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Numerical Simulations of Nonlinear Evolution Equations in Mathematical Physics

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Abstract: Reduced differential transform method (RDTM) is used for constructing the explicit and numerical solutions of a coupled nonlinear evolution equations. The approximate solutions of the equations is calculated in the form of a series with easily computable components. Comparing the methodology with some other known techniques shows that the present approach is effective and powerful. Two models of special interst from mathematical physics are discussed to illustrate the effectiveness the proposed method.

Keywords: Reduced differential transform method;Adomian decomposition method; Variational iteration method; Hirota-Satsuma coupled KdV system

1 Introduction

Since the world around us is inherently onlinear,nonlinear evolution equations are widely used to describe complex phenomena in various fields of sciences,especially in physics such as,plasma physics,fluid mechanics,optical .fibers,solid state physics,nonlinear optics and so on. One of the most exciting advances of nonlinear science and theoretical physics has been a development of methods to look for exact solutions for nonlinear partial differential equations. A search of directly seeking for exactly solutions of nonlinear equations [1-10] has been more interest in recent years because of the availability of symbloic computation Mathematica or Maple.These computer systems allow us to perform some complicated and tedious algebraic and differential calculations on a computer.

Recently much attention has been devoted to the numerical methods which do not require discretization of space-time variables or linearization of the nonlinear equations, among which are the Adomian decomposition method (ADM) [11] and the variational iteration method, which is suggested by Ji- Huan He [12], and the Homotopy perturbation method (HPM) [13].The numerical methods can provide approximate solutions rather than analytic solutions of the problems.The differential transform method [14−17] is a numerical method for solving differential equations or system of the differential equations.The reduced Differential transform method for solving differential equations has recently renewed interest due to many important applications [18 − 21].

The paper has been organized as follows.**In Section 2**,we begin by introducing the definition and the basic mathematical operations of reduced differential transform method .**In Section 3**,two models are chosen to illustrate the validity of the reduced differential Transform Method (RDTM) .Finally,discussions and conclusions are presented in **Section 4**.

2 Methodology

In what follows Summarize the setps reports in Reduced differentional transforme method [18-21].Based on the properties of differential transform,function $u(x,t)$ can be represented as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k, \tag{1}$$

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where $U_k(x)$ is called t-dimensional spectrum function of $u(x,t)$.

The basic definitions of reduced differentional transform (RDTM) method $[18-21]$ are intoduced as

Definition 1 *.If function $u(x,t)$ is analytic and differentiated continuously with respect to t and x in the domian of interst,then let*

$$U_k(x) = \frac{1}{k}[[\frac{d^k u(x,t)}{dt^k}]]|_{t=0}, \tag{2}$$

where the t dimensional spectrum function $u_k(x)$ is the transformed function.In this paper,the lowercase $u(x,t)$ represent the original function while the uppercase $U_k(x)$ stand for the transformed function.

The differential inverse transform of $U_k(x)$ is defined as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k \tag{3}$$

Then combining Eqs.(1) and (3) we write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k}[\frac{d^k u(x,t)}{dt^k}]]|_{t=0} t^k \tag{4}$$

With the aid above definitions ,it can be found that the concept of the reduced differential transform is derived from the power series expansion.

For the purpose of illustration of the methodlology to the proposed method,we write nonlinear equation in the standered operator form as follows

$$L(u(x,t)) + R(u(x,t)) + N(u(x,t)) = g(x,t), \tag{5}$$

with initial condition

$$u(x,0) = f(x), \tag{6}$$

where $L = \frac{\partial}{\partial t}$,$R$ is a linear operator which has partial derivatives ,$Nu(x,t)$ is a nonlinear term and $g(x,t)$ is an inhomogeneous term.

According to the RDTM and Table[1],we can constructing the following iteration formulas

$$(k+1)U_{k+1}(x) = G_k(x) - RU_k(x) - NU_k(x), \tag{7}$$

where $U_k(x)$,$RU_k(x)$,$NU_k(x)$ and $G_k(x)$ are transformation of the functions $Lu(x,t)$,$Ru(x,t)$,$Nu(x,t)$ and $g(x,t)$ respectively.From the initial condition,we write

$$U_0(x) = f(x), \tag{8}$$

Substituting Eq.(8) and (7) and by a straight forward iterative formula,we get the following $U_k(x)$ values.Then the inverse transformation of the set of values $U_k(x)^n{}_{k=0}^n$ gives approximation solution as,

$$u^*(x,t) = \sum_{k=0}^{n} U_k(x) t^k \tag{9}$$

where n is order of approximation solution.Therefore,the exact solution of problem is given by

$$u(x,t) = lim_{n\longrightarrow\infty} u_n^*(x,t), i = 1,2,....,n \tag{10}$$

3 Applications

To illustrate the effectiveness and the advantages of the proposed method,three models of nonlinear evolution equations arising in mathematical physics are chosen,namely,the generalized Hirota-Satsuma coupled KdV system and coupled system of diffusion-reaction equation.

3.1 Example[1].Generalized Hirota-Satsuma coupled KdV system

Firstly,we consider the Hirota-Satsuma coupled KdV system [6]

$$u_t - \frac{1}{2}u_{xxx} + 3uu_x - (vw)_x,$$

$$v_t - \frac{1}{2}v_{xxx} - 3uv_x,$$

$$w_t + w_{xxx} - 3uw_x, \tag{11}$$

with initial conditions

$$u_0(x) = \frac{1}{3}(\beta - 2k^2) + 2k^2 tanh^2(kx),$$

$$v_0(x) = \frac{-4k^2 c_0(\beta + k^2)}{3c_1^2} + \frac{4k^2(\beta + k^2)}{3c_1} tanh(kx),$$

$$w_0(x) = c_0 + c_1 tanh(kx), \tag{12}$$

where k,β,c_0 and c_1 is constant.For the solution procedure,we first take the differential transform of Eqs.(11) by the use of Table[1] and have the following equations

$$(k+1)U_{k+1}(x) = \frac{1}{2}\frac{\partial^3}{\partial x^3}U_k(x) - 3A_k + 3B_k,$$

$$(k+1)V_{k+1}(x) = \frac{\partial^3}{\partial x^3}V_k(x) + 3C_k(x),$$

$$(k+1)W_{k+1}(x) = -\frac{\partial^3}{\partial x^3}W_k(x) - 3D_k(x), \tag{13}$$

where the t-dimensional spectrum function $U_k(x)$,$V_k(x)$ and $W_k(x)$ are the transformed function, $A_k(x)$,$B_k(x)$,$C_k(x)$ and $D_k(x)$ are transformed form of the nonlinear terms.From the initial conditions (12),we write

$$U_0(x) = \frac{1}{3}(\beta - 2k^2) + 2k^2 tanh^2(kx),$$

$$V_0(x) = \frac{-4k^2 c_0(\beta + k^2)}{3c_1^2} + \frac{4k^2(\beta + k^2)}{3c_1} tanh(kx),$$

$$W_0(x) = c_0 + c_1 tanh(kx) \tag{14}$$

For the easy to follows of the reader,the first five nonlinear term are

$$A_0 = U_0U_{0x}, A_1 = U_1U_{0x} + U_0U_{1x}, A_2 = U_2U_{0x} + U_1U_{1x} + U_0U_{2x}, A_3 = U_3U_{0x} + U_2U_{1x} + U_1U_{2x} + U_0U_{3x},$$

$$C_0 = U_0V_{0x}, C_1 = U_1V_{0x} + U_0V_{1x}, C_2 = U_2V_{0x} + U_1V_{1x} + U_0V_{2x}, C_3 = U_3V_{0x} + U_2V_{1x} + U_1V_{2x} + U_0V_{3x},$$

$$B_0 = (V_0W_0)_x, B_1 = (V_1W_1)_x, B_2 = (V_2W_2)_x, B_3 = (V_3W_3)_x, D_0 = U_0W_{0x}, D_1 = U_1W_{0x} + U_0W_{1x},$$

$$D_2 = U_2W_{0x} + U_1W_{1x} + U_0W_{2x}, D_3 = U_3W_{0x} + U_2W_{1x} + U_1W_{2x} + U_0W_{3x}, \tag{15}$$

Then substituting Eqs.(14),(15) into Eqs.(13),we can directly evaluated the values of $U_k(x)$,$V_k(x)$ and $W_k(x)$.Explicit form for $U_k(x)$,$V_k(x)$ and $W_k(x)$ are obtained for 5 order of approximation.For simplicity should be omitte here.

Then,the inverse transform of of the set of values $U_k(x)$,$V_k(x)$ and $W_k(x)$ gives five term approximate solutions as

$$u^*(x,t) = \sum_{k=0}^{5} U_k(x)t^k, \tag{16}$$

$$v^*(x,t) = \sum_{k=0}^{5} V_k(x)t^k, \tag{17}$$

$$w^*(x,t) = \sum_{k=0}^{5} W_k(x)t^k \tag{18}$$

Therefore,the approximate solutions of the problem are given by

$$u(x,t) = lim_{n\longrightarrow\infty} u_n^*(x,t), i = 1,2,....,n \tag{19}$$

$$v(x,t) = lim_{n\longrightarrow\infty} v_n^*(x,t), i = 1,2,....,n \tag{20}$$

$$w(x,t) = lim_{n\longrightarrow\infty} w_n^*(x,t), i = 1,2,....,n \tag{21}$$

The numerical behaviour of the approximate solutions of $u(x,t)$,$v(x,t)$ and $w(x,t)$ obtained by RDTM with the exact solutions by Variational iteration method [6] are shown graphically in Figs.[(1.(a-f)] for a different values of time t with a fixed values of $k = \beta = 1 = c_0 = c_1 = 1$,which proofs the two solutions are quite good.It is to be noted that the exact solutions of $u(x,t)$,$v(x,t)$ and $w(x,t)$ can be written as

$$u(x,t) = \frac{1}{3}(\beta - 2k^2) + 2k^2 tanh^2(k(x+\beta t)),$$

$$v(x,t) = \frac{-4k^2 c_0(\beta + k^2)}{3c_1^2} + \frac{4k^2(\beta + k^2)}{3c_1} tanh(k(x+\beta t)),$$

$$w(x,t) = c_0 + c_1 tanh(k(x+\beta t)), \tag{22}$$

where β is constant.

3.2 Example (2)

The coupled system of diffusion-reaction equation in this case we shall deal with coupled system of the diffusion-reaction equation [22]

$$\frac{\partial u(x,t)}{\partial t} = u(1-u-v) + u_{xx}, t > 0 \tag{23}$$

$$\frac{\partial v(x,t)}{\partial t} = v_{xx} - uv, \tag{24}$$

with initial conditions

$$u(x,0) = \frac{e^{kx}}{[1+e^{0.5kx}]^2}, \tag{25}$$

$$v(x,0) = \frac{1}{[1+e^{0.5kx}]}, \tag{26}$$

where k is constant.The above for two coupled nonlinear equations of reaxtio diffusion type arising in chemical reactions or ecology, and other fields of physics.This model can be used to describe the ecological process of two specimens.The differential transform of Eqs.(23) and (24) and initial conditions (25) and (26) are readily found to be

$$(k+1)U_{k+1}(x) = -U_k(x) - C_k - B_k + \frac{\partial^2}{\partial x^2}U_k(x), \tag{27}$$

$$(k+1)V_{k+1}(x) = \frac{\partial^2}{\partial x^2}V_k(x) - B_k(x), \tag{28}$$

where the t-dimensional spectrum function $U_k(x)$ and $V_k(x)$ are the transformed function, $A_k(x)$,$B_k(x)$ and $C_k(x)$ are transformed form of the nonlinear terms.From the initial conditions,we write

$$U_0(x) = \frac{e^{kx}}{[1+e^{0.5kx}]^2}, \tag{29}$$

$$V_0(x) = \frac{1}{[1+e^{0.5kx}]} \tag{30}$$

The first three component of the nonlinear term can be written as follows

$$A_0 = U_0U_{0x}, A_1 = U_1U_{0x} + U_0U_{1x}, A_2 = U_2U_{0x} + U_1U_{1x} + U_0U_{2x},$$

$$A_3 = U_3U_{0x} + U_2U_{1x} + U_1U_{2x} + U_0U_{3x}, C_0 = U_0V_{0x}, C_1 = U_1V_{0x} + U_0U_{1x},$$

$$C_3 = U_3V_{0x} + U_2V_{1x} + U_1V_{2x} + U_0V_{3x}, C_2 = U_2V_{0x} + U_1V_{1x} + U_0V_{2x},$$

$$B_0 = V_0V_{0x}, B_1 = V_1V_{0x} + V_0V_{1x}, B_2 = V_2V_{0x} + V_1V_{1x} + V_0V_{2x}, B_3 = V_3V_{0x} + V_2V_{1x} + V_1V_{2x} + V_0V_{3x} \tag{31}$$

By means of Eqs.(29-31) into Eqs.(27) and (28),the values of $U_k(x)$ and $V_k(x)$ are obtained.Explicit form for $U_k(x)$ and $V_k(x)$ obtained by RDTM are obtained for 5 order of approximation.For simplicity should be omite here.

The inverse transform of of the set of values of $U_k(x)$ and $V_k(x)$ gives five term approximate solutions as follows

$$u^*(x,t) = \sum_{k=0}^{5} U_k(x)t^k, \tag{32}$$

$$v^*(x,t) = \sum_{k=0}^{5} V_k(x)t^k, \tag{33}$$

Therefore,the approximate solutions of the problem are given by

$$u(x,t) = lim_{n\longrightarrow\infty} u_n^*(x,t), i = 1,2,....,n \tag{34}$$

$$v(x,t) = lim_{n\longrightarrow\infty} v_n^*(x,t), i = 1,2,....,n \tag{35}$$

It is to be noted that,the exact solutions of $u(x,t)$ and $v(x,t)$ can be written as [22]

$$u(x,t) = \frac{e^{k(x+ct)}}{(1+e^{0.5k(x+ct)})^2}, \tag{36}$$

$$v(x,t) = \frac{1}{(1+e^{0.5k(x+ct)})}, \tag{37}$$

where k and c are constants.Figs.[2.(a-d)] shows the comparison of the RDTM approximation solution of order six and exact solutions obtained by Adomian decomposition method [22].From the figures ,it is clearly seen that the RDTM approximation and the exact solutions are quite good.

3.3 Conclusions and Discussion

In the present work,The proposed method (RDTM) has been successfully used for constructing the approximate and numerical solutions of two nonlinear evolution equations arising in mathematical physics, namely, generalized Hirota Satsuma coupled KdV system and diffusion reaction equation" The RDTM was clearly very efficient and powerful technique in finding the solutions of the proposed nonlinear equations. It is clear that this method avoids linearization and biologically unrealistic assumptions,and .provides an efficient numerical solution.

It is worth noting that the new solutions obtained by the proposed method confirm the correctness of those obtained by other methods.The method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in mathematical physics.This is our task in future work.

Table 1: Reduced Differentional Transformation

$FunctionalFrom$	$TransformedForm$
$u(x,t)$	$U_k = \frac{1}{k\vert}[\frac{\partial u(x,t)}{dt^k}]_{t=0}$
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \alpha u(x,t)$	$W_k(x) = \alpha U_k(x)$,α is a constant
$w(x,t) = x^m t^n$	$W_k(x) = x^m \delta(k-n)$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U(k-n)$
$w(x,t) = u(x,t)v(x,t)$	$W_k(x) = \sum_{r=0}^{k} U_r(x) V_{k-r}(x)$
$w(x,t) = \frac{\partial^r}{\partial t^r} u(x,t)$	$W_k(x) = (k+1)....(k+r)U_{k+1}(x) = \frac{(k+r)!}{k!} U_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x} u(x,t)$	$W_k(x) = \frac{\partial}{\partial x} U_k(x)$

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References

[1] S.A.El-Wakil,M.A.Abdou.New exact travelling wave solutions using Modified extended tanh function method. *Chaos,Solitons and Fractals*, 31(2007):840-848.

[2] S.A.El-Wakil,M.A.Abdou.Modified extended tanh function method for solving nonlinear partial differential equations.*Chaos,Solitons and Fractals*, 31(2007):1256-1264.

[3] M.J.Ablowitz,P.A.Clarkson.Solitons,Nonlinear Evolution Equations and Inverse Scattering Transform.Cambridge University Press,Cambridge,1990.

[4] S.A.El-Wakil,M.A.Abdou,A.Elhanbaly.New solitons and periodic wave solutions for nonlinear evolution equations.*Phys.Lett.A*, 353(2006):40-47.

[5] M.A.Abdou and A.Elhanbaly.Construction of periodic and solitary wave solutions by the extended Jacobi elliptic function expansion method.*Commun.in Non.Sci. and Numer.Simu.*12 (2007):1229-1241.

[6] M A Abdou.On the variational iteration method.*Phys.Lett.A*,366(2007):61-68.

[7] M A abdou,A A Soliman.New applications of Variational iteration method.*Physica D*, 211(2005):1-8.

[8] Ji-Huan He.variational iteration method:a kind of nonlinear analytical technique:some examples.*Interna.J.of Non-Linear Mech.*, 34(4)(1999):699-708.

[9] M.A.Abdou.The extended F-expansion method and its application for nonlinear evolution equations.*Chaos Solitons and Fractals*, 31(2007):95-104.

[10] M A Abdou,S Zhang.New periodic wave solutions via extended mapping method.*Communcation in Nonlinear Science and Numerical Simulation*,14(2009):2-11.

[11] G.Adomian.Solving Frontier Problems of Physics: The Decomposition Method,Kluwer,1994.

[12] J.H.He.Variational iteration method-a kind of non-linear analytical technique: Some examples. *International Journal of Non-Linear Mechanics*, 34(1999):699-708.

[13] He J.H..Homotopy perturbation method: A new nonlinear technique. *Applied Mathematics and Computation*, 135(2003):73-79.

[14] G.E.Pukhov.Differential transformations and mathematical modelling of physical processes,Kiev,1986.
[15] J.K.Zhou,Differential transform and its Applications for Electrical Circuits,Huazhong University Press, Wuhan, China,1986.
[16] G.E.Pukhov,Differential transformations and mathematical modelling of physical processes,Kiev,1986.
[17] J.K.Zhou,Differential transform and its Applications for Electrical Circuits,Huazhong University Press, Wuhan, China,1986.
[18] Y.Keskin, Ph.D.Thesis,University of Selcuk(to appear).
[19] Keskin Y.,Oturanc G.,Reduced Differential Transform Method for Partial Differential Equations.*International Journal of Nonlinear Sciences and Numerical Simulation*,10(2009):741-749.
[20] Keskin Y.,Oturanc G.,Numerical simulations of systems of PDEs by reduced differential transform method, Communications in Nonlinear Science and Numerical Simulations(In Press).
[21] Keskin Y.,Oturanc G..Reduced differential transform method for fractional parial differential equations. *Non.Sci.Lett.A*, 1(2010):61-72.
[22] S A El-Wakil and M A Abdou,New applications of Adomian decomposition method. *Chaos Solitons & Fractals*,33(2007):513-522.

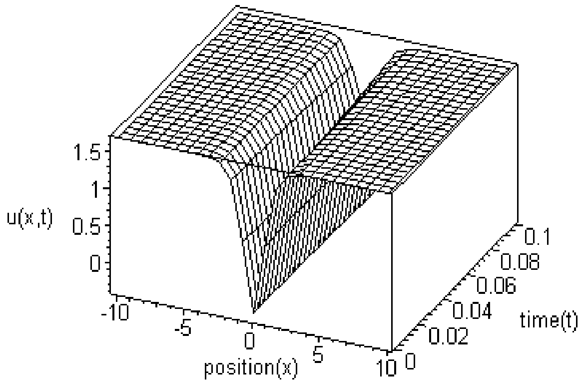

Fig.[1.a].Numerical solution of u(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=1 and β=1 for a different values of t

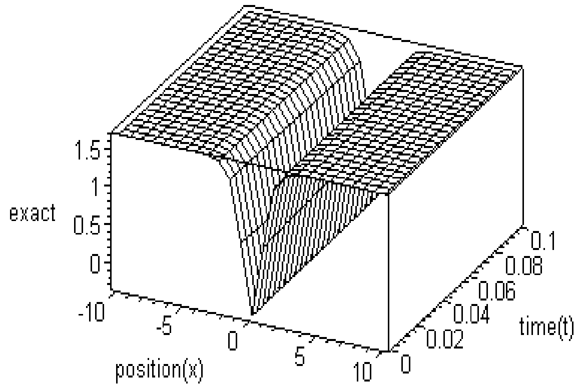

Fig.[1.b].Exact solution of u(x,t) Eq.[22] obtained **by Variational iteration method** with a fixed values of k=1 and β=1 for a different values of t

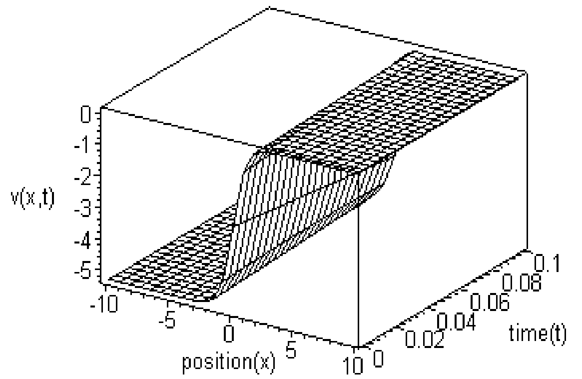

Fig.[1.c].Numerical solution of v(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=1 and β=1 for a different values of t

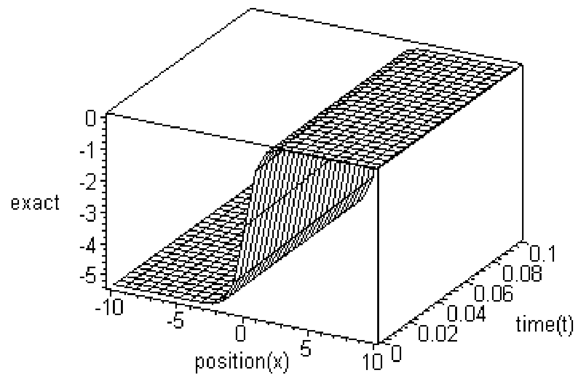

Fig.[1.d]. Exact solution of v(x,t) Eq.[22] obtained **by Variational iteration method** with a fixed values of k=1 and β=1 for a different values of t

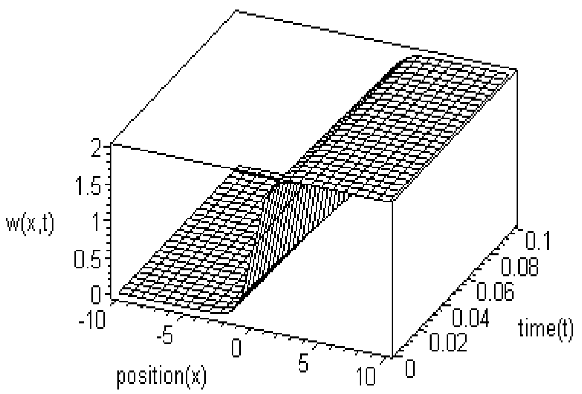

Fig.[1.e].Numerical solution of w(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=1 and β=1 for a different values of t

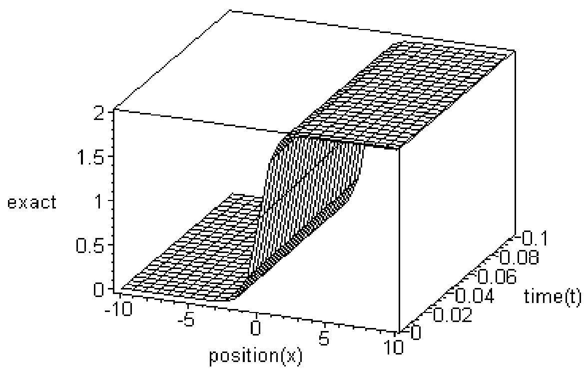

Fig.[1.f]. Exact solution of w(x,t) Eq.[22] obtained **by Variational iteration method** with a fixed values of k=1 and β=1 for a different values of t

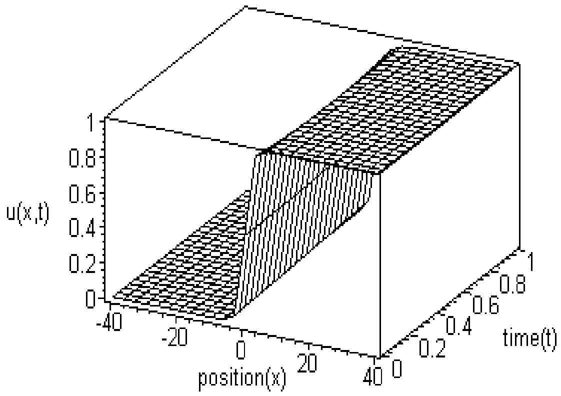

Fig.[2.a].Numerical solution of u(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=1 and c=1 for a different values of t

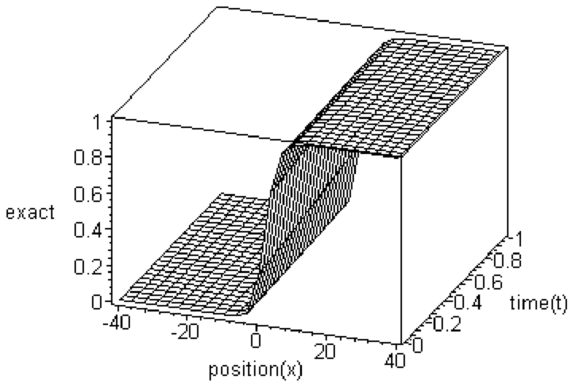

Fig.[2.b].Exact solution of u(x,t) Eq.[36] obtained by **Adomian decomposition method** with a fixed values of k=1 and c=1 for a different values of t

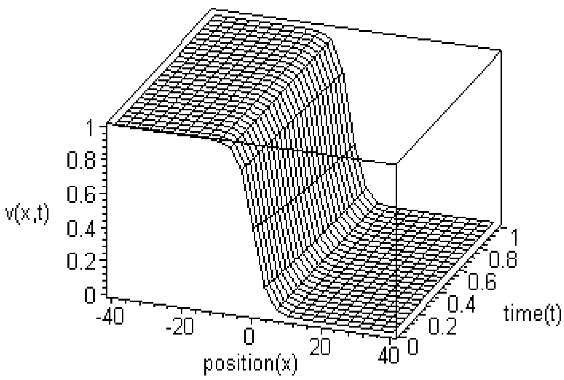

Fig.[2.c].Numerical solution of v(x,t) obtained by **Reduced transform method** (DRTM) with a fixed values of k=1 and c=1 for a different values of t

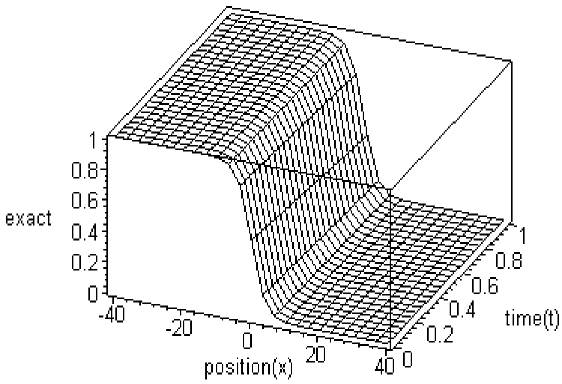

Fig.[2.d].Exact solution of v(x,t) Eq.[37] obtained by **Adomian decomposition method** (ADM) with a fixed values of k=1 and c=1 for a different values of t