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Production Function under Environmental Constraint

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Abstract: In this paper, we mainly discuss the relationship between the growth rate of pollution intensity and economic growth rate. We consider the production function under environmental constraints. The economy that we analyze includes three departments: the final goods sector, the intermidate goods sector and the R&D department. For the aggregate production function of the final goods sector, we use the extended D-S model to consider the amount of human capital, the number of domestic intermediate goods, and the quantity of intermediate goods provided by foreign enterprises in the output of the final goods. After that, we mainly discuss the quantity of domestic intermediate goods. We consider the environmental quality, labor, capital and pollution intensity in the production function of intermediate goods. For R&D department, its R&D output depends on the input of human capital and the stock of technical knowledge. According to the first-order conditions obtained in the intermediate goods sector, we find that the output level of the goods rises with a better natural environment and a higher technical level also makes a higher tax rate of production technology. Then, we consider the representative household utility related to the level of consumption and environmental quality by constructing Hamilton function in order to solve the dynamic optimization problem of the representative family decision-making problem. It is found that the economic growth rate is inversely related to the growth rate of pollution intensity.

Keywords: Environmental constraints; Production function; Hamilton function

1 Introduction

In 1940, Harrod and Domar gave a model of long-term economic growth. Harrod [1] and Domar [2] assume that the input factor capital and labor are irreplaceable. Solow [3] relaxed the hypothesis and put forward neoclassical economic growth model. In 1980, Romer [4] and Lucas [5] created the new growing theory. Subsequently, many scholars put forward more models to supply and consummate the new growing theory.

The production function model is a powerful tool for economic growth analysis. The production function can be divided into four types in form: the Cobb-Douglas production function model, variable elasticity of substitution VES production function model, variable elasticity of substitution CES production function model and Translog production function model. Benhabib and Farmer [6] considered capital and labor into production function based on the Cobb-Douglas production function, Farmer and Guo [7] also considered the interference of productivity. Bovenberg and Smulders [8] considered the capital stock, the private capital stock created by the people, and the environmental resources harvested by the consumer goods sector into production function. Basu and Fernald [9] considered the capital, labor, the purchase of intermediate goods, energy and materials as well as the level of technology into production function. Moretti [10] also considered capital and labor, but divided labor into two parts: skilled labor and unskilled labor. Peng and Bao [11] considered the number of unskilled labor, human capital, the level of innovation or technical knowledge, natural resources into the production function, but the human capital is divided into two parts: put into the final good sector and put into the R&D department. In the production function, only the human capital invested into the final good sector is considered. Zhang and Zuo [12] considered energy input, environmental quality, capital stock and effective labor into production function.

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Manash and Trishita [13] considered the stock of physical capital, the congestion effect adjust effective benefit derived from the public infrastructural input, the stock of health capital into production function. Hsun and Lai [14] considered capital, labor, emission inputs and environmental quality into production function. Simon, Hallonsten and Ziesemer [15] considered level parameter, private physical capital, human capital and unskilledlabour inputs into production function.

On the basis of previous papers, this paper mainly discusses the relationship between the growth rate of pollution intensity and economic growth rate. This paper considers human capital and intermediate goods in the production function of final goods sector, and we divide the intermediate goods into two parts: domestic intermediate goods and foreign intermediate goods. In addition to the environmental quality, labor and capital stock, the production function of intermediate goods in China also consider the pollution intensity. According to the first-order conditions obtained in the intermediate goods sector in this paper, we find that the output level of the goods rises with a better natural environment and a higher technical level also makes a higher tax rate of production technology. Then, we consider the representative household utility related to the level of consumption and environmental quality by constructing Hamilton function in order to solve the dynamic optimization problem of the representative family decision-making problem. Finally, we find that the economic growth rate is inversely related to the growth rate of pollution intensity.

2 Model

We analyze the economy which consists of three sectors: the final goods sector, the intermediate goods sector and the R&D department.

2.1 Final goods sector

Following Romer [16], the total production function of the final goods sector can be written as an extended D-S function form

$$Y = AH_Y^{\alpha}\big[\int_0^M x_i^{\beta}di + \int_0^{M^*} x_i^{*\beta}di\big], \alpha, \beta > 0, \alpha + \beta = 1, \tag{1}$$

where Y is the output of the final goods, $A > 0$ is the technical level parameter, H_Y is the human capital invested into the production sector of the final goods, M is the types of domestic intermediate goods, x_i is the number of the ith domestic intermediate goods, M^* is the types of foreign intermediate goods, x_i^* is the number of the i^*th foreign intermediate goods, α and β are the output elasticity of human capital and intermediate products in the final goods sector, respectively.

Let π_Y denote the profit of the final goods sector, p_Y be the price of the final good, p_{x_i} be the price of the ith domestic intermediate goods, $p_{x_i^*}$ be the price of the i^*th foreign intermediate goods, and p_{H_Y} be the price of the human capital. The maximization problem of the final goods can be expressed as

$$\underset{H_Y, x_i, x_i^*}{Max}\, \pi_Y = p_Y Y - \int_0^M p_{x_i} x_i di - \int_0^{M^*} p_{x_i^*} x_i^* di^* - p_{H_Y} H_Y. \tag{2}$$

In our paper, we only consider the domestic intermediate goods. The first-order condition for this problem yields the demand function of the domestic intermediate goods

$$x_i = \left(\frac{p_{x_i}}{p_Y A H_Y^{\alpha}\beta}\right)^{\frac{1}{\beta - 1}}. \tag{3}$$

2.2 Intermediate goods sector

The production function of the ith domestic intermediate goods is given by

$$x_i = A(N)k_i^{\gamma} l_{x_i}^{\theta} z, \gamma, \theta > 0, \gamma + \theta = 1, z \in [0, 1], A'(N) > 0, \tag{4}$$

where A is an environment-productivity function, N is the environmental quality, k_i and l_{x_i} are the capital and labor used by the ith domestic intermediate goods, respectively, and z is the pollution intensity which is used to Measure the pollution level of existing production technologies. To reflect the positive production externality arising from the environmental quality, Eq.(5) specifies that the output level of the intermediate goods rises with a better natural environment. The profit of the domestic intermediate goods sector π_i can be expressed as

$$\pi_i = p_{x_i} x_i - rk_i - \omega l_{x_i} - st_i, \tag{5}$$

where r is the captial rental rate, ω is the real wage, s is the tax rate paid by production technology, t_i is the pollution input, and st_i is the taxes on pollution control. Here, $z = \lambda t_i$. The accumulation equation of physical capital is

$$\dot{k}_i = x_i - C - \delta k_i, \tag{6}$$

where δ is the depreciation of capital, C is the level of consumption. We regard the environmental quality N as a kind of capital. With the increase of pollution, it will be exhausted, but it also has regeneration capacity. The regeneration rate $\psi > 0$ is set up, and the pollution emission P is the increasing function of the output level and pollution intensity with the form $P(x_i, z) = x_i z^{\psi}$. We find that the greater ψ, the smaller actual pollution emission of qualitative technology. The equation of motion of environmental quality is

$$\dot{N} = -x_i z^{\psi} - \upsilon N. \tag{7}$$

The first-order conditions of intermediate goods sector are as follows

$$P_Y A H_Y^{\alpha} \beta^2 \gamma \frac{x_i^{\beta}}{k_i} = r, \tag{8}$$

$$P_Y A H_Y^{\alpha} \beta^2 \theta \frac{x_i^{\beta}}{l_{x_i}} = \omega, \tag{9}$$

$$P_Y A H_Y^{\alpha} \beta^2 \lambda \frac{x_i^{\beta}}{t_i} = s. \tag{10}$$

Eqs.(8)-(10) indicate that, given the environmental quality and the pollution intensity, immediate good sector equate the marginal revenue of the capital, labor and pollution to their respective marginal cost. Of particular note, the higher level of technology makes the higher tax rate.

2.3 R&D departments

Because of the development of new technology, we need to introduce foreign existing technology in addition to the needs of the homeland existing technology. Therefore, on the basis of Lai at.el [17], we have further improvement of the production function of R&D as follow

$$\dot{H} = \delta H_Y^{\chi} [\varphi H + (1 - \varphi) H^*], \tag{11}$$

where $\dot{H}$ is the increment of the technical knowledge, H is the existing technical knowledge stock in China, H^* is the The existing stock of the foreign technical knowledge, δ is the productivity parameters of R&D department, H_Y is the human capital. χ is the productivity parameter of human capital in R&D department, and φ is the absorptive capacity of Technology.

Here, our technological knowledge increment depends not only on the existing technical knowledge stock in China, but also on the existing stock of the foreign technical knowledge. When the R&D departments develop new technologies, only a part of the domestic technology knowledge increment is absorbed by the new technologies, while the remaining part absorbs the foreign technical knowledge increment.

3 Preference

We set U as a general preference, which is related to the level of consumption C and environmental quality N. The representative family utility is

$$U = \int_0^{\infty} \frac{(CN^{\eta})^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \tag{12}$$

where σ is the coefficient of relative risk aversion, ρ is the subjective time preference rate, and η denotes the weight in terms of the utility attached to the environment.

4 Solution

In view of the above, we find that the decision making problem of representative families based on the consumption level C is a dynamic optimization problem, and the optimal growth problem is as follows

$$max \int_0^\infty \frac{(CN^\eta)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

s.t.

$$\begin{aligned} \dot{k}_i &= x_i - C - \delta k_i, \\ \dot{H} &= \delta H_Y^\chi [\varphi H + (1-\varphi) H^*], \\ \dot{N} &= -x_i z^\psi - \upsilon N. \end{aligned} \tag{13}$$

Based on Eq.(13), we construct the Hamilton function

$$\hat{H} = \frac{(CN^\eta)^{1-\sigma}}{1-\sigma} e^{-\rho t} + \lambda_1 (x_i - C - \delta k_i) + \lambda_2 \delta H_Y^\chi [\varphi H + (1-\varphi) H^*] + \lambda_3 (-x_i z^\psi - \upsilon N). \tag{14}$$

Here, $\lambda_1, \lambda_2, \lambda_3$ is the Hamilton multiplier. The first order conditions of optimality can be obtained

$$C^{-\sigma} = \lambda_1, \tag{15}$$

$$\lambda_1 = \lambda_3 (\psi + 1) z^\psi, \tag{16}$$

$$\lambda_1 (\frac{\gamma x_i}{k_i} - \delta) = \lambda_3 \frac{\gamma x_i z^\psi}{k_i}. \tag{17}$$

In our model, for long-run growth to be feasible and sustainable, the growth rate in the steady state is characterized by

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y}. \tag{18}$$

According to Eqs.(15)-(18), we can get the rate of economic growth

$$g = -\frac{\psi}{\sigma} \frac{\dot{z}}{z}. \tag{19}$$

Here, $\frac{\dot{z}}{z}$ is the growth rate of pollution intensity. So, we find that the economic growth rate is inversely related to the growth rate of pollution intensity.

5 Conclusions

Environmental factors are endogenous factors that affect economic growth, and thus have a certain constraint on the production function. In this paper, we mainly discuss the relationship between the growth rate of pollution intensity and economic growth rate. This paper considers human capital and intermediate goods in the production function of final goods sector, and we divide the intermediate goods into two parts: domestic intermediate goods and foreign intermediate goods. In addition to the environmental quality, labor and capital stock, the production function of intermediate goods in China also consider the pollution intensity. According to the first-order conditions obtained in the intermediate goods sector in this paper, we find that the output level of the goods rises with a better natural environment and a higher technical level also makes a higher tax rate of production technology. Then, we consider the representative household utility related to the level of consumption and environmental quality by constructing Hamilton function in order to solve the dynamic optimization problem of the representative family decision-making problem. Finally, we find that the economic growth rate is inversely related to the growth rate of pollution intensity.

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