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Existence for Delay Integrodifferential Equations of Sobolev Type with Nonlocal Conditions

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Abstract: This paper is concerned with delay integrodifferential equations of Sobolev type with nonlocal conditions in Banach spaces. Using the theory of semigroups and the method of fixed points, existence results are obtained for mild solutions.

Keywords: measure of noncompactness; integrodifferential equations; fixed point; nonlocal condition; linear semigroups; mild solutions

1 Introduction

In this paper we discuss the nonlocal initial value problem

$$(Eu(t))' + Au(t) = f(t, u(\sigma_1(t)), \int_0^t k(t,s)h(s, u(\sigma_2(s)))\,\mathrm{d}s), \quad t \in [0,b], \tag{1.1}$$

$$u(0) + g(u) = u_0, \tag{1.2}$$

where A, E are two closed, linear operators such that $-AE^{-1}$ generates a semigroup of bounded linear operators $T(t)$ in Banach space X, $f : [0,b] \times X^2 \to X$, $k : [0,b] \times [0,b] \to R$, $h : [0,b] \times X \to X$, $\sigma_i : [0,b] \to [0,b], i = 1,2$, and $g : C(0,b;X) \to D(E)$.

The study of abstract nonlocal semilinear initial value problems was initiated by Byszewski [9–11]. Because it is demonstrated that the nonlocal problems have better effects in applications than the classical Cauchy problems. Such problems with nonlocal conditions have been extensively studied in the literature [1, 2, 5-19, 22-24, 26-30]. Sobolev type equations arise in various applications such as in the flow of fluid through fissured rocks, thermodynamics, and shear in second-order fluids. For more details, we refer the reader to [4, 20, 21]. Recently, Balachandran et al. [7] studied the nonlocal problems for this kind of equation. Using Schaefer's fixed point theorem, they established the existence results for mild solutions and strong solutions of (1.1), (1.2) under the compactness assumptions on the semigroup and the nonlocal item.

Motivated by the above approach, the goal of this paper is to use the fixed point argument to obtain the mild solution of (1.1), (1.2) under the conditions in respect of the Hausdorff's measure of noncompactness. Here, we do not need the compactness of the semigroup. And we can deal with the nonlocal item g in two cases, i.e., g is a compact or Lipchitz continuous mapping.

2 Preliminaries

Let $(X, \|\cdot\|)$ be a real Banach space. We denote by $C(0,b;X)$ the space of X-valued continuous functions on $[0,b]$ with the norm $\|u\| = \sup\{\|u(t)\|, t \in [0,b]\}$ and by $L^1(0,b;X)$ the space of X-valued Bochner integrable functions on $[0,b]$ with the norm $\|f\|_{L^1} = \int_0^b \|f(t)\|\,\mathrm{d}t$.

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Let us recall the following definition.

Definition2.1 *A continuous solution $u(t)$ of the integral equation*

$$u(t) = E^{-1}T(t)E(u_0 - g(u)) + \int_0^t E^{-1}T(t-s)f(s, u(\sigma_1(s)), \int_0^s k(s,\tau)h(\tau, u(\sigma_2(\tau)))\,\mathrm{d}\tau)\,\mathrm{d}s$$

is called a mild solution of (1.1), (1.2) on $[0,b]$, where $T(t)$ is the semigroup generated by $-E^{-1}A$.

Next, we introduce the Hausdorff's measure of noncompactness $\chi(\cdot)$ defined on each bounded subset Ω of Banach space Y by

$$\chi(\Omega) = \inf\{\varepsilon > 0;\ \Omega\ has\ a\ finite\ \varepsilon - net\ in\ Y\}.$$

Some basic properties of $\chi(\cdot)$ are given in the following lemmas.

Lemma2.2([3]). *Let Y be a real Banach space and $B, C \subseteq Y$ be bounded, the following properties are satisfied:*
(1). B is pre-compact if and only if $\chi(B) = 0$;
(2). $\chi(B) = \chi(\overline{B}) = \chi(convB)$, where $\overline{B}$ and $convB$ mean the closure and convex hull of B, respectively;
(3). $\chi(B) \le \chi(C)$ when $B \subseteq C$;
(4). $\chi(B + C) \le \chi(B) + \chi(C)$, where $B + C = \{x + y; x \in B, y \in C\}$;
(5). $\chi(B \cup C) \le \max\{\chi(B), \chi(C)\}$;
(6). $\chi(\lambda B) = |\lambda|\chi(B)$ for any $\lambda \in R$;
(7). If the map $Q : D(Q) \subseteq Y \to Z$ is Lipschitz
continuous with constant k, then $\chi(QB) \le k\chi(B)$ for any bounded subset $B \subseteq D(Q)$, where Z be a Banach space.

Lemma2.3([3]) *Let $\{f_n\}_{n=1}^{+\infty}$ is a sequence of functions in $L^1(0, b; X)$. If there exists μ in $L^1(0, b; R^+)$ such that $\sup_{n\ge1} \|f_n(t)\| \le \mu(t)$ a.e. $t \in [0, b]$. Then $\chi(\{f_n(t)\}_{n=1}^{+\infty})$ is measurable and*
$\chi(\{\int_0^t f_n(s)\,\mathrm{d}s\}_{n=1}^{+\infty}) \le 2\int_0^t \chi(\{f_n(s)\}_{n=1}^{+\infty})\,\mathrm{d}s.$

Lemma2.4([3]) *If $D \subseteq C(0, b; X)$ be bounded, then we have $\sup_{t\in[0,b]} \chi(D(t)) \le \chi(D)$. Furthermore, if D is equicontinuous on $[0, b]$, then $\chi(D(t))$ is continuous on $[0, b]$ and $\sup_{t\in[0,b]} \chi(D(t)) = \chi(D)$.*

The following fixed point theorem, a nonlinear alternative of Monch type, plays a key role in our existence of mild solutions for nonlocal Cauchy problem (1.1), (1.2) (see Theorem 2.2 in [25]).

Theorem2.5 *Let Y be a Banach space, U an open subset of Y and $0 \in U$. Suppose that $F : \overline{U} \to Y$ is a continuous map which satisfies Monch's condition (that is, if $D \subseteq \overline{U}$ is countable and $D \subseteq \overline{co}(\{0\} \cup F(D))$, then $\overline{D}$ is compact) and assume that*

$$x \ne \lambda F(x) \quad for\ x \in \partial U\ and\ \lambda \in (0, 1)$$

holds. Then F has a fixed point in $\overline{U}$.

3 Main results

In this Section, we give the existence of the mild solutions for nonlocal Cauchy problem (1.1), (1.2).

We first give the following assumptions:

(HAE) A and E are closed, linear operators. $D(E) \subseteq D(A)$ and E is bijective. $E^{-1} : X \to D(E)$ is continuous. Moreover, $-AE^{-1}$ generates an equicontinuous semigroup $T(t), t \ge 0$, of bounded linear operators on Banach space X;

$(Hf1)$ $f : [0, b] \times X^2 \to X$, for a.e. $t \in [0, b]$, the function $f(t, \cdot, \cdot) : X^2 \to X$ is continuous and for all $x, y \in X$, the function $f(\cdot, x, y) : [0, b] \to X$ is measurable;

$(Hf2)$ there exists a function $\theta_1 \in L^1(0, b; R^+)$ and a nondecreasing continuous function $\Omega_1 : R^+ \to R^+$ such that

$$\|f(t, x, y)\| \le \theta_1(t)\Omega_1(\|x\| + \|y\|)$$

for all $x, y \in X$ and $t \in [0, b]$;

$(Hf3)$ there exists a function $\eta_1 \in L^1(0,b;R^+)$ such that for every bounded $D_1, D_2 \subset X$,

$$\chi(f(t,D_1,D_2)) \leq \eta_1(t)(\chi(D_1)+\chi(D_2))$$

for a.e. $t \in [0,b]$;

$(Hh1)$ $h:[0,b]\times X \to X$, for each $t\in[0,b]$, the function $h(t,\cdot):X\to X$ is continuous and for each $x \in X$, the function $h(\cdot,x):[0,b]\to X$ is strongly measurable;

$(Hh2)$ there exists a function $\theta_2 \in L^1(0,b;R^+)$ and a nondecreasing continuous function $\Omega_2 : R^+ \to R^+$ such that

$$\|h(t,x)\| \leq \theta_2(t)\Omega_2(\|x\|)$$

for all $x\in X$ and $t \in [0,b]$;

$(Hh3)$ there exists a function $\eta_2 \in L^1(0,b;R^+)$ such that for every bounded $D \subset X$,

$$\chi(h(t,D)) \leq \eta_2(t)\chi(D)$$

for a.e. $t\in[0,b]$;

(Hk) $k:[0,b]\times[0,b]\to R$ is a measurable function such that there exists a constant $K>0$ such that $|k(t,s)|\leq K$, for $t\geq s\geq 0$;

(Hg) $g: C(0,b;X)\to D(E)$ is a continuous compact map such that $\|g(u)\|\leq c\|u\|+d$, $\forall u\in C(0,b;X)$, for some positive constants c and d.

From (HAE) and Banach inverse operator theorem, we know that E is a bounded operator and denote $\alpha = \|E^{-1}\|$, $\beta=\|E\|$. Now, we give the existence results for (1.1), (1.2).

Theorem3.1 *Assume that the conditions (HAE),$(Hf1)-(Hf3)$,$(Hh1)-(Hh3)$, (Hk) and (Hg) are satisfied. Then for every $u_0\in D(E)$ the nonlocal Cauchy problem (1.1), (1.2) has at least one mild solution on $[0,b]$ provided that there exists a constant $N>0$ with*

$$\frac{(1-\alpha\beta Mc)N}{\alpha\beta M(d+\|u_0\|)+\alpha M\|\theta_1\|_{L^1}\Omega_1(N+K\|\theta_2\|_{L^1}\Omega_2(N))} > 1, \tag{3.1}$$

and that

$$2\alpha\|\eta_1\|_{L^1}M(1+2K\|\eta_2\|_{L^1})<1, \tag{3.2}$$

where M equals to $\sup_{0\leq t\leq b}\|T(t)\|$.

Proof. We consider the operator $R: C(0,b;X)\to C(0,b;X)$ defined by

$$(Ru)(t) = (R_1u)(t)+(R_2u)(t) \tag{3.3}$$

with

$$(R_1u)(t) = E^{-1}T(t)E(u_0-g(u)) \tag{3.4}$$

$$(R_2u)(t) = \int_0^t E^{-1}T(t-s)f(s,u(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u(\sigma_2(\tau)))\,\mathrm{d}\tau)\,\mathrm{d}s \tag{3.5}$$

for all $t\in[0,b]$.

It is easy to see that the fixed point of R is the mild solution of nonlocal Cauchy problem (1.1), (1.2). Subsequently, we will prove that R has a fixed point by using Theorem2.5.

Firstly, we claim that the operator R is continuous on $C(0,b;X)$. For this purpose, we assume that $u_n \to u$ in $C(0,b;X)$. Then by $(Hh1)$ we get that

$$h(\tau,u_n(\sigma_2(\tau)))\to h(\tau,u(\sigma_2(\tau))),\ \forall\, t\in[0,b].$$

Since $(Hh2)$, (Hk) hold, by the dominated convergence theorem, for every $s\in[0,b]$ we have

$$\int_0^s k(s,\tau)h(\tau,u_n(\sigma_2(\tau)))\,\mathrm{d}\tau \to \int_0^s k(s,\tau)h(\tau,u(\sigma_2(\tau)))\,\mathrm{d}\tau,\ (n\to+\infty).$$

So by the same reason, we obtain that

$$\begin{aligned}&f(s,u_n(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u_n(\sigma_2(\tau)))\,\mathrm{d}\tau)\\ \to& f(s,u(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u(\sigma_2(\tau)))\,\mathrm{d}\tau),\quad (n\to+\infty),\ \ \forall s\in[0,b].\end{aligned}$$

Thus,

$$\|Ru_n - Ru\| \le \alpha\beta M\|g(u)-g(u_n)\|$$

$$+alphaM\int_0^b \|f(s,u(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u(\sigma_2(\tau)))\,\mathrm{d}\tau)-$$

$$f(s,u_n(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u_n(\sigma_2(\tau)))\,\mathrm{d}\tau)\|\,\mathrm{d}s \ \to\ 0,\qquad as\ \ n\to+\infty, \tag{3.6}$$

i.e., R is continuous.

Secondly, we claim that the Monch's condition holds.

Suppose that $D\subseteq B_r$ is countable and $D\subseteq\overline{co}(\{0\}\cup R(D))$, we show that $\chi(D)=0$, where B_r is the open ball of the radius r centered at the zero in $C(0,b;X)$.

Without loss of generality, we may suppose that $D=\{u_n\}_{n=1}^{+\infty}$. By using the condition (HAE) and (Hg), we can easily verify that $\{Ru_n\}_{n=1}^{+\infty}$ is equicontinuous (also see [7]). So, $D\subseteq\overline{co}(\{0\}\cup R(D))$ is also equicontinuous.

Now, from (Hg), $(Hf3)$, $(Hh3)$, Lemma 2.2-2.4 and the continuity of $E^{-1}T(t)E$, it follows that

$$\begin{aligned}&\chi(\{Ru_n\}_{n=1}^{+\infty})\\ \le&\sup_{t\in[0,b]}(\chi(\{E^{-1}T(t)Eg(u_n)\}_{n=1}^{+\infty})+\\ &\chi(\{\int_0^t E^{-1}T(t-s)f(s,u_n(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u_n(\sigma_2(\tau)))\,\mathrm{d}\tau)\,\mathrm{d}s\}_{n=1}^{+\infty}))\\ \le&2\alpha M\int_0^t\chi(\{f(s,u_n(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u_n(\sigma_2(\tau)))\,\mathrm{d}\tau)\}_{n=1}^{+\infty})\,\mathrm{d}s\\ \le&2\alpha M\int_0^t\eta_1(s)(\chi(\{u_n(\sigma_1(s))\}_{n=1}^{+\infty})+\chi(\int_0^s k(s,\tau)h(\tau,\{u_n(\sigma_2(\tau))\}_{n=1}^{+\infty})\,\mathrm{d}\tau))\,\mathrm{d}s\\ \le&2\alpha M\int_0^t\eta_1(s)(\chi(\{u_n(\sigma_1(s))\}_{n=1}^{+\infty})+2K\int_0^s\chi(h(\tau,\{u_n(\sigma_2(\tau))\}_{n=1}^{+\infty})\,\mathrm{d}\tau))\,\mathrm{d}s\\ \le&2\alpha M\int_0^t\eta_1(s)(\chi(\{u_n(\sigma_1(s))\}_{n=1}^{+\infty})+2K\int_0^s\eta_2(\tau)\chi(\{u_n(\sigma_2(\tau))\}_{n=1}^{+\infty})\,\mathrm{d}\tau)\,\mathrm{d}s\\ \le&2\alpha M\chi(\{u_n\}_{n=1}^{+\infty})(\|\eta_1\|_{L^1}+2K\int_0^t\eta_1(s)\int_0^s\eta_2(\tau)\,\mathrm{d}\tau\,\mathrm{d}s)\\ \le&2\alpha\|\eta_1\|_{L^1}M(1+2K\|\eta_2\|_{L^1})\chi(\{u_n\}_{n=1}^{+\infty}).\end{aligned}$$

Thus, we get that

$$\chi(D)\le\chi(\overline{co}(\{0\}\cup R(D)))=\chi(R(D))\le 2\alpha\|\eta_1\|_{L^1}M(1+2K\|\eta_2\|_{L^1})\chi(D),$$

which implies that $\chi(D)=0$, since the condition (3.2) holds.

Now let $\lambda\in(0,1)$ and $u=\lambda R(u)$. Then for $t\in[0,b]$

$$\begin{aligned}u(t)=&\lambda E^{-1}T(t)E(u_0-g(u))+\\ &\lambda\int_0^t E^{-1}T(t-s)f(s,u(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u(\sigma_2(\tau)))\,\mathrm{d}\tau)\,\mathrm{d}s,\end{aligned}$$

and one has

$$\begin{aligned}\|u(t)\|\le&\alpha\beta M(\|u_0\|+c\|u\|+d)+\alpha M\int_0^b\theta_1(s)\Omega_1(\|u\|+K\|\theta_2\|_{L^1}\Omega_2(\|u\|))\,\mathrm{d}s\\ \le&\alpha\beta M(\|u_0\|+c\|u\|+d)+\alpha M\|\theta_1\|_{L^1}\Omega_1(\|u\|+K\|\theta_2\|_{L^1}\Omega_2(\|u\|)).\end{aligned}$$

Consequently,

$$\frac{(1-\alpha\beta Mc)\|u\|}{\alpha\beta M(d+\|u_0\|)+\alpha M\|\theta_1\|_{L^1}\Omega_1(\|u\|+K\|\theta_2\|_{L^1}\Omega_2(\|u\|))}\leq 1.$$

Then by (3.1) there exists N such that $\|u\|\neq N$. Set

$$U=\{u\in C(0,b;X):\|u\|<N\}.$$

From the choice of U there is no $u\in\partial U$ such that $u=\lambda R(u)$ for some $\lambda\in(0,1)$. Thus we get a fixed point of R in $\overline{U}$ due to Theorem 2.5, which is a mild solution to (1.1), (1.2). The proof is completed. ■

Remark 1 *We note that if* $\alpha\beta Mc<1$ *and* $\Omega_1(l)=k_1l^\gamma+k_2$, $\Omega_2(l)=k_3l+k_4$, $\forall l>0$ *for some* $k_1,\ k_2,\ k_3,\ k_4>0$ *and* $0<\gamma<1$*, then condition (3.1) is automatically satisfied.*

Now, we will give the existence for (1.1), (1.2) when the nonlocal item g has no compactness. Assume the following holds:

(Hg') $g:C(0,b;X)\to D(E)$ is Lipschitz continuous with constant L.

Theorem3.2 *Assume that the conditions* (HAE),$(Hf1)-(Hf3)$,$(Hh1)-(Hh3)$, (Hk) *and* (Hg') *are satisfied. Then for every* $u_0\in D(E)$ *the nonlocal Cauchy problem (1.1), (1.2) has at least one mild solution on* $[0,b]$ *provided that there exists a constant* $N>0$ *with*

$$\frac{(1-\alpha\beta ML)N}{\alpha\beta M(\|g(0)\|+\|u_0\|)+\alpha M\|\theta_1\|_{L^1}\Omega_1(N+K\|\theta_2\|_{L^1}\Omega_2(N))}>1, \tag{3.7}$$

and that

$$\alpha\beta ML+2\alpha\|\eta_1\|_{L^1}M(1+2K\|\eta_2\|_{L^1})<1, \tag{3.8}$$

where M *equals to* $\sup_{0\leq t\leq b}\|T(t)\|$.

Proof. On account of Theorem3.1, we can prove that operator R defined by (3.3) is continuous on $C(0,b;X)$.

We now prove that R satisfies the Monch's condition.

For this purpose, Let $D\subseteq B_r$ be countable and $D\subseteq\overline{co}(\{0\}\cup R(D))$. We will show that $\chi(D)=0$.

Without loss of generality, we may suppose that $D=\{u_n\}_{n=1}^{+\infty}$. By using the condition (HAE), we can easily verify that $\{R_2u_n\}_{n=1}^{+\infty}$ is equicontinuous. Moreover, $R_1:D\to C(0,b;X)$ is Lipschitz continuous with constant $\alpha\beta ML$ due to the condition (Hg'). In fact, for $u,v\in D$, we have

$$\begin{aligned}\|R_1u-R_1v\|=&\sup_{t\in[0,b]}\|E^{-1}T(t)Eg(u)-E^{-1}T(t)Eg(v)\|\\ \leq&\alpha\beta M\|g(u)-g(v)\|\\ \leq&\alpha\beta ML\|u-v\|.\end{aligned}$$

So, from (Hg'), $(Hf3)$, $(Hh3)$ and Lemma 2.2-2.4, it follows that

$$\begin{aligned}&\chi(\{Ru_n\}_{n=1}^{+\infty})\\ \leq&\chi(\{R_1u_n\}_{n=1}^{+\infty})+\chi(\{R_2u_n\}_{n=1}^{+\infty})\\ \leq&\alpha\beta ML\chi(\{u_n\}_{n=1}^{+\infty})+\\ &\sup_{t\in[0,b]}\chi(\{\int_0^t E^{-1}T(t-s)f(s,u_n(\sigma_1(s)),\int_0^s k(s,\tau)h(\tau,u_n(\sigma_2(\tau)))\,\mathrm{d}\tau)\,\mathrm{d}s\}_{n=1}^{+\infty})\\ \leq&\alpha\beta ML\chi(\{u_n\}_{n=1}^{+\infty})+2\alpha\|\eta_1\|_{L^1}M(1+2K\|\eta_2\|_{L^1})\chi(\{u_n\}_{n=1}^{+\infty})\\ \leq&(\alpha\beta ML+2\alpha\|\eta_1\|_{L^1}M(1+2K\|\eta_2\|_{L^1}))\chi(\{u_n\}_{n=1}^{+\infty}).\end{aligned}$$

Thus, we get that

$$\chi(D)\leq\chi(\overline{co}(\{0\}\cup R(D)))=\chi(R(D))\leq\alpha M(\beta L+2\|\eta_1\|_{L^1}(1+2K\|\eta_2\|_{L^1}))\chi(D),$$

which implies that $\chi(D)=0$, since the condition (3.8) holds.

Now, with analogous arguments as in the proof of Theorem3.1 , we can get an open ball U by the condition (3.7), and there is no $u\in\partial U$ such that $u=\lambda R(u)$ for some $\lambda\in(0,1)$. Thus we get a fixed point of R in $\overline{U}$ due to Theorem2.5 , which just is the mild solution to (1.1), (1.2). The proof is completed. ■

Remark 2 *We note that if $\alpha\beta ML < 1$ and $\Omega_1(l) = k_1 l^\gamma + k_2, \Omega_2(l) = k_3 l + k_4, \forall l > 0$ for some $k_1,\ k_2,\ k_3,\ k_4 > 0$ and $0 < \gamma < 1$, then condition (3.7) is automatically satisfied. In addition, we try to make use of the properties of Hausdorff's noncompact measure χ in proof, which enables us to avoid the difficulties associated with unbounded operators when $t = 0$.*

Remark 3 *In [12], the authors discuss a related semilinear nonlocal problem when g is convex and compact on a given ball. From the above theorem, however, we can see many key conditions in [12] are not required, such as the compactness of semigroup $T(t)$ and the convexity of g.*

In this paper, we require f to satisfy a compactness condition $(Hf3)$, but do not require the compactness of semigroup $T(t)$. Note that if f is compact or Lipschitz continuous with respect to the second and third arguments, then condition $(Hf3)$ is satisfied. Therefore, this work extends and improves many previous results such as those in [5–7, 12, 23, 26, 27, 29] and so on, where they need the compactness of $T(t)$ or f, or the Lipschitz continuity of f.

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