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Robustness of Multiplex Networks under Localized Attack

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Abstract: In recent years, multilayer networks start to draw scholars' attention due to their social realistic significance. We build a two layer networks model to study the robustness under two kinds of attacking strategies. One way is removing a p fraction of nodes severally and locally, remaining a giant component of each one, then combining two partial networks to one network. The other way is to combine two layer networks at first and then attack a p fraction of nodes locally. In this paper, we studied two models, which are the single clustering network and the multiplex network composed of two independent clustering networks as layers. Finally, we find the robustness of both networks become weaker with the increase of the clustering coefficient and the decrease of the connected density for monoplex network and multiplex network. By analyzing multiplex networks, we concluded that the first way of doing an attack was more efficient than the second way of an attack.

Keywords: multiplex network; robustness; localized attack

1 Introduction

In recent years, the robustness of multilayer networks is one of the focal problems to complex networks analysis, due to its widespread existence in present world [1–10]. In social networks, its subjects are a group of people and the ways of interpersonal communication are developing from simplification to diversification. Nowadays, there are ways that social softwares and technologies get in touch with each other, such as WeChat, WhatsApp, QQ, e-mail, phone, Facebook and so on. People also have many ways for traveling and moving around, such as taxi, bus, airplane, train, and so forth. Sometimes, people need more than two kinds of vehicles to get to a destination. So the public transport network also are complex and multiple, which include railway network and aviation network, and so forth. So the public transport network also are complex and multiple. If we take every way to travel as a independent network, the public transport network can be regarded as a multi-layer network. In fact, Du et al. has done such research by using the Chinese Airline Network as a multilayer network via a "k-core decomposition " method [11]. Moreover, Zanin et al. thought that multiple relationships exist widely and multilayer networks should not be neglected [9]. Feng et al. made a study about the robustness of two layer interdependent networks and N-layer interdependent networks theoretically and numerically [12]. Guha et al. studied the site-bond percolation of multilayer networks, in which each layer has a q fraction of nodes that [3]. At the same time, many achievements of localized attack has been published. In this paper, we mainly study the robustness of multiplex networks under localized attack [13–15].

A network contains nodes and edges connecting two nodes. We firstly remove a fraction of nodes and inner edges between removed nodes, then we make a statistics to calculate the size of the giant component in the remaining network to explicitly get its robustness. The giant component is the biggest cluster, which includes the maximum number of nodes among all clusters, and any node has a link with another node. Multilayer networks consist of multiple single networks, which have the same nodes, but every layer has different connecting edges. The useful method to compare the robustness of multiplex networks is to choose different ways to remove a fraction of nodes.

For example, in China, airports are mainly distributed in first-tier cities, so the residents in other cites usually select train or bus to travel between the cities without airports. Considering this situation, we establish a two layer network, in

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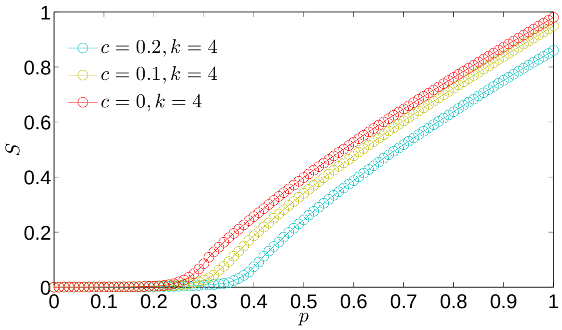


Figure 1: The giant component S as a function of p. The average degree of single cluster network is 4. The solid line means simulation result.

which nodes mean cities, edges of one layer represent air lines and the edges of the other layer indicate rail ways. First, we study the multilayer network composed of two clustering networks having the same mean average degree, which are completely independent. For this model, we attack both networks by two different methods [16]. One is removing a p fraction of nodes from the two layers respectively, and then the two remaining networks are reflected to a new network to calculate the size of giant component. The other way is to combine the two layer networks into a new network, then remove a p fraction of nodes from the new network to calculate the size of giant component. We have performed computer simulations to investigate the relation between the robustness of the network and the attack intensity.

2 Monoplex network

Here we suppose a single clustering network has N nodes and the average degree is $\langle k \rangle$. And it obeys a double Poisson distribution,

$$p(k) = e^{-\langle s \rangle} \frac{\langle s \rangle^s}{s!} e^{-\langle t \rangle} \frac{\langle t \rangle^t}{t!}, \tag{1}$$

where $\langle k \rangle = \langle s \rangle + 2\langle t \rangle$. Clustering network consists of normal line and triangle construction, which means three nodes linked with each other to form a triangle. Due to the special structure, we use two parameters to describe its degree distribution, where $\langle s \rangle$ is the average number of single edge for whole nodes and $\langle t \rangle$ is the mean number of triangles belonging to all nodes. If one node has a triangle that means its has two single edges at least, so the average degree of a network is $\langle k \rangle = \langle s \rangle + 2\langle t \rangle$. At the same time, the clustering coefficient can be expressed as,

$$c = \frac{2\langle t \rangle}{2\langle t \rangle + \langle k \rangle^2}. \tag{2}$$

For a clustering network, we use localized attack way to attack it. The detailed processes are following. First, we randomly choose a node of the giant component as “root” node. Second, finding all neighboring nodes of ”root” nodes as second shell. with that, making neighboring nodes of second shell as third shell but not include these nodes that has been in second shell. The next step is to repeat the last step until all nodes in the shells expect isolated nodes. Then, you remove a $1 - p$ fraction of nodes from “root” nodes to other nodes shell by shell [17]. Last, we calculate the giant component of remaining network.

Fig. 1 shows the giant component become bigger with increasing p, which means the remaining nodes in the network. For different clustering coefficient c, the critical point p_c increases when it increasing. It means that increasing c can decrease the robustness of network. Simultaneously, Fig. 2 agrees well with the former solution. p_c decreases with average degree $\langle k \rangle$ increasing. This illustrates that the robustness of clustering network become stronger with increasing link density.

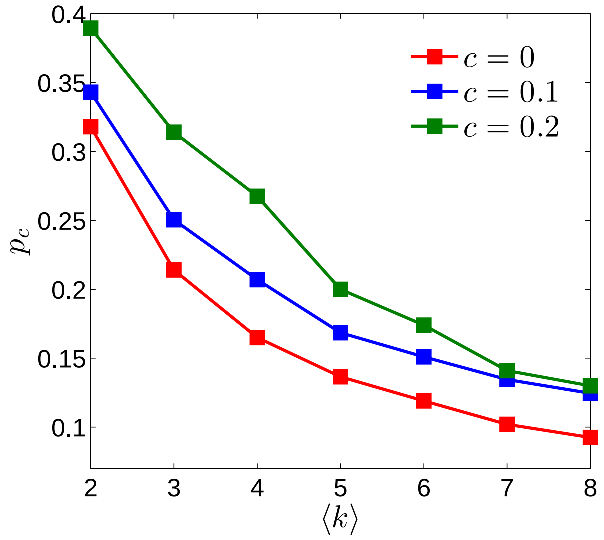


Figure 2: The critical threshold p_c as a function of $\langle k_1 \rangle$ and $\langle k_2 \rangle$ for three clustering coefficients $c = 0$, 0.1 and 0.2.

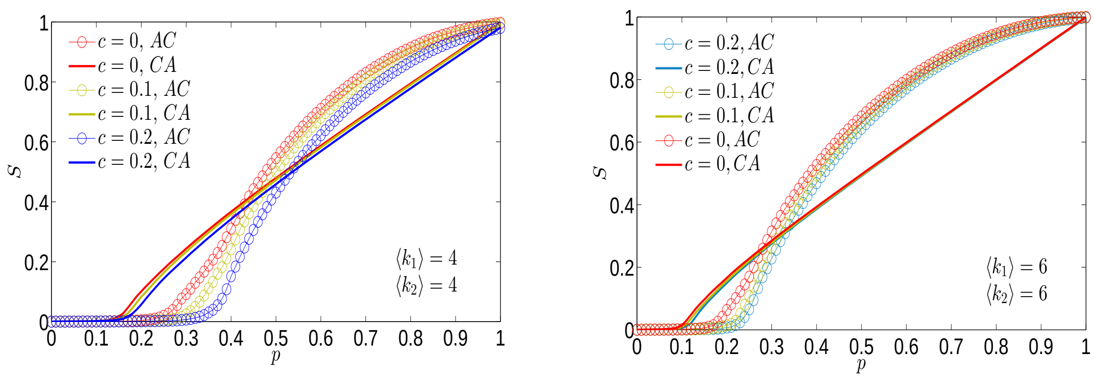


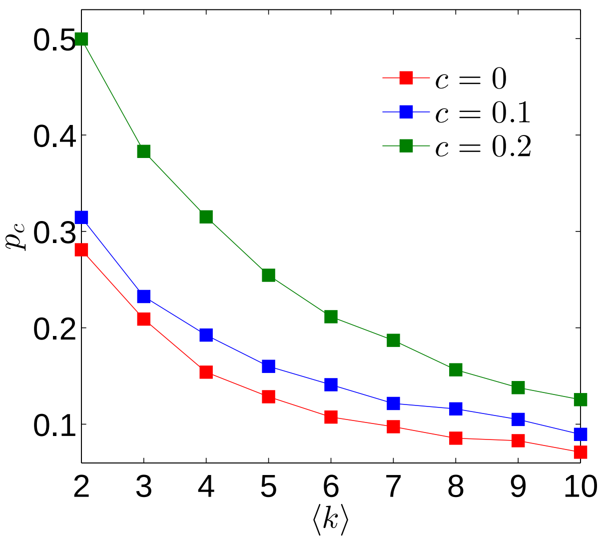
Figure 3: The giant component S as a function of p. The symbols means the results under the first attack way (AC), and the solid line means the result under the second attack way (CA). (Left) Both of the two clustering networks have the same average degree 4. (Right) Both of the two clustering networks have the same average degree 6.

3 Multiplex network consisted of two clustering networks

This network model consists of two layers, each layer is a clustering network. Let $\langle k_1 \rangle$ and $\langle k_2 \rangle$ be the average degrees of two layers, respectively. We randomly choose a node, which has multidegree vector $\vec{k} = (k_1, k_2)$, where k_α is the degree of the node in layer α, and $p(k_\alpha) = \sum_{s_i+2t_i=k_\alpha} p(s_i, t_i)$. Then the multidegree distribution of the multiplex network can be defined as

$$p(\vec{k}) = e^{-\langle k_1 \rangle} \frac{\langle k_1 \rangle^{k_1}}{k_1!} e^{-\langle k_2 \rangle} \frac{\langle k_2 \rangle^{k_2}}{k_2!}. \tag{3}$$

Then we use two methods to attack the model respectively to study the difference of robustness. The first attacking way (AC) is to remove a fraction $1 - p$ of nodes by localized attack way from two layer networks and find their giant component respectively. Then combine the rest fraction of two layers into a new network. Finally, the size of the giant component of the new network as the giant component of whole system. The second attacking way (CA) is to project two obtained clustering networks into a monoplex network by ignoring the connected edges of two layers. Then the giant component of the complex network is obtained under localized attack.



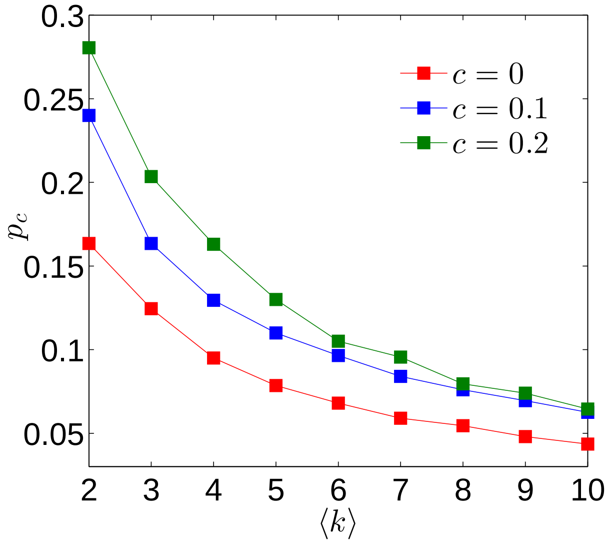


Figure 4: Critical threshold p_c as a function of $\langle k \rangle$ for different clustering coefficients. (Left) p_c under the first attack way (AC). (Right) p_c under the second attack way (CA).

The two figures in Fig. 3 respectively have average degrees 4 and 6. By comparing two figures, we can obtain the differences of p_c in the middle of diverse c reduce with $\langle k \rangle$ increasing for both attack ways. Otherwise, the robustness of multiplex network under first attack way is more weaker than under second attack way for same $\langle k \rangle$. In addition, the robustness also became weaker with increasing clustering coefficient for both attack ways.

From Fig. 4, p_c decreases as $\langle k \rangle$ increases, which agrees well with Fig. 3. Moreover, comparing the two figures of Fig. 4, we get that change around the three clustering coefficients of AC is more obvious than CA, which is also coincident with Fig. 3.

4 Conclusions

In this paper, we study the size of the giant component and the critical threshold of attacking intensity of two layer clustering networks under two kinds of attacking approaches. One is to attack two layer networks severally and get the giant component of each layer, then we combine two networks into a new network. The other way is to project two layer networks into one network at first, then attack the new network. Starting from one clustering network, we get the giant component by simulation results. The next part is to set up network model consisting of two independent clustering networks. Based on the above, we get the simulation results of the giant components and critical thresholds. From the results, we can conclude that increasing the connected density or decreasing the clustering coefficient can make complex networks more robust. Meanwhile, the first attack way (AC) causes more damage to the connections of the multiplex networks than the second way. By comparing the results of the two ways of attack, we find separately attacking makes the system more vulnerable.

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