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Linear State Feedback Stabilization for Controlled Chaotic Systems

Ping He[1] *, Fei Tan[2]
[1]School of Automation and Electronic Information, Sichuan University of Science & Engineering, Zigong, Sichuan, 643000, P. R. China
[2]Artificial Intelligence of Key Laboratory of Sichuan Province, Sichuan University of Science & Engineering, Zigong, Sichuan, 643000, P. R. China
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Abstract: In this paper, the problem of the linear state feedback stabilization of controlled chaotic systems is investigated. The Lyapunov stability theory and Riccati algebra matrix equation in the matrix theory are used to design a controller by linear state feedback control laws and prove the controlled closed-loop system is asymptotically stable. Finally, by control and simulation of unified chaotic system show that the control law is effective for this class of systems. The results presented in this paper improve and generalize the corresponding results of recent works.

Keywords: controlled chaotic system; linear state feedback stabilization; *Riccati* algebra matrix equation; unified chaotic system

1 Introduction

Stabilization is one of the basic tasks in control design. The asymptotic stability and stabilization of nonlinear systems has won extensive attention from researchers [1]. The problem of stabilization of nonlinear systems has widely been studied in the last three decades [2]. Several approaches have been proposed to deal with this problem like the small gain theorem, Popov's criterion, Lyapunov theory, feedback linearization and backstepping, to mention a few of them. And some achievements have been made [3].

Chaos is a very interesting nonlinear phenomenon. High sensitivity to initial conditions is a main characteristic of chaotic systems. Accordingly, these systems are difficult for synchronization, stabilization or control [4]. Due to complex behavior, and coupling, the control and stabilization task of chaotic nonlinear systems have been one of the major issues in control engineering area. Chaos control is crucial in applications of chaos. It attracted a great deal of attention from various fields since Huber published the first paper on chaos control in 1989 [5], so in the past decade, a great efforts has been devoted towards the chaos control, including stabilization of unstable equilibrium points, and more generally, unstable periodic solutions [5]. Particularly, in case of chaos suppression of known chaotic systems, some useful methods have been developed. These include adaptive control, adaptive fuzzy control, sliding mode control, Robust control, time-delayed feedback control, double delayed feedback control, bang-bang control, optimal control, intelligent control, etc [4-16].

In 2005, Yassen designed different linear feedback controllers to locally stabilize the equilibria of the new chaotic system [17]. In this present paper, we prove that the linear state feedback controller can globally stabilize the corresponding equilibria of the chaotic system, thus eliminating chaos.

The unified chaotic system is investigated in [12]. The new system reprensents the continued transition from the *Lorenz* system to the Chen system and is chaotic over the entire spectrum of the key system parameter $\alpha \in [0, 1]$. In the end, in order to show that the approach of this paper is effective and convenient to this sort of control system. We give some illustrative examples and simulation results on unified chaotic system.

The foregoing discussion, the main purpose of this paper is to provide a new design technique of linear state feedback control, and the linear state feedback control by designed ensures that the dynamic responds of controlled closed-loop

*__Corresponding author__. *E-mail address*: pinghecn@yahoo.cn.

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chaotic systems is asymptotically stable, it is extended to a general and large of chaotic systems. To obtain the desirable properties, we combine the *Lyapunov* theory with *Riccati* algebra matrix equation in the matrix theory.

This paper is organized as follows. Section 2 includes some basic mathematical analysis of linear systems and controlled chaotic systems, which will be employed to accomplish main work of this paper, commodiously and easily. Several useful lemmas and definitions are briefly introduced, which will be employed in the remainder of this paper. The linear state feedback controller is proposed to stabilize of controlled chaotic systems in section 3. Results of some illustrative examples and numerical simulations on *unified* chaotic system are given in section 4, which are show that the approach to chaotic systems is effective and convenient. The paper will be closed by a conclusion in section 5.

In this paper, if not specially illustrated, $\|\cdot\|$ refers to *Euclid norm*, $\|\cdot\|_F$ refers to *Frobenius norm*, $|\cdot|$ refers to scalar function or absolute value of function.

2 Systems analysis and preliminaries

The modern trend in engineering systems is toward greater complexity, due mainly to the requirements of complex tasks and good accuracy. So we first consider the analysis problem of complex systems with multiple inputs and multiple outputs linear system.

We begin by recalling some well-known linearization basic approach of nonlinear system control and analysis to accomplish main work of this paper, commodiously and easily [18].

In the conventional approach to the design of a multiple inputs and multiple outputs control system, we design a controller (compensator) such that the dominant closed-loop poles have a design damping ratio ξ and an undamped natural frequency ω_n. Note that in this approach we assume the effects on the responses of non-dominant closed-loop poles to be negligible.

Consider a control system

$$\begin{cases} \dot{x} = Ax + Bu, \\ \dot{y} = Cx + Du. \end{cases} \tag{1}$$

where $x \in R^n$ is the system state variables group, $u \in R^p$ is the system input variables group, $y \in R^q$ is the system output variables group, A is called the state matrix $(n \times n)$, B is called the input matrix $(n \times p)$, C is called the output matrix $(q \times n)$, D is called the direct transmission matrix $(q \times p)$.

A block diagram representation of system (2.1) is show in Fig. 1. We shall choose the state feedback control law as

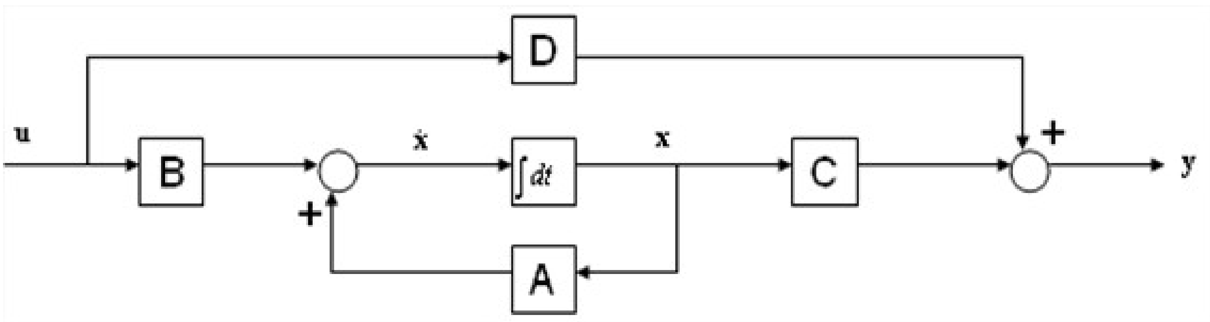

Figure 1: Block diagram of the linear, continuous-time control system.

follows

$$u(t) = Kx(t). \tag{2}$$

This means that the control signal $u(t)$ is determined by an instantaneous state. Such a scheme is called state feedback. The $(n\times n)$ matrix K is called the state feedback gain matrix. We assume that all state variables are available for feedback. In the following analysis we assume that $u(t)$ is unconstrained. A block diagram for this system is shown in Fig. 2. This closed-loop system has no input. Its objective is to maintain the zero output. Because of the disturbances that may be present, the output will deviate from zero. The nonzero output will be returned to the zero reference input because of the state feedback scheme of system. Such a system where the reference input is always zero is called a regulator system (That is, the system is always a nonzero constant, the system is also called a regulator system.).

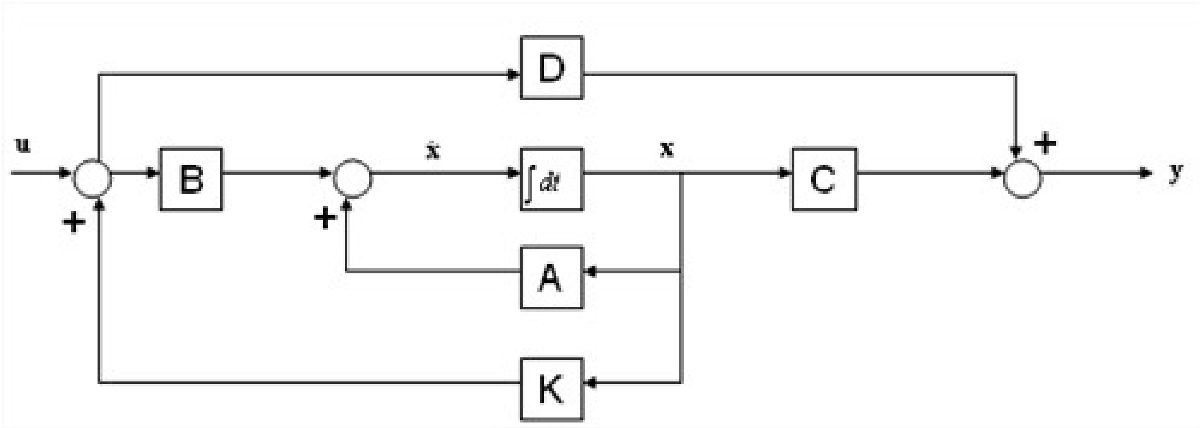

Figure 2: Closed-loop control system with $u(t) = Kx(t)$.

After substituting the state feedback control law (2.2) into the system (2.1) we see that

$$\dot{x} = (A + BK)x$$

which implies

$$x = e^{(A+BK)}x(0)$$

where $x(0)$ is the initial state caused by external disturbances. The stability and transient-response characteristics are determined by the eigenvalues of matrix $A + BK$. If matrix K is chosen properly, the matrix $A + BK$ can be made an asymptotically stable matrix, and for all $x(0) \neq 0$, it is possible to make $x(t)$ approach 0 as t approaches infinity. The eigenvalues of matrix of $A + BK$ are called the regulator poles. If these regulator poles are placed in the left-half s plane [18], then $x(t)$ approaches 0 as t approaches infinity. The problem of placing the regulator poles (closed-loop poles) at the desired location is called a pole-placement problem.

In what follows, we shall see that arbitrary pole placement for chaotic systems with input control is possible and established that stabilization of chaotic systems with input control if and only if the system is completely state controllable.

In order to apply this technique to a chaotic system, we rewrite the dynamical system in the form

$$\dot{x} = f(x) + Bu. \tag{3}$$

where $x \in R^n$ is a state vector of system, $u \in R^m$ is a input vector of systems, $f : D \to R^n$ is a Lipschitz map from a domain $D \subset R^n$ into R^n and D is a neighborhood of the origin. $B \in R^{n\times m}$ is a constant matrix.

Suppose $\tilde{x} \in D$ is an equilibrium point of controlled chaotic system (2.3); that is, $f(\tilde{x}) = 0$.

Remark 1 *Our goal is to characterize and study the stability of $\tilde{x}$. For convenience, we state all definitions and theorems for the case when the equilibrium point of controlled chaotic system (2.3) is at the origin of R^n; that is, $\tilde{x} = 0$. There is no loss of generality in doing so because any equilibrium point can be shifted to the origin via a change of variables.*

Remark 2 *Suppose $\tilde{x} \neq 0$ is a equilibrium point of controlled chaotic system (2.3) and consider the change of variables $y = x - \tilde{x}$. The derivative of y is given by*

$$\dot{y} = \dot{x} = f(x) + Bu = f(y + \tilde{x}) = s(y).$$

where $s(0) = 0$. In the new variable y, the system has equilibrium at the origin. Therefore, without loss of generality, we will always assume that $f(x)$ satisfies $f(0) = 0$ and study the stability of the origin $x = 0$.

For the nonlinear function $f(x)$ of system (2.3) using Taylor series which is given by

$$f(x) = f(0) + \frac{\partial f(x)}{\partial x}\ |_{x=0} \cdot x + g(x). \tag{4}$$

where $g(x)$ satisfies $\lim\limits_{\|x\|\to 0} \frac{\|g(x)\|}{\|x\|} = 0$.

After substitution $f(0) = 0$ into the controlled chaotic system (2.3) and combining with (2.4) we see that

$$\dot{x} = Ax + g(x) + Bu. \tag{5}$$

where $A = \frac{\partial f(x)}{\partial x} \mid_{x=0}$.

Now, several useful lemmas and definitions are briefly introduced, which be found in some place [18-19], are the key to the proof of main theorems of this paper, and will be useful in handling under investigation and proof of this paper.

Definition 1 *([18])The linear system (2.1) is completely state controllability if and only if the vectors $B, AB, \cdots, A^{n-1}B$ are linearly independent, that is, $rank[B, AB, \cdots, A^{n-1}B] = n$.*

Lemma 3 *([19,20]). Let $x = 0$ be an equilibrium point for the nonlinear system*

$$\dot{x} = f(x)$$

where $f : D \to R^n$ is continuously differentiable and D is a neighborhood of the origin. Let

$$A = \frac{\partial f(x)}{\partial x} \mid_{x=0} .$$

Then

(i) The origin is asymptotically stable if $Re\{\lambda_i\} < 0$ for all eigenvalues of A,

(ii) The origin is unstable if $Re\{\lambda_i\} > 0$ for one or more of the eigenvalues of A.

Lemma 4 *([18]). The necessary and sufficient condition of arbitrary pole-placement for a given system is that the system be completely state controllable.*

Definition 2 *The controlled chaotic system (2.3) is asymptotically stable in origin if and only if its zero solution approaches 0 as t approaches infinity.*

Definition 3 *The system (2.5) is called the linearization systems of controlled chaotic system (2.3).*

Definition 4 *The controlled chaotic system (2.3) is linear state feedback stabilization if there is a linear state feedback control law such that the controlled closed-loop system of the controlled chaotic system (2.3) is asymptotically stable.*

3 Main results

Simple global stabilization criterion, based on *Lyapunov* stability theory and *Riccati* algebra matrix equation in the matrix theory is used to design a successful scheme to control chaos. The proposed control method is designed to achieve global stabilization of the unstable equilibria of some dynamical systems. This control technique is efficient and ease of implementation in most real systems.

In this section, so, we first prove the main theorem of this paper, then, the controller arithmetic for chaotic system is given.

3.1 Controller of chaotic system design

Theorem 5 *For the linearization system (2.5) of controlled chaotic system (2.3) satisfies (A, B) is completely state controllable. Then there is a linear state feedback control law*

$$u = -B^T Px. \tag{6}$$

such that the controlled chaotic system (2.3) is asymptotically stable in origin, where P is a unique positive definite symmetry solution for Riccati *algebra matrix equation which is given by*

$$PA + A^T P - PBB^T P + Q = 0. \tag{7}$$

where Q is a arbitrarily positive definite matrix.

Proof. The solution matrix P of *Riccati algebra matrix equation* (3.2) is a positive definite matrix so because $Q \in R^{n\times n}$ is a arbitrarily positive definite matrix.

After substitution the linear state feedback control law (3.1) into the linearization system (2.5) we see that

$$\dot{x} = (A - BB^T P)x + g(x). \tag{8}$$

We can construct a *Lyapunov* functional

$$V(x) = x^T Px.$$

then its derivative along the trajectory of system (3.3) is

$$\begin{aligned} \dot{V}(x) \mid_{(3.3)} &= [(A - BB^T P)x + g(x)]^T Px + x^T P[(A - BB^T P)x + g(x)] \\ &= x^T (PA + A^T P - PBB^T P)x + x^T (-PBB^T P)x + 2x^T Pg(x) \\ &= x^T (-Q)x + 2x^T Pg(x) + x^T (-PBB^T P)x. \end{aligned}$$

Let $V_1 = x^T(-Q)x + 2x^T Pg(x)$ and $V_2 = x^T(-PBB^T P)x$.

According to the theory of matrices, it's easy to know that

$$V_2 < 0. \tag{9}$$

thus, $\dot{V}(x) \mid_{(3.3)} \leq 0$ if $V_1 < 0$.

From (2.4), we can know that the function $g(x)$ satisfies

$$\lim_{\| x \| \to 0} \frac{\| g(x) \|}{\| x \|} = 0.$$

Thus, there must be $\exists \delta > 0$ for $\forall \varepsilon > 0$ such that

$$\frac{\| g(x) \|}{\| x \|} < \varepsilon.$$

at $\| x \| < \delta$. That is,

$$\| g(x) \| < \varepsilon \| x \| .$$

And

$$x^T Qx \geq \lambda_{min}(Q) \| x \|^2 .$$

And

$$\begin{aligned} 2x^T Pg(x) &\leq 2 \mid x^T Pg(x) \mid \\ &\leq 2 \| x^T P \| \cdot \| g(x) \| \\ &= 2\sqrt{x^T PPx} \cdot \| g(x) \| \\ &\leq 2\sqrt{\lambda_{max}(P^2)} \cdot \| x \| \cdot \| g(x) \| . \end{aligned}$$

where $\lambda_{min}(\cdot)$ is the smallest characteristic of the matrix, $\lambda_{max}(\cdot)$ is the maximum characteristic of the matrix.

From Q and P are positive definite matrices, thus, $\lambda_{min}(Q)$ and $\lambda_{max}(P^2)$ are positive.

According to this, we have

$$V_1 < -[\lambda_{min}(Q) - 2\varepsilon\sqrt{\lambda_{max}(P^2)}] \| x \|^2 .$$

So, we have

$$V_1 < 0. \tag{10}$$

at $\varepsilon < \frac{\lambda_{min}(Q)}{\sqrt{\lambda_{max}(P^2)}}$.

And $\varepsilon < \frac{\lambda_{min}(Q)}{\sqrt{\lambda_{max}(P^2)}}$ is always available whenever ε is sufficient small.

Combined (3.4) with (3.5) it's easy to know that

$$\dot{V}(x) \mid_{(3.3)} < 0.$$

According to *Lyapunov* second method, we easy to know that the linearization system (2.5) is asymptotically stabilization by the linear state feedback control law (3.1).

According to Lemmas 2.1-2.2 and Definitions 2.1-2.4 , we easy to know that the controlled chaotic system (2.3) is asymptotically stabilization by the linear state feedback control law (3.1).

The proof is completed. ■

Remark 6 *The proof of Theorem is valid only on situation that the controlled chaotic system (2.3) is completely state controllability.*

Remark 7 *From the previous work, we can easy see that some pre-existing techniques for chaotic systems is only consider single situation, for examples,* Lorenz *chaotic systems is only considered in [10], etc. From the analysis of this paper, however, arbitrary controlled chaotic systems are able to stabilization whenever the controlled chaotic system is completely state controllability.*

Remark 8 *Some relatively complex techniques are used in [4,6] or others, for examples, these include adaptive control, adaptive fuzzy control, sliding mode control, Robust control, time-delayed feedback control, double delayed feedback control, bang-bang control, optimal control, intelligent control, etc. From the analysis of this paper, however, the linear state feedback is only used,* Lyapunov *second method and* Riccati *algebra matrix equation are only regard as mathematics tools.*

Remark 9 *The same problem is investigated [21], however, the specific control scheme is not proposed, and the scope investigated is extremely restricted. Obviously, it is a special case of this paper.*

3.2 Controller arithmetic for chaotic system

The foregoing discussion shows that the procedure for controller of chaotic system can be given as follows.

Step 1. The linearization system (2.5) is given by linearizing the chaotic system (2.3).

Step 2. Choosen a matrix B such that the linearization system (2.5) is completely state controllability.

Step 3. The positive definite symmetry matrix P is obtained by soluting *Riccati* algebra matrix equation (3.2).

Step 4. The linear state feedback control law u is obtained by calculating expression (3.1).

Step 5. The controlled closed-loop chaotic system is given by substituting the state feedback control law (3.1) into chaotic system (2.3).

4 Control of unified chaotic system and simulations

In this section, in order to show that the approach of this paper is effective and convenient to this sort of control system. Without loss of generality, we give some illustrative examples and simulation results on *unified* chaotic system.

4.1 Unified chaotic system

The unified chaotic system is described by

$$\begin{cases} \dot{x} = (25\alpha + 10)(-x + y), \\ \dot{y} = (28 - 35\alpha)x - xz + (29\alpha - 1)y, \\ \dot{z} = xy - \frac{8+\alpha}{3}z. \end{cases} \tag{11}$$

where $[x, y, z]^T$ is the unified chaotic system state variables group and $\alpha \in [0, 1]$ is a parameter [12].

Due to the system (4.1) is chaotic for arbitrarily $\alpha \in [0, 1]$ and the system (4.1) belongs to the *generalized Lorenz* chaotic system for $0 \leq \alpha < 0.8$, the system (4.1) belongs to the *Lü* chaotic system at $\alpha = 0.8$ and the system (4.1) belongs to the *generalized Chen* chaotic system for $0.8 < \alpha \leq 1$, so the system (4.1) is regarded as unified chaotic system.

The *Jacobic* matrix A of unified chaotic system (4.1) in origin is given by

$$\begin{bmatrix} -(25\alpha + 10) & (25\alpha + 10) & 0 \\ (28 - 35\alpha) & (29\alpha - 1) & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}.$$

Eigenvalues of unified chaotic system are

$$\begin{aligned} \lambda_1 &= -\tfrac{8+\alpha}{3}, \\ \lambda_2 &= -2\alpha - \tfrac{11}{2} + \tfrac{1}{2}\sqrt{-584\alpha^2 + 2372\alpha + 1201}, \\ \lambda_3 &= -2\alpha - \tfrac{11}{2} - \tfrac{1}{2}\sqrt{-584\alpha^2 + 2372\alpha + 1201}. \end{aligned}$$

and we can know that one of eigenvalue is positive, that is, λ_2, so the linearization system of unified chaotic system (11) is unstable.

Let

$$B = Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

we can know $rank[B, AB, \cdots, A^2B] = 3$, so it's easy to know that the linearization system $\dot{x} = Ax + g(x) + Bu$ of chaotic system (4.1) is completely state controllability. Thus, the controlled unified chaotic system is given by

$$\begin{cases} \dot{x} = (25\alpha + 10)(-x + y) + u_1, \\ \dot{y} = (28 - 35\alpha)x - xz + (29\alpha - 1)y + u_2, \\ \dot{z} = xy - \frac{8+\alpha}{3} z + u_3. \end{cases} \tag{12}$$

where $u = [u_1, u_2, u_3]^T$.

According to this paper on controller arithmetic for chaotic system, our object is to design a suitable feedback gain matrix $K = -B^T P$ such that *Theorem 3.1* is satisfied. Let the linear state feedback control law $u = -B^T P[x, y, z]^T$ such that the chaotic system (4.1) is asymptotically stable in origin.

4.2 Control of *generalized Lorenz* chaotic system and simulation

The foregoing discussion shows that we can know that the chaotic system (4.1) is a *generalized Lorenz* chaotic system at $\alpha = 0$.

Eigenvalues of *generalized Lorenz* chaotic system are

$$\lambda_1 = -2.6667, \\ \lambda_2 = 11.8277, \\ \lambda_3 = -22.8277.$$

and we can know that one of eigenvalues is positive, so the linearization system of *generalized Lorenz* chaotic system and its original system are unstable.

According to this paper on controller arithmetic for chaotic system, let a linear state feedback control law is given by

$$u = -B^T P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -14.7479 & -11.4912 & 0 \\ -11.4912 & -8.9888 & 0 \\ 0 & 0 & -0.1813 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

which such that the *generalized Lorenz* chaotic system is asymptotically stable in origin.

Let the initial value of simulation is $[x, y, z]^T = [1, 0, -1]^T$. State vector history chart for *generalized Lorenz* chaotic system controlled before and after at $t = 60s$ is shown in Figure 3.

Stability track of controlled *generalized Lorenz* chaotic system at phase space is shown in Figure 4. From Figures 3-4, we can know that the controlled closed-loop of *generalized Lorenz* chaotic system is asymptotically stable in origin.

4.3 Control of *Lü* chaotic system and simulation

The foregoing discussion shows that we can know that the chaotic system (4.1) is a *Lü* chaotic system at $\alpha = 0.8$.

Eigenvalues of *Lü* chaotic system are

$$\lambda_1 = -2.9333, \\ \lambda_2 = 22.2, \\ \lambda_3 = -30.$$

and we can know that one of eigenvalues is positive, so the linearization system of *Lü* chaotic system and its original system are unstable.

According to this paper on controller arithmetic for chaotic system, let a linear state feedback control law is given by

$$u = -B^T P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.0167 & -0.0096 & 0 \\ -0.0096 & -44.4354 & 0 \\ 0 & 0 & -0.1658 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

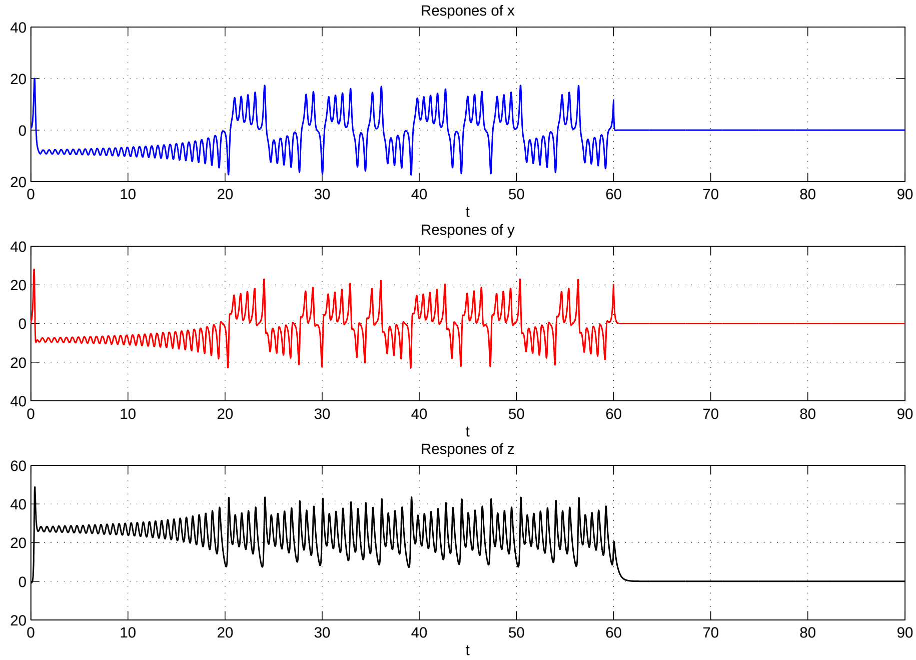

Figure 3: State vector history chart for *generalized Lorenz* chaotic system controlled before and after.

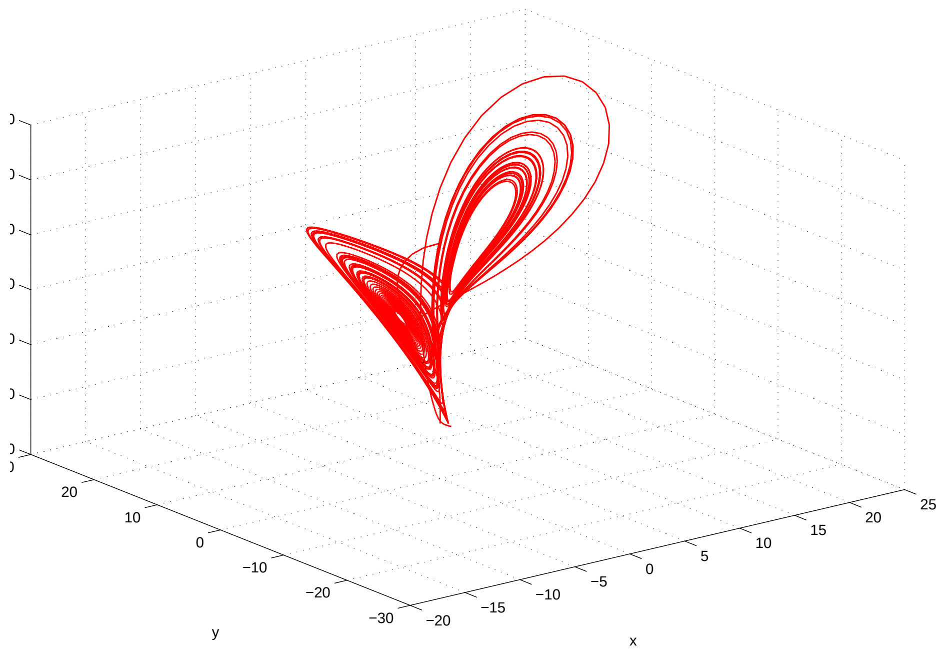

Figure 4: Stability track of controlled *generalized Lorenz* chaotic system at phase space.

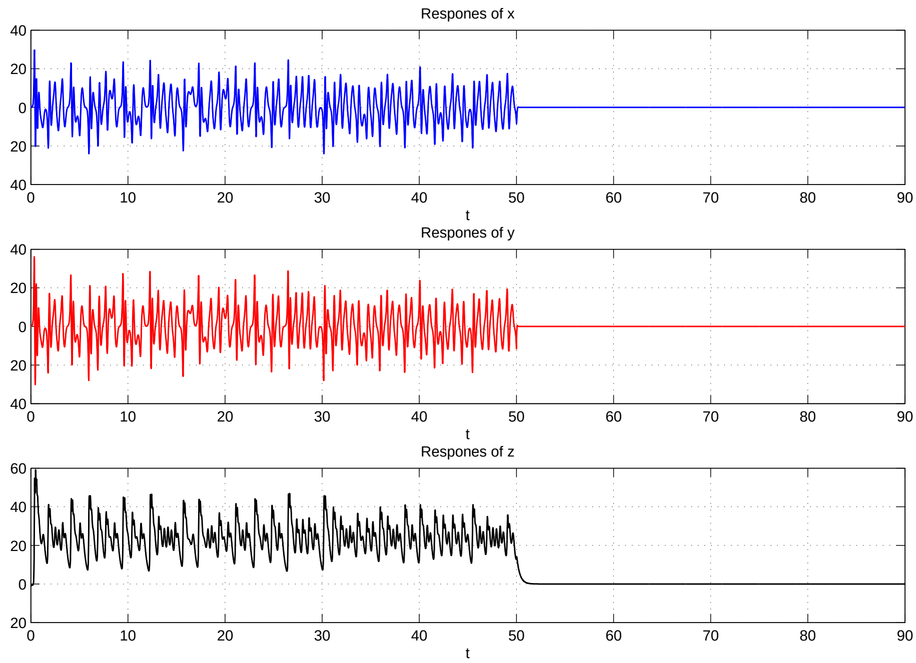

Figure 5: State vector history chart for $L\ddot{u}$ chaotic system controlled before and after.

which such that the $L\ddot{u}$ *chaotic system* is asymptotically stable in origin.

Let the initial value of simulation is $[x, y, z]^T = [1, 0, -1]^T$. State vector history chart for $L\ddot{u}$ chaotic system controlled before and after at $t = 50s$ is shown in Figure 5.

Stability track of controlled $L\ddot{u}$ chaotic system at phase space is shown in Figure 6. From Figures 5-6, we can know that the controlled closed-loop $L\ddot{u}$ chaotic system is asymptotically stable in origin.

4.4 Control of generalized Chen chaotic system and simulation

The foregoing discussion shows that we can know that the chaotic system (11) is a *generalized Chen* chaotic system at $\alpha = 1$.

Eigenvalues of *generalized Chen* chaotic system are

$$\lambda_1 = -3, \quad \lambda_2 = 23.8359, \quad \lambda_3 = -30.8359.$$

and we can know that one of eigenvalues is positive, so the linearization system of *generalized Chen* chaotic system and its original system are unstable.

According to this paper on controller arithmetic for chaotic system, let a linear state feedback control law is given by

$$u = -B^T P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.6790 & 5.5836 & 0 \\ 5.5836 & -47.0518 & 0 \\ 0 & 0 & -0.1623 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

which such that the *generalized Chen* chaotic system is asymptotically stable in origin.

Let the initial value of simulation is $[x, y, z]^T = [1, 0, -1]^T$. State vector history chart for *generalized Chen* chaotic system controlled before and after at $t = 40s$ is shown in Figure 7. Stability track of controlled *generalized Chen* chaotic system at phase space is shown in Figure 8. From Figures 7-8, we can know that the controlled closed-loop of *generalized Chen* chaotic system is asymptotically stable in origin.

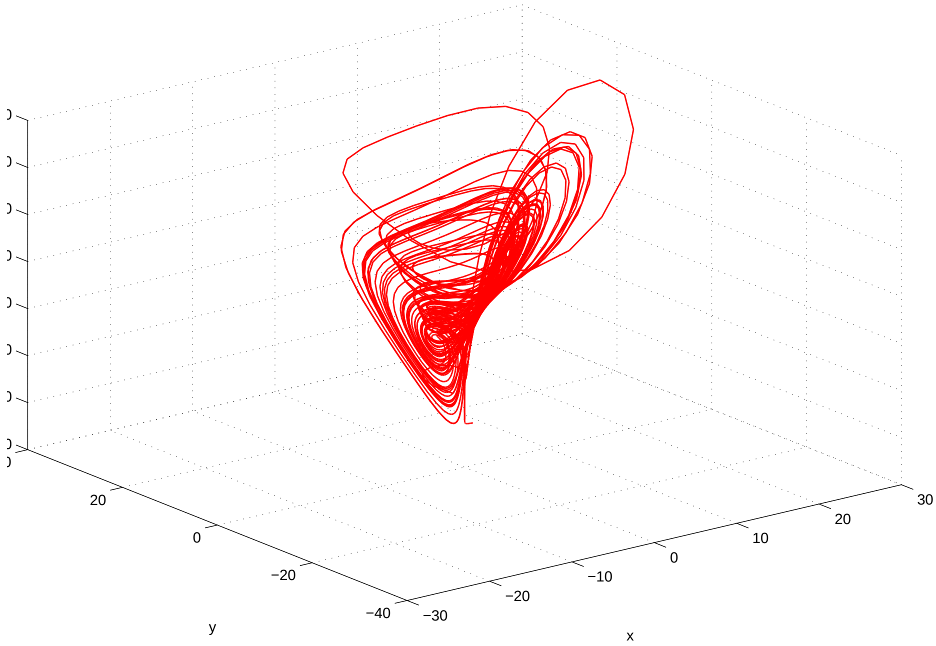

Figure 6: Stability track of controlled $L\ddot{u}$ chaotic system at phase space.

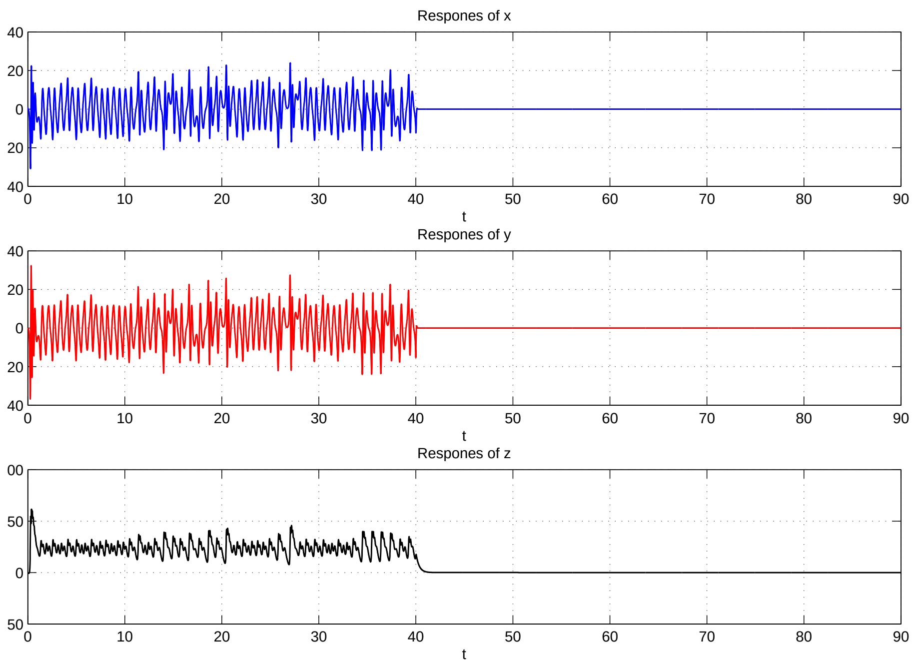

Figure 7: State vector history chart for *generalized Chen* chaotic system controlled before and after.

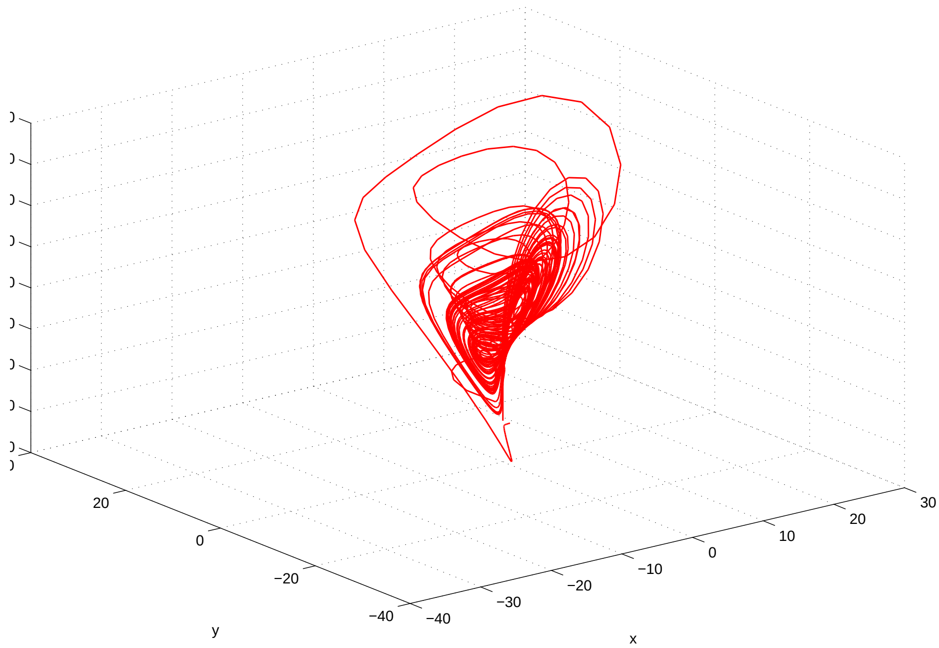

Figure 8: Stability track of controlled *generalized Chen* chaotic system at phase space.

Remark 10 *Yassen designed different linear feedback controllers to locally stabilize the equilibria of the new chaotic system [17], however, we prove that the linear state feedback controller can globally stabilize the corresponding equilibria of the chaotic system, thus eliminating chaos. Obviously, it is a special case of this paper.*

5 Conclusions

Based on some work of pioneers, a new design technique of linear state feedback control law for controlled chaotic system in this paper is investigated by *Lyapunov* second method and *Riccati* algebra matrix equation in the matrix theory. The linear state feedback control law for controlled chaotic systems is ensures that the dynamic responds of controlled closed-loop chaotic systems is asymptotically stable and established in this paper is extended to a general and large of chaotic systems.

From the previous work, we can easy see that some pre-existing techniques for chaotic systems have some situations that need improve. However, the approach in this paper is significantly expanded compared to pre-existing technique for some chaotic systems, and the stabilization problem of systems is become more generalized.

In order to show that the approach of this paper to chaotic systems is effective and convenient, without loss of generality, the *unified* chaotic system is simulated in the end.

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