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# Constructing Infinite Number of Exact Traveling Wave Solutions of Nonlinear Evolution Equations Via an Extended Tanh–Function Method

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**Abstract:** Two class of fractional type solutions of Riccati equation are constructed from its three known solutions. These fractional type solutions are used to propose an approach for constructing infinite number of exact traveling wave solutions of nonlinear evolution equations by means of the extended tanh–function method. The infinite number of exact traveling wave solutions of the long–short wave interaction system and the generalized variable coefficients Korteweg–de Vries equation are obtained by the approach. The obtained new solutions not only can recover old solutions but also can give infinite number of new exact solutions for nonlinear evolution equations. The method also can be applied to find infinite number of exact traveling wave solutions of the other nonlinear evolution equations.

**Keywords:** Riccati equation; extended tanh–function method; fractional type solution; traveling wave solution

## 1 Introduction

Seeking new exact solutions for a given nonlinear evolution equation(NLEE) is an interesting hot topic in the soliton theory. Some direct methods, such as the extended $\tanh$-function method (ETFM)[1–4], F–expansion method[5, 6], $G'/G$–expansion method[7, 8], auxiliary equation method[9–12], $\exp(-\varphi(\xi))$–expansion method[13, 14] and so on, had been carried on this topic and found some new exact solutions of the considered NLEEs. All these methods are connected with two elements, i.e, an auxiliary ordinary differential equation(s) (AODEs) and an expansion expression involving the solution of the chosen AODEs. Therefore, new solutions of a given NLEE can be found by modifying one or both of the elements. The modification on the AODEs contains two situation that is to give new AODEs or new solutions for the old AODEs. In this work, we shall consider how to construct new solutions of the NLEEs by using the ETFM. In order that the Riccati equation used by Fan in ETFM[1] of the form

$$f'(\xi) = b + f^2(\xi), \tag{1}$$

is taken as the AODE and then try to construct its new solutions. If it is possible to find some new solutions of the Riccati equation (1), then we can construct new solutions for the given NLEEs by using the obtained new solutions of the Riccati equation (1) to replace the old solutions which used before in the extended tanh–function method.

Based on the above idea, in Section 2, we shall give two class of fractional type solutions with two parameters of the Riccati equation (1) starting from its three known solutions. We find our fractional type solutions not only can recover the old solutions given by Fan as below

$$f(\xi) = \begin{cases} -\sqrt{-b}\tanh\left(\sqrt{-b}\,\xi\right), & b<0, \\ -\sqrt{-b}\coth\left(\sqrt{-b}\,\xi\right), & b<0, \\ \sqrt{b}\tan\left(\sqrt{b}\,\xi\right), & b>0, \\ -\sqrt{b}\cot\left(\sqrt{b}\,\xi\right), & b>0, \end{cases} \tag{2}$$

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but also can give infinite number of new solutions for Riccati equation (1) as its parameters varying.

In Section 3, we shall use our fractional type solutions to construct infinite number of new exact traveling wave solutions of the long–short wave interaction system (LSWIS) and the generalized variable coefficients Korteweg–de Vries(GVCKDV) equation through use of the ETFM . As a result, the old solutions are recovered and infinitely many new exact traveling wave solutions are also constructed. Finally, in Section 4 give some conclusions.

# 2 Two Parametric Solutions for Riccati Equation

As we know that the Riccati equation has fractional type solution[15] which can be constructed from its three known solution $f_1, f_2, f_3$ by the formula[16]

$$f = f_1 - \frac{(f_1 - f_2)(f_1 - f_3)/(f_2 - f_3)}{(f_1 - f_3)/(f_2 - f_3) + c}, \tag{3}$$

where $c$ is an arbitrary constant. This formula allows us to construct the non–trivial and non–degenerate fractional type solutions of the Riccati equation (1). In order that, when $b < 0$ we choose three known solutions of the Riccati equation (1) as

$$f_1(\xi) = -\sqrt{-b}\tanh\left(\sqrt{-b}\,\xi\right), f_2(\xi) = -\sqrt{-b}\coth\left(\sqrt{-b}\,\xi\right), f_3(\xi) = -\sqrt{-b}, \tag{4}$$

then using formula (3) we get

$$f(\xi) = \frac{\sqrt{-b}\left(c\tanh\left(\sqrt{-b}\,\xi\right) - 1\right)}{\tanh\left(\sqrt{-b}\,\xi\right) - c}. \tag{5}$$

Replacing $c$ and $-1$ in (5) by $r_1$ and $r_2$, respectively, we obtain

$$f(\xi) = -\frac{\sqrt{-b}\left(r_1\tanh\left(\sqrt{-b}\,\xi\right) + r_2\right)}{r_1 + r_2\tanh\left(\sqrt{-b}\,\xi\right)},\ b < 0. \tag{6}$$

where $\Delta = \sqrt{-b}(r_2^2 - r_1^2) \neq 0$ and $r_1, r_2$ are two real parameters. Direct calculation shows that (6) is just the solution of the Riccati equation (1) and which is non–trivial and non–degenerate if and only if the two real parameters $r_1$ and $r_2$ must be satisfy the condition

$$r_2 \neq \pm r_1, r_1^2 + r_2^2 \neq 0. \tag{7}$$

Clearly, if $r_1 = 1, r_2 = 0$, then (6) is reduced to the old solution

$$f(\xi) = -\sqrt{-b}\tanh\left(\sqrt{-b}\,\xi\right),\ b < 0,$$

and if $r_1 = 0, r_2 = 1$, then (6) gives the old solution

$$f(\xi) = -\sqrt{-b}\coth\left(\sqrt{-b}\,\xi\right),\ b < 0.$$

When $(r_1, r_2) \neq (1, 0), (0, 1)$, the fractional type solution (6) is a new class of solutions for the Riccati equation (1).

In the case of $b > 0$, we observe that $\sqrt{-b} = i\sqrt{b}$, $\tanh\left(i\sqrt{b}\,\xi\right) = i\tan\left(\sqrt{b}\,\xi\right)$ and set $r_3 = r_1, r_4 = ir_2$, then we obtain the second class of fractional type solutions of the Riccati equation (1)

$$f(\xi) = -\frac{\sqrt{b}\left(r_4 - r_3\tan\left(\sqrt{b}\,\xi\right)\right)}{r_3 + r_4\tan\left(\sqrt{b}\,\xi\right)},\ b > 0, \tag{8}$$

in which $\Delta = \sqrt{b}(r_3^2 + r_4^2) \neq 0$ and $r_3, r_4$ are real parameters. This solution is non–trivial and non–degenerate if and only if

$$r_3^2 + r_4^2 \neq 0. \tag{9}$$

Also we can see that, if $r_3 = 1, r_4 = 0$, then (8) is reduced to the old solution

$$f(\xi) = \sqrt{b}\tan\left(\sqrt{b}\,\xi\right),\ b > 0,$$

and if $r_3 = 0, r_4 = 1$, then (8) gives the old solution

$$f(\xi) = -\sqrt{b}\cot\left(\sqrt{b}\,\xi\right),\ b > 0.$$

In addition, by setting $(r_1, r_2) = (1, \sqrt{-b}), (r_3, r_4) = (1, 1)$ and $(r_1, r_2) = (-3, 1), (r_3, r_4) = (3, 1)$ in (6) and (8) we obtain solutions No. 7–No. 10 in Ref.[2].

As shown above, from the solutions (6) and (8) we not only can recover the all old solutions but also can give infinite number of new solutions of the Riccati equation (1). Therefore, following the procedures of the ETFM equipped with solutions (6) and (8) we can get infinitely many new two parametric solutions for a given NLEE.

# 3 Illustrative Examples

According to the discussion in Section 2, if we use the fractional type solution (6) or (8) to replace the solution (2), then we can obtain new type of exact traveling wave solutions of the given NLEEs from the ETFM. To illustrate this, we now consider two examples as below.

## 3.1 The long–short–wave interaction system

Now let us consider the long–short–wave interaction system (LSWIS)[17]

$$\begin{cases} i\psi_t + \psi_{xx} - \psi v = 0, \\ v_t + v_x + \left(|\psi|^2\right)_x = 0, \end{cases} \tag{10}$$

in which $\psi(x, t)$ is a complex function and $v(x, t)$ is a real function. This system describes the interaction between one long longitudinal wave and one short transverse wave propagating in a generalized elastic medium. The exact solutions of the system (10) were obtained in the literature [18–20].

Let

$$\psi(x, t) = u(\xi)\exp(i\eta), v(x, t) = v(\xi), \xi = x + \omega t, \eta = \alpha x + \beta t, \tag{11}$$

where $\omega, \alpha, \beta$ are constants to be determined. Substituting (11) into (10) and setting the real part and the imaginary part to zero, we get

$$u'' - (\beta + \alpha^2 + c)u + \frac{u^3}{1 - 2\alpha} = 0, \tag{12}$$

and

$$v = \frac{u^2}{2\alpha - 1} + c, \omega = -2\alpha, \tag{13}$$

where $c$ is an integration constant. Balancing the highest order derivative term $u''$ with the highest power nonlinear term $u^3$ in (12) gives leading order $n = 1$. Therefore, we may take the solution of (12) as

$$u(\xi) = a_0 + a_1 f(\xi), \tag{14}$$

where $a_0, a_1$ are constants to be determined. Substituting (14) into (12) and setting the coefficients of $f^j(\xi)$ $(j = 0, 1, 2, 3)$ to zero yields a set of algebraic equations

$$\begin{aligned} &\frac{3a_0a_1^2}{1 - 2\alpha} = 0, \\ &2a_1 + \frac{a_1^3}{1 - 2\alpha} = 0, \\ &-(\beta + \alpha^2 + c)a_0 + \frac{a_0^3}{1 - 2\alpha} = 0, \\ &(2b - \beta - \alpha^2 - c)a_1 + \frac{3a_0^2a_1}{1 - 2\alpha} = 0. \end{aligned}$$

Solving this set of algebraic equations we obtain

$$a_0 = 0, a_1 = \pm\sqrt{2(2\alpha-1)}, b = \frac{1}{2}(\beta+\alpha^2+c). \tag{15}$$

Taking (15) with (6),(8) into (14) and using the relations (13) and (11) we obtain the exact traveling wave solutions of the LSWIS (10) as follows

$$\psi_1(x,t) = \pm\exp\left[i(\alpha x+\beta t)\right]\sqrt{(2\alpha-1)(-\beta-\alpha^2-c)}\,\frac{r_1\tanh\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t)+r_2}{r_2\tanh\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t)+r_1}, \tag{16}$$

$$v_1(x,t) = -(\beta+\alpha^2+c)\left(\frac{r_1\tanh\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t)+r_2}{r_2\tanh\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t)+r_1}\right)^2+c, \tag{17}$$

$$\psi_2(x,t) = \pm\exp\left[i(\alpha x+\beta t)\right]\sqrt{(2\alpha-1)(\beta+\alpha^2+c)}\,\frac{r_4-r_3\tan\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t)}{r_4\tan\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t)+r_3}, \tag{18}$$

$$v_2(x,t) = (\beta+\alpha^2+c)\left(\frac{r_4-r_3\tan\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t)}{r_4\tan\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t)+r_3}\right)^2+c. \tag{19}$$

The above two class of solutions will give infinite number of exact traveling wave solutions of the LSWIS (10) with the variation of their parameters $r_i$ $(i = 1,2,3,4)$. Especially, when choosing $r_1 = 1, r_2 = 0$ and $r_1 = 0, r_2 = 1$, the solution (16)–(17) can recover the known solution (16)–(17) and (18)–(19) in [20], respectively, they are

$$\psi_{1a}(x,t) = \pm\exp\left[i(\alpha x+\beta t)\right]\sqrt{(2\alpha-1)(-\beta-\alpha^2-c)}\,\tanh\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t),$$

$$v_{1a}(x,t) = -(\beta+\alpha^2+c)\tanh^2\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t)+c,$$

$$\psi_{1b}(x,t) = \pm\exp\left[i(\alpha x+\beta t)\right]\sqrt{(2\alpha-1)(-\beta-\alpha^2-c)}\,\coth\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t),$$

$$v_{1b}(x,t) = -(\beta+\alpha^2+c)\coth^2\sqrt{\frac{1}{2}(-\beta-\alpha^2-c)}(x-2\alpha t)+c.$$

When $r_3 = 1, r_4 = 0$ and $r_3 = 0, r_4 = 1$,the solution (18)–(19) is reduced to the old solution (21)–(22) in [20]

$$\psi_{2a}(x,t) = \pm\exp\left[i(\alpha x+\beta t)\right]\sqrt{(2\alpha-1)(\beta+\alpha^2+c)}\,\tan\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t),$$

$$v_{2a}(x,t) = (\beta+\alpha^2+c)\tan^2\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t)+c,$$

and

$$\psi_{2b}(x,t) = \pm\exp\left[i(\alpha x+\beta t)\right]\sqrt{(2\alpha-1)(\beta+\alpha^2+c)}\,\cot\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t),$$

$$v_{2b}(x,t) = (\beta+\alpha^2+c)\cot^2\sqrt{\frac{1}{2}(\beta+\alpha^2+c)}(x-2\alpha t)+c.$$

## 3.2 The generalized variable coefficients KdV equation

Now we consider the generalized variable coefficients KdV (GVCKDV) equation

$$u_t+2\beta(t)u+\left[\alpha(t)+\beta(t)x\right]u_x-3c\gamma(t)uu_x+\gamma(t)u_{xxx}=0, \tag{20}$$

where the wave amplitude $u(x,t)$ is a function of the scaled space $x$ and scaled time $t$,$\alpha(t), \beta(t), \gamma(t)$ are differentiable functions of $t$ and $c$ is a constant. The exact solutions of the GVCKDV equation have been obtained by the Exp–function method [21], the F–expansion method [22] and the polynomial expansion method [23], etc. The N–soliton solutions of the GVCKDV equation were constructed by means of the Wronskian technique in [24].

By balancing the highest order derivative term $u_{xxx}$ with highest power nonlinear term $uu_x$ in (20) gives the leading order $n = 2$. Therefore, according to the procedure of ETFM, we can set

$$u(x,t) = a_0(t) + a_1(t)f(\xi) + a_2(t)f^2(\xi), \xi = p(t)x + q(t), \tag{21}$$

where $a_0(t), a_1(t), a_2(t), p(t), q(t)$ are undetermined functions. Taking (21) into (20) and then setting the coefficients of $x, x^i f^j$ $(i = 0,1; j = 0,1,2,3,4)$ to be zero, we obtain the following ordinary differential equations (ODEs)

$$a_1(t)b^i\left(\frac{dp(t)}{dt} + \beta(t)p(t)\right) = 0, i = 0,1, \tag{22}$$

$$2a_2(t)b^i\left(\frac{dp(t)}{dt} + \beta(t)p(t)\right) = 0, i = 0,1, \tag{23}$$

$$3p(t)\gamma(t)a_1(t)\left(2p^2(t) - 3ca_2(t)\right) = 0, \tag{24}$$

$$6p(t)\gamma(t)a_2(t)\left(4p^2(t) - ca_2(t)\right) = 0, \tag{25}$$

$$-3cp(t)\gamma(t)a_0(t)a_1(t) - 9bcp(t)\gamma(t)a_1(t)a_2(t) + \frac{da_2(t)}{dt} + \alpha(t)p(t)a_1(t) + \frac{dq(t)}{dt}a_1(t) + 2\beta(t)a_2(t) = 0, \tag{26}$$

$$-6cp(t)\gamma(t)a_0(t)a_2(t) - 6bcp(t)\gamma(t)a_2^2(t) - 3cp(t)\gamma(t)a_1^2(t) + 24bp^3(t)\gamma(t)a_2(t) + 2\alpha(t)p(t)a_2(t) + 2\frac{dq(t)}{dt}a_2(t) = 0, \tag{27}$$

$$-3bcp(t)\gamma(t)a_0(t)a_1(t) + 2\beta(t)a_0(t) + \frac{da_0(t)}{dt} + b\alpha(t)p(t)a_1(t) + 10b^2p^3(t)\gamma(t)a_1(t) + b\frac{dq(t)}{dt}a_1(t) = 0, \tag{28}$$

$$-3bcp(t)\gamma(t)a_1^2(t) - 6bcp(t)\gamma(t)a_0(t)a_2(t) + \frac{da_1(t)}{dt} + 2b\alpha(t)p(t)a_2(t) + 32b^2p^3(t)\gamma(t)a_2(t) + 2b\frac{dq(t)}{dt}a_2(t) + 2\beta(t)a_1(t) = 0. \tag{29}$$

We find this set of ODEs is solvable if and only if

$$a_1(t) = 0. \tag{30}$$

Because $a_2(t) \neq 0$,thus (23) leads

$$\frac{dp(t)}{dt} + \beta(t)p(t) = 0, \tag{31}$$

which solves

$$p(t) = \exp\left(-\int_{t_0}^{t}\beta(\tau)d\tau\right). \tag{32}$$

Under (30), the relation (24) is automatically satisfied and (25) gives rise to

$$a_2(t) = \frac{4}{c}p^2(t) = \frac{4}{c}\exp\left(-2\int_{t_0}^{t}\beta(\tau)d\tau\right). \tag{33}$$

Because of (30), the equation (28) is reduced to

$$\frac{da_0(t)}{dt} + 2\beta(t)a_0(t) = 0, \tag{34}$$

which solves

$$a_0(t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau). \quad (35)$$

Under (30) and (33), the equation (26) is automatically satisfied, (27) and (29) are reduced to the following ODEs

$$-16bp^3(t)\gamma(t) + 6cp(t)\gamma(t)a_0(t) - 2\alpha(t)p(t) - 2\frac{dq(t)}{dt} = 0, \quad (36)$$

$$-8bp^3(t)\gamma(t) + 3cp(t)\gamma(t)a_0(t) - \alpha(t)p(t) - \frac{dq(t)}{dt} = 0. \quad (37)$$

Its solution is found to be

$$q(t) = \int_{t_0}^{t}\left[(3c-8b)\gamma(s)\exp(-3\int_{s_0}^{s}\beta(\tau)d\tau) - \alpha(s)\exp(-\int_{s_0}^{s}\beta(\tau)d\tau)\right]ds. \quad (38)$$

Substitution of (30), (33), (35), (6), (8) into (21) gives two infinite number of exact traveling wave solutions of the GVCKDV equation as below

$$u_1(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 - \frac{4b}{c}\left(\frac{r_1\tanh(\sqrt{-b}(p(t)x+q(t))) + r_2}{r_2\tanh(\sqrt{-b}(p(t)x+q(t))) + r_1}\right)^2\right], \quad (39)$$

$$u_2(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 + \frac{4b}{c}\left(\frac{r_4 - r_3\tan(\sqrt{b}(p(t)x+q(t)))}{r_4\tan(\sqrt{b}(p(t)x+q(t))) + r_3}\right)^2\right], \quad (40)$$

where $p(t)$ and $q(t)$ are determined by expression (32) and (38), (39) and (40) also satisfy the condition (7) and (9), respectively.

In particular, when $r_1 = 1, r_2 = 0$ and $r_1 = 0, r_2 = 1$, the solution (39) is reduced to the old solitary wave like solutions [23]

$$u_{1a}(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 - \frac{4b}{c}\tanh^2(\sqrt{-b}(p(t)x+q(t)))\right], b<0, \quad (41)$$

$$u_{1b}(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 - \frac{4b}{c}\coth^2(\sqrt{-b}(p(t)x+q(t)))\right], b<0. \quad (42)$$

where $p(t)$ and $q(t)$ is given by (32) and (38).

When $r_3 = 1, r_4 = 0$ and $r_3 = 0, r_4 = 1$, the solution (40) is reduced to the old trigonometric periodic like solutions [23]

$$u_{2a}(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 + \frac{4b}{c}\tan^2(\sqrt{b}(p(t)x+q(t)))\right], b>0, \quad (43)$$

$$u_{2b}(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 + \frac{4b}{c}\cot^2(\sqrt{b}(p(t)x+q(t)))\right], b>0, \quad (44)$$

in which $p(t)$ and $q(t)$ are determined by (32) and (38).

Finally, if we consider the case $b = 0$,then the solution of Riccati (1) is found to be

$$f(\xi) = -\frac{1}{\xi} \quad (45)$$

and correspondingly the set of equations (22)–(29) solves

$$\begin{aligned}
(1)\ a_0(t) &= a_1(t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau), a_2(t) = 0, p(t) = 0, q(t) = q,\\
(2)\ a_0(t) &= a_2(t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau), a_1(t) = 0, p(t) = 0, q(t) = q,\\
(3)\ a_0(t) &= a_1(t) = a_2(t) = \exp(-\int_{t_0}^{t}\beta(\tau)d\tau), p(t) = 0, q(t) = q,\\
(4)\ a_0(t) &= \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau), a_1(t) = 0, a_2(t) = \frac{4}{c}\exp(-2\int_{t_0}^{t}\beta(\tau)d\tau),\\
p(t) &= \exp(-\int_{t_0}^{t}\beta(\tau)d\tau), q(t) = \int_{t_0}^{t}p(\tau)\left[3c\gamma(\tau)a_0(\tau) - \alpha(\tau)\right]d\tau.
\end{aligned}$$

which leads the exact solutions of the GVCKDV equation as follows

$$u(x,t) = \left(1 - \frac{1}{q}\right)\exp(-2\int_{t_0}^{t}\beta(\tau)d\tau), \tag{46}$$

$$u(x,t) = \left(1 + \frac{1}{q^2}\right)\exp(-2\int_{t_0}^{t}\beta(\tau)d\tau), \tag{47}$$

$$u(x,t) = \left(1 - \frac{1}{q} + \frac{1}{q^2}\right)\exp(-2\int_{t_0}^{t}\beta(\tau)d\tau), \tag{48}$$

$$u(x,t) = \exp(-2\int_{t_0}^{t}\beta(\tau)d\tau)\left[1 + \frac{4}{c\left(x\exp(-\int_{t_0}^{t}\beta(\tau)d\tau) + q(t)\right)^2}\right], \tag{49}$$

where $q$ in (46), (47) and (48) is constant, $q(t)$ in (49) is given by

$$q(t) = \int_{t_0}^{t}\left[3c\gamma(s)\exp(-3\int_{s_0}^{s}\beta(\tau)d\tau) - \alpha(s)\exp(-\int_{s_0}^{s}\beta(\tau)d\tau)\right]ds. \tag{50}$$

# 4 Conclusions

Two class of fractional type solutions of the Riccati equation were obtained from its three known solutions. By the ETFM we constructed the infinite number of exact traveling wave solutions of the LSWIS and the GVCKDV equation by using the obtained fractional type solutions (6) and (8) to replace the solution (2) of the Riccati equation (1). As a result, some old solutions are recovered and the approach is applicable to find infinite number of new exact traveling wave solutions for other NLEEs. It is noted that the approach is an essential generalization of the ETFM and the ETFM is a only a special case of the approach.

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