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$J - A_2$ Methodology to Quantify the Crack-Tip Stress Fields of Unirradiated and Irradiated Materials

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Abstract: This study applies the $J - A_2$ approach to quantify the crack tip stress fields of unirradiated and irradiated materials. For quantifying the constraints, the $J - A_2$ two-parameter fracture methodology, which is capable of accurately characterizing the geometric effects on fracture as shown in the past, is adopted to quantify the crack tip constraints of the unirradiated and irradiated materials. Finite element analysis is performed with the ABAQUS program to calculate the crack-tip stress fields, from which the constraint parameters are estimated for specimens under unirradiated and irradiated conditions, respectively. As an example, the compact specimens of HSST weld 73W are used in this study.

Keywords: $J - A_2$ methodology; crack-tip stress fields; FEA

1 Introduction

The theory of the constraint effect in fracture mechanics is now reasonably well understood. For a stationary crack in an elastic-plastic material, effect of constraint on crack-tip fields has been extensively investigated for different specimen geometry and loading configurations. The three most commonly used methodologies to quantify crack-tip constraints are (1) the $J - T$ methodology [1], (2) the $J - Q$ methodology[2, 3], and (3) the $J - A_2$ methodology [4,5]. In these methods, T, Q and A_2 are the parameters for quantifying the constraint effect, respectively. However, the applicability of these methods is different. The $J - T$ methodology only exists in linear elastic field and the $J - Q$ is particularly suitable for small applied loads or small scale yielding (SSY). The $J - A_2$ is best used for materials under large scale yielding (LSY) at fracture when constraint effect is included in the interpretation. The $J - A_2$ methodology is based on a rigorous asymptotic mathematical solution for elastic-plastic power law materials. Moreover, A_2 as a constraint parameter which is nearly independent of its position near the crack-tip [6] has been successfully used to quantify the constraint effect on fracture toughness for different geometry and loading configurations [7]. The $J - A_2$ approach has been successfully used to demonstrate the shift of the ductile-brittle curve due to difference in constraint in deep and shallow flaws[8]. The methodology therefore holds salient promise for a two-parameter fracture testing as outlined by references [9, 10]. The present paper adopts the $J - A_2$ approach for quantifying the crack tip stress fields of unirradiated and irradiated materials.

2 $J - A_2$ three-term solution

Our attention is focused on mode-I cracks in elastic-plastic materials under plane strain conditions. The material behavior described by the Ramberg-Osgood power-law stress-strain relation in uni-axial tension can be given by

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n, \tag{1}$$

where ε is the uni-axial strain, σ is the uni-axial stress, σ_0 is a reference stress, $\varepsilon_0 = \sigma_0/E$ is a reference strain with E as the Young's modulus (for actual elastic-plastic solids, σ_0 and ε_0 may be taken as the yield stress and the yield strain of

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the material, respectively), n is a material strain hardening exponent and α is a material strain coefficient. By use of J_2 deformation theory of plasticity, the uni-axial stress-strain relation (1) can be generalized to multi-axial state as

$$\frac{\varepsilon_{ij}}{\varepsilon_0} = (1+v)\frac{\sigma_{ij}}{\sigma_0} - v\frac{\sigma_{kk}}{\sigma_0}\delta_{ij} + \frac{3}{2}\alpha(\frac{\sigma_e}{\sigma_0})^{n-1}\frac{S_{ij}}{\sigma_0}, \tag{2}$$

where v is the Poisson's ratio, δ_{ij} is the Kronecker symbol, S_{ij} is the deviatoric stress and σ_e is the Von Mises effective stress defined as $\sigma_e = \sqrt{3S_{ij}S_{ij}/2}$.

Making use of rigorous asymptotic mathematical analysis, Yang et al. [4] and Chao et al. [5] developed the $J - A_2$ two-parameter fracture methodology. Consider a plane-strain idealization of a fracture specimen or component containing an initially sharp crack. The crack-tip stress fields can be written as

$$\frac{\sigma_{ij}}{\sigma_0} = A_1\left[\left(\frac{r}{L}\right)^{S_1}\tilde{\sigma}_{ij}^{(1)}(\theta,n) + A_2\left(\frac{r}{L}\right)^{S_2}\tilde{\sigma}_{ij}^{(2)}(\theta,n) + A_2^2\left(\frac{r}{L}\right)^{S_3}\tilde{\sigma}_{ij}^{(3)}(\theta,n)\right], \tag{3}$$

where a polar coordinate system (r,θ) centered at the crack tip is used, the angular functions $\tilde{\sigma}_{ij}^{(k)}(\theta,n)(k=1,2,3)$ and the stress power exponents S_k ($S_1 < S_2 < S_3$) are only dependent of the hardening exponent n and independent of other material constants (i.e. σ_0,ε_0) and applied loads. L is a characteristic length parameter which can be chosen as the crack length a, the specimen width W, the thickness B or unity. The parameters A_1 and S_1 can be written as

$$A_1 = \left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n L}\right)^{-S_1} S_1 = -\frac{1}{n+1}, \tag{4}$$

and $S_3 = 2S_2 - S_1$ for $n \geq 3$. The dimensionless integration constant I_n is only dependent of the hardening exponent n and independent of other material constants (i.e. σ_0,ε_0) and applied loads. Plane strain mode-I dimensionless functions$\tilde{\sigma}_{ij}^{(k)}$,$S_k$ and I_n have been calculated and tabulated by Chao and Zhang [11].

The three-term series solution of the crack-tip stress fields, Eq. (3), includes only two amplitudes, J and A_2. The amplitude J–integral is a parameter for quantifying the magnitude of applied loading. And A_2 is a function of specimen or component geometry and loading configuration which can be determined using a simple weight average technique [12]. It therefore can be adopted as a parameter to quantify the geometry constraint level existing at the crack-tip. When A_2 = 0, Eq. (3) reduces to the leading-term HRR singularity field.

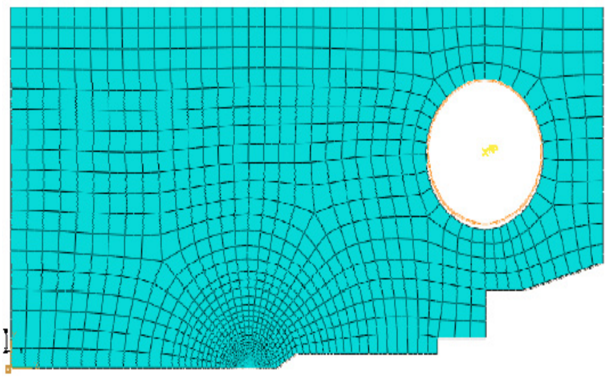

Figure 1: Finite element mesh for CT specimen ($a/W = 0.5, W = 50$mm)

3 Finite element modeling

Consider an ASTM standard CT specimens having width $W = 50$mm, thickness $B = W/2 = 25$mm, and crack length $a = W/2 = 25$mm. Finite element analysis was performed using ABAQUS. By use of symmetry condition, only one half of each specimen was modeled. A typical FEA mesh for simulating a crack-tip is shown in Figure 1. The FEA mesh consists of eight-node plane strain, second order isoparametric elements with reduced integration. The innermost ring of elements has one side collapsed onto the crack tip. A fine mesh with the smallest element size of 0.005mm is focused on the crack-tip with increasingly coarse mesh generated elsewhere. Furthermore, 10 rings of elements around the crack tip are used in the model.

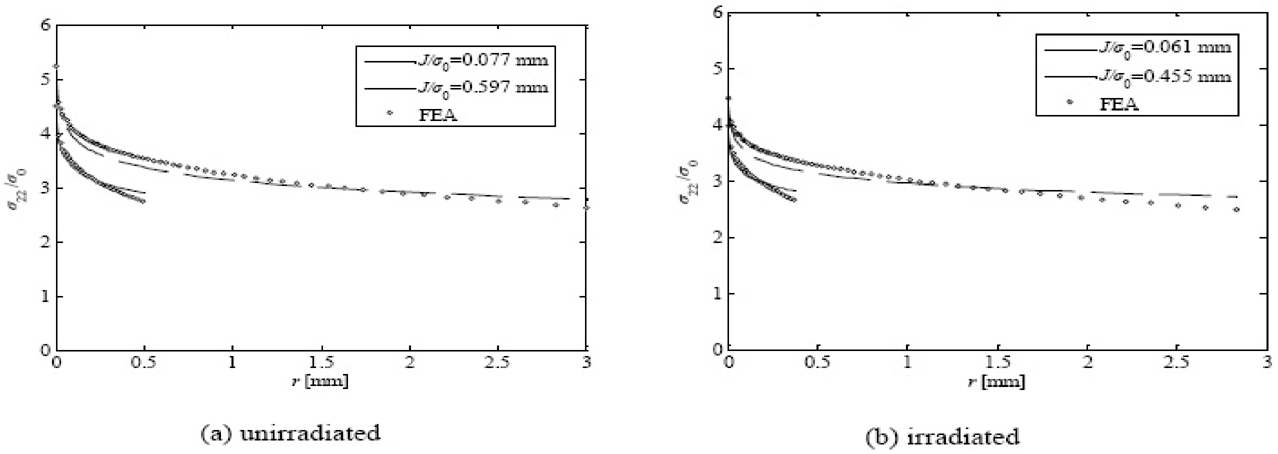

Figure 2: Comparison of the predicted crack-tip stress fields by Eq. (3) and FEA results for CT specimen of HSST weld 73W

Table 1: Dimensionless parameters

n	$\tilde{\sigma}^{(1)}_{\theta\theta}(0)$	$\tilde{\sigma}^{(2)}_{\theta\theta}(0)$	$\tilde{\sigma}^{(3)}_{\theta\theta}(0)$	S_1	S_2	S_3	I_n
12(unirradiated)	2.5542	0.3067	-7.0021	-0.07692	0.06653	0.20999	4.44111
15.8(irradiated)	2.6258	0.2953	-8.0775	-0.05952	0.05945	0.17842	4.31422

4 Estimation of crack-tip stress fields

In this section, Consider the HSST weld 73W. At the reference temperature T_0=-65^{o}C, its yield stress is $\sigma_0 = 557$MPa under unirradiated condition and σ_0= 700MPa under irradiated condision at 228^{o}C to an average fast fluence of 1.6×10^{19}n/cm^2 ($>$ 1MeV)[13], respectively.

For a group of irradiated RPV steels, the yield stress σ_0 and strain hardening n were determined and correlated into a curve [14] as

$$\frac{1}{n} = 0.1075 \left(\frac{\sigma_0}{\sigma_{0,\text{ref}}}\right)^{-1.202}, \tag{5}$$

where σ_0 is normalized by $\sigma_{0,ref} = 450$MPa. The relation (5) is applicable to both unirradiated and irradiated RPV forging, plate and weld materials, including HSST welds tested by ORNL [13].

For the unirradiated material, n=12 and n=15.8 after it was irradiated to a fluence of 1.6×10^{19}n/cm^2 from Eq. (5). Then the dimensionless parameters can be found from reference [10] as shown in Table 1.

To demonstrate the applicability of $J-A_2$ methodology, we considered two conditions. In unirradiated condition, FEA calculations were performed, which gives $A_2 = -0.1515$ according to $J/\sigma_0 = 0.077$mm and $A_2 = -0.1685$ according to $J/\sigma_0 = 0.597$mm using a simple weight average technique. In irradiated condition, FEA calculations were performed, which gives $A_2 = -0.1534$ according to $J/\sigma_0 = 0.061$mm and $A_2 = -0.1504$ according to $J/\sigma_0 = 0.455$mm using a simple weight average technique. From Eq. (3), the crack tip opening stress ahead of the crack tip can be determined as shown in Figure 2.

5 Conclusion

Figure 2 shows that the predicted crack-tip stress fields by Eq. (3) compare very well with FEA results in the region of $J/\sigma_0 \leq r \leq 5J/\sigma_0$ for both unirradiated and irradiated materials. It indicated that the $J - A_2$ methodology is also valid to predict the crack-tip stress fields for irradiated materials.

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