerical solution of v(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=0051 and c=.05 for a different values of t

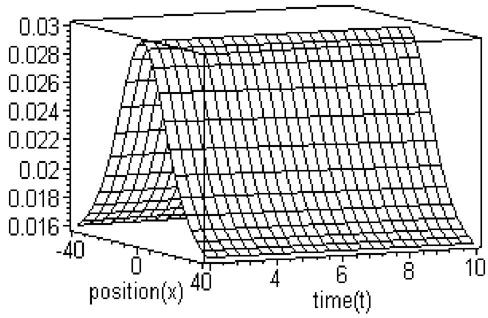

Fig.[2.d].the exact solution of v(x,t) Eq.(32) obtained by **Adomian decomposition method** with a fixed values of k=0.05 and c=0.05 for a different values of t