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Estimation of Viscosity and Controller Strengthening in Boundary Stabilization of Korteweg-de Vries-Burgers' Equation

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Abstract: In this paper, we propose a fortified boundary control law and an adaptation law for KdV-Burgers' equation with unknown viscosity, where no a prior knowledge of a lower bound on viscosity is needed. Using the Lyapunov method, we prove that the closed loop system, including the parameter estimator as a dynamic component, is globally H^1 stable and well posed.

Keywords: KdV-Burgers' equation; boundary control; stabilization

1 Introduction

Recently, extensive attention has been paid to the problem of controllability and stabilization for the Burgers' and KdV-Burgers' equation [1][2][3][4][5][6][7]. In this paper, we are concerned with the problem of the boundary control of the KdV-Burgers' equation

$$\begin{cases} u_t - \varepsilon u_{xx} + \delta u_{xxx} + uu_x = 0, & 0 < x < 1, t > 0, \\ u_x(0,t) = u_x(1,t) = 0, & t > 0, \\ u_{xx}(0,t) = \varphi_0, & t > 0, \\ u_{xx}(1,t) = \varphi_1, & t > 0, \\ u(x,0) = u^0(x). & 0 < x < 1. \end{cases} \tag{1.1}$$

where ε and δ are positive parameters. In this problem, φ_0 and φ_1 are control inputs and $u^0(x)$ is an initial state in an appropriate function space.

Our objective is to find feedback functions φ_0 and φ_1 such that the equilibrium $u(x) \equiv 0$ is globally stable and $u(x,t) \longrightarrow 0$ as $t \longrightarrow \infty$ for all $x \in (0,1)$. Since ε and δ take arbitrary positive values, this objective cannot be achieved by static feedback, hence, we need to design an adaptive controller which incorporates a parameter estimator as a dynamic component of the control law.

The paper is organized as follows: main result is given in section 2. In section 3, using the Lyapunov method, we prove that the closed-loop system, including the parameter estimator as a dynamic component, is globally H^1 stable and well posed.

Throughout this paper, we denote by $H^s(0,1)$ the usual Sobolev space [8] for any $s \in R$. Set

$$H_0^1(0,1) = \{\varphi \in H^1(0,1) : \varphi(0) = 0\} \tag{1.2}$$

$$H_0^2(0,1) = \{\varphi \in H^2(0,1) : \varphi_x(0) = \varphi_x(1) = 0\} \tag{1.3}$$

Let X be a Banach space and $T > 0$. We denote by $C^n([0,\ T];\ X)$ the space of n times continuously differentiable functions defined on $[0,T]$ with values in X , and write $C([0,T];X)$ for $C^0([0,T];X)$. The norm on $L^2(0,1)$ is denoted by$\|\cdot\|$.

In what follows, for simplicity, we omit the indication of the varying rang of x and t in equations and we understand that x varies from 0 to 1 and t from 0 to ∞.

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2 Main result

For notational convenience,we denote

$$\omega_0 = u|_{x=0}, \quad \omega_1 = u|_{x=1} \tag{2.1}$$

and

$$\widetilde{\eta}_0 = \eta_0 - \frac{1}{6\delta}, \quad \widetilde{\eta}_1 = \eta_1 - \frac{1}{6\delta} \tag{2.2}$$

where η_0 and η_1 shall be used as estimates of $\frac{1}{6\delta}$.

We define the energy function by

$$E = \int_0^1 u^2 dx \tag{2.3}$$

and the Lyapunov function V as the energy function E augmented by the estimation error $\widetilde{\eta}_0^2 + \widetilde{\eta}_1^2$, that is

$$V = E + \frac{\delta}{\gamma}(\widetilde{\eta}_0^2 + \widetilde{\eta}_1^2) \tag{2.4}$$

where γ is a positive constant. Let us calculate the time derivative of V. Using the equation (1.1) and integrating by parts, we obtain

$$\dot{V} \le -2\varepsilon \int_0^1 u_x^2 dx - 2\delta\omega_1\varphi_1 + 2\delta\omega_0\varphi_0 + \frac{1}{3}(\omega_0^2 + \omega_0^4 + \omega_1^2 + \omega_1^4) + \frac{2\delta}{\gamma}(\eta_0 - \frac{1}{6\delta})\dot{\eta}_0 + \frac{2\delta}{\gamma}(\eta_1 - \frac{1}{6\delta})\dot{\eta}_1$$

$$= -2\varepsilon \int_0^1 u_x^2 dx - 2\delta\omega_1[\varphi_1 - \eta_1(\omega_1^3 + \omega_1) + 2\delta\omega_0[\varphi_0 + \eta_0(\omega_0^2 + \omega_0)$$

$$+2\delta(\frac{1}{6\delta} - \eta_0)(\omega_0^4 + \omega_0^4 - \frac{\dot{\eta}_0}{\gamma}) + 2\delta(\frac{1}{6\delta} - \eta_1)(\omega_1^4 + \omega_1^2 - \frac{\dot{\eta}_1}{\gamma}) \tag{2.5}$$

This leads us to select the adaptive feedback control

$$\dot{\eta}_0 = \gamma(\omega_0^2 + \omega_0^4) \tag{2.6}$$

$$\dot{\eta}_1 = \gamma(\omega_1^2 + \omega_1^4) \tag{2.7}$$

$$\varphi_0 = -k(\omega_0 + \omega_0^9) - \eta_0(\omega_0 + \omega_0^3) \tag{2.8}$$

$$\varphi_1 = k(\omega_1 + \omega_1^9) + \eta_1(\omega_1 + \omega_1^3) \tag{2.9}$$

where k is any positive constant, then we get

$$\dot{V} \le -2\varepsilon \int_0^1 u_x^2 dx - 2\delta k(\omega_0^2 + \omega_0^{10} + \omega_1^2 + \omega_1^{10}) \tag{2.10}$$

which implies the L^2 stability. Furthermore, the closed-loop system

$$\begin{cases} u_t - \varepsilon u_{xx} + \delta u_{xxx} + u u_x = 0, \\ u_x(0,t) = u_x(1,t) = 0, \\ u_{xx}(0,t) = -k(\omega_0 + \omega_0^9) - \eta_0(\omega_0 + \omega_0^3), \\ u_{xx}(1,t) = k(\omega_1 + \omega_1^9) + \eta_1(\omega_1 + \omega_1^3), \\ \dot{\eta}_0 = \gamma(\omega_0^2 + \omega_0^4), \\ \dot{\eta}_1 = \gamma(\omega_1^2 + \omega_1^4), \\ u(x,0) = u^0(x),\ \eta_0(0) = \eta_0^0,\ \eta_1(0) = \eta_1^0 \end{cases} \tag{2.11}$$

satisfies the following theorem.

Theorem 1 *Suppose that $k > 0$ and $\gamma > 0$. Then, for the initial conditions $u^0 \in H_0^2(0,1)\cap H^3(0,1)$ and $\eta_0^0 \geq 0, \eta_1^0 \geq 0$, the problem (2.11) has a unique global classical solution $(u, \eta_0,\ \eta_1)$ with*

$$u \in C([0,\ \infty);\ H_0^2(0,\ 1) \cap H^3(0,\ 1)) \cap C^1([0,\ \infty); L^2(0,1)),$$

$$\eta_0,\ \eta_1 \in C^1([0,\ \infty); R)$$

Furthermore
1. the equilibrium $u(x) \equiv 0, \widetilde{\eta}_0 = \widetilde{\eta}_1 = 0$ is globally L^2-stable, i.e.

$$\| u(t) \|^2 + \frac{\delta}{\gamma}\tilde{\eta}_0(t)^2 + \frac{\delta}{\gamma}\tilde{\eta}_1(t)^2 \leq \| u^0 \|^2 + \frac{\delta}{\gamma}\tilde{\eta}_0(0)^2 + \frac{\delta}{\gamma}\tilde{\eta}_1(0)^2 \tag{2.12}$$

for all $t \geq 0$, and u is regulated to zero in L^2 sense:

$$\lim_{t\to\infty} \| u(t) \| = 0 \tag{2.13}$$

2. the equilibrium $u(x) \equiv 0, \widetilde{\eta}_0 = \widetilde{\eta}_1 = 0$ is globally H^1-stable, i.e.

$$\begin{aligned} \| u(t) \|_{H^1}^2 \leq C(\| u^0 \|_{H^1}^2 + \| u^0 \|_{H^1}^{18} + \tilde{\eta}_0(0)^2 + \tilde{\eta}_0(0)^4 + \tilde{\eta}_1(0)^2 + \tilde{\eta}_1(0)^4) \\ \times exp(C(\| u^0 \|^2 + \tilde{\eta}_0(0)^2 + \tilde{\eta}_1(0)^2)) \end{aligned} \tag{2.14}$$

for all $t \geq 0$, where $C = C(k,\ \varepsilon,\ \delta,\ \gamma)$ is a positive constant, and u is regulated to zero for all $x \in (0,1)$:

$$\lim_{t\to\infty} \max_{x\in[0,1]} | u(x,t) | = \lim_{t\to\infty} \| u(t) \|_{H^s} = 0 \tag{2.15}$$

for any $s < 3$.

3 Proof

To prove the regulation result (2.13) independent of (2.15), we establish the following alternative to Barbalat's lemma[9] .

Lemma 2 *Suppose that the function $f(t)$ defined on $[0,\infty)$ satisfies the following conditions:*
(i) $f(t) \geq 0$ for all $t \in [0,\infty)$,
(ii) $f(t)$ is differentiable on $[0,\infty)$ and there exists a constant M such that

$$f'(t) \leq M,\ \ \forall t \geq 0,$$

(iii) $\int_0^\infty f(t)dt < \infty$ Then we have

$$\lim_{t\to\infty} f(t) = 0$$

Proof. See [1]

We are now in the position to prove our main result.
Step1: Well-posedness. By (2.11), we first have

$$\eta_0 = \eta_0^0 + \gamma\int_0^1 [\omega_0(s)^4 + \omega_0(s)^2]ds \tag{3.1}$$

$$\eta_1 = \eta_1^0 + \gamma\int_0^1 [\omega_1(s)^4 + \omega_1(s)^2]ds \tag{3.2}$$

Substituting η_0, η_1 into the boundary condition of (2.11), it becomes the following problem

$$\begin{cases} u_t - \varepsilon u_{xx} + \delta u_{xxx} + uu_x = 0, \\ u_x(0,t) = u_x(1,t) = 0, \\ u_{xx}(0,t) = -g(\omega_0, \eta_0^0)(t), \\ u_{xx}(1,t) = g(\omega_1, \eta_1^0)(t), \\ u(x,0) = u^0(x). \end{cases} \tag{3.3}$$

where

$$g(\omega,r)(t)=k[\omega(t)+\omega(t)^9]+(r+\gamma\int_0^1[\omega(s)^4+\omega(s)^2]ds)[\omega(t)^3+\omega(t)]$$

for any function $\omega=\omega(t)$ and $r\in R$. Thus , it suffices to prove that problem (3.3) has an unique solution.

Let the theta function be defined by [10]

$$\theta(x,t)=\frac{1}{\sqrt{4\pi\delta t}}\sum_{n=-\infty}^{+\infty}exp(-\frac{(x+2n)^2}{4\delta t})=\frac{1}{2}+\sum_{n=1}^{+\infty}cos(n\pi x)e^{-\delta n^2\pi^2 t}$$

It is well known that the problem (3.3) is equivalent to the following integral equation

$$u(x,t)=G(u(x,t))=\int_0^1[\theta(x-y,t)+\theta(x+y,t)]u^0(y)dy$$

$$+\frac{1}{2}\int_0^1\int_0^1[\theta_x(x+y,t-\tau)-\theta_x(x-y,t-\tau)]u(y,\tau)^2dyd\tau$$

$$+\int_0^1\theta(x-1,t-\tau)[2g(\omega_1,\eta_1^0)(\tau)-\omega_1(\tau)^2]d\tau$$

$$+\int_0^1\theta(x,t-\tau)[2g(\omega_0,\eta_0^0)(\tau)+\omega_0(\tau)^2]d\tau \tag{3.4}$$

We introduce the function space $C=C([0,T];C[0,1])$ with the norm

$$\| u \|_C=\max_{0\le x\le 1,\ 0\le t\le T}| u(x,t) |$$

In order to prove that (3.3) has an unique solution solution, it suffices to prove that the mapping G defined by (3.4) has an unique fixed point in C. By using the standard properties of the theta function[10], we can prove that G is contractive if T is small enough. By Banach contraction fixed point theorem, G has an unique fixed point $u\in C([0,T];C[0,1])$ if T is small enough. Furthermore, in view of the following prior estimates which we are going to establish, we conclude that the local solution is an actually global classical solution.

Step2: Stability Estimate(2.12): By (2.10), we obtain

$$\| u(t) \|^2+\frac{\delta}{\gamma}\tilde\eta_0(t)^2+\frac{\delta}{\gamma}\tilde\eta_1(t)^2\le\| u^0 \|^2+\frac{\delta}{\gamma}\tilde\eta_0(0)^2+\frac{\delta}{\gamma}\tilde\eta_1(0)^2 \tag{3.5}$$

and

$$2\varepsilon\int_0^\infty\int_0^1u_x^2dxdt+2\delta k\int_0^\infty[\omega_0(t)^2+\omega_0(t)^{10}+\omega_1(t)^2+\omega_1(t)^{10}]dt$$

$$\le\| u^0 \|^2+\frac{\delta}{\gamma}\tilde\eta_0(0)^2+\frac{\delta}{\gamma}\tilde\eta_1(0)^2 \tag{3.6}$$

Hence (2.12) is established.

Step3: Regulation(2.13). To prove (2.13), it suffices to verify conditions (ii) and (iii) of Lemma 3.1. By (3.6), we obtain

$$\int_0^\infty\| u(t) \|^2\,dt\le C(\| u_0 \|^2+\tilde\eta_0(0)^2+\tilde\eta_1(0)^2)<\infty \tag{3.7}$$

Here and in the sequel, $C=C(\varepsilon,\delta,\gamma,k)$ denotes various positive constants.

On the other hand , we have

$$\frac{d}{dt}(\| u(t) \|^2)=2\int_0^1u(\varepsilon u_{xx}-\delta u_{xxx}-uu_x)dx$$

$$\le-2\delta k(\omega_0^2+\omega_0^{10}+\omega_1^2+\omega_1^{10})-2\delta[\eta_0(\omega_0^2+\omega_0^4)+\eta_1(\omega_1^2+\omega_1^4)]+\frac{1}{3}(\omega_0^2+\omega_0^4+\omega_1^2+\omega_1^4)$$

$$\leq -2\delta k(\omega_0^2+\omega_0^{10}+\omega_1^2+\omega_1^{10})+\delta k(\omega_0^2+\omega_0^{10}+\omega_1^2+\omega_1^{10})+C(\delta,k)(\eta_0^2+\eta_1^2)$$

$$+\delta k(\omega_0^2+\omega_0^{10}+\omega_1^2+\omega_1^{10})+C(\delta,k)$$

$$\leq C(\delta,k)(1+\eta_0^2+\eta_1^2) \tag{3.8}$$

which combined with (3.5), implies condition (ii) of Lemma 3.1.

Step4: Stability Estimation (2.14). We first note that the solution u of (3.3) is infinitely differentiable in $(0,1)\times(0,T)$ since it satisfies the integral equation (3.4) and the theta function θ is infinitely differentiable. This guarantees that the calculations performed below are valid.

Using the equation (2.11), we obtain

$$\frac{d}{dt}\int_0^1 u_x^2dx=-2\int_0^1 u_{xx}(\varepsilon u_{xx}-\delta u_{xxx}-uu_x)dx$$

$$=-2\varepsilon\int_0^1 u_{xx}^2dx+\delta[k(\omega_1+\omega_1^9)+\eta_1(\omega_1+\omega_1^3)]^2-\delta[k(\omega_0+\omega_0^9)+\eta_0(\omega_0+\omega_1^3)]^2$$

$$+2\int_0^1 uu_xu_{xx}dx \tag{3.9}$$

Since

$$\mid u(x,t)\mid=\mid\omega_0(t)+\int_0^x u_\xi(\xi,t)d\xi\mid\leq\mid\omega_0(t)\mid+\parallel u_x(t)\parallel \tag{3.10}$$

it follows from Young's inequality that

$$2\int_0^1 uu_xu_{xx}dx\leq\frac{1}{\varepsilon}(\omega_0^2+\int_0^1 u_x^2dx)\int_0^1 u_x^2dx+\varepsilon\int_0^1 u_{xx}^2dx \tag{3.11}$$

Therefore we have

$$\frac{d}{dt}\int_0^1 u_x^2dx\leq 8\delta k^2(\omega_0^2+\omega_0^{18}+\omega_1^2+\omega_1^{18})+\frac{8\delta}{k^2}(\eta_0^4+\eta_1^4)+\frac{1}{\varepsilon}(\omega_0^2+\int_0^1 u_x^2dx)\int_0^1 u_x^2dx \tag{3.12}$$

Integrating with respect to t, we deduce from (2.11) that

$$\parallel u_x(t)\parallel^2\leq C(\parallel u^0\parallel_{H^1}^2+\parallel u^0\parallel_{H^1}^{18}+\tilde{\eta}_0(0)^4+\tilde{\eta}_1(0)^4)+\frac{1}{\varepsilon}\int_0^1(\omega_0(s)^2+\parallel u_x(s)\parallel^2)\parallel u_x(s)\parallel^2 ds \tag{3.13}$$

By (3.6) and Gronwall's inequality , we obtain that for $t\geq 0$

$$\parallel u_x(t)\parallel^2\leq C(\parallel u^0\parallel_{H^1}^2+\parallel u^0\parallel_{H^1}^{18}+\tilde{\eta}_0(0)^4+\tilde{\eta}_1(0)^4)exp(C(\parallel u^0\parallel^2+\tilde{\eta}_0(0)^2+\tilde{\eta}_1(0)^2)) \tag{3.14}$$

Thus (2.14) gets from (3.5) and (3.14).

Step5: Regulation(2.15). In order to prove (2.15), we first estimate $\parallel u\parallel_{H^3}$. Integrating by parts, we have

$$\frac{d}{dt}\int_0^1 u_t^2dx=2\int_0^1 u_t(\varepsilon u_{xxt}-\delta u_{xxxt}-u_tu_x-uu_{xt}dx$$

$$=-2\varepsilon\int_0^1 u_{xt}^2dx-2\delta k\omega_{1t}^2(1+9\omega_1^8)-2\delta\omega_{1t}^2\eta_1(1+3\omega_1^2)-2\delta\omega_{1t}\dot{\eta}_1(\omega_1+\omega_1^3)-2\delta k\omega_{0t}^2(1+9\omega_0^8)$$

$$-2\delta\omega_{0t}^2\eta_0(1+3\omega_0^2)-2\delta\omega_{0t}\dot{\eta}_0(\omega_0+\omega_0^3)+2\int_0^1 u_t(u_tu_x+uu_{xt})dx \tag{3.15}$$

Since we have assumed that $\eta_0^0,\ \ \eta_1^0\geq 0$, we have $\eta_0,\ \ \eta_1\geq 0$ and then

$$\frac{d}{dt}\int_0^1 u_t^2dx\leq-2\varepsilon\int_0^1 u_{xt}^2dx-2\delta k\omega_{1t}^2(1+9\omega_1^8)-2\delta\omega_{1t}\dot{\eta}_1(\omega_1+\omega_1^3)-2\delta k\omega_{0t}^2(1+9\omega_0^8)$$

$$-2\delta\omega_{0t}\dot{\eta}_0(\omega_0+\omega_0^3)+2\int_0^1 u_t(u_tu_x+uu_{xt})dx \tag{3.16}$$

Furthermore, since

$$\begin{aligned}2\dot{\eta}_i\omega_{it}(\omega_i+\omega_i^3) &\le \dot{\eta}_i(\frac{k}{2\gamma}\omega_{it}^2(\omega_i^2+1)+\frac{2\gamma}{k}(\omega_i^4+\omega_i^2))\\ &= \frac{k}{2}\omega_{it}^2\omega_i^2(\omega_i^2+1)^2+\frac{2\gamma^2}{k}(\omega_i^4+\omega_i^2)^2\\ &\le k\omega_{it}^2(\omega_i^6+\omega_i^2)+\frac{4\gamma^2}{k}(\omega_i^8+\omega_i^4)\\ &\le 2k\omega_{it}^2(1+\omega_i^8)+\frac{8\gamma^2}{k}(\omega_i^2+\omega_i^{10}),\quad i=0,1\end{aligned} \tag{3.17}$$

and by (3.10) we obtain

$$\begin{aligned}&\mid 2\int_0^1 u_t(u_tu_x+uu_{xt})dx\mid\\ &\le 2(\mid\omega_{0t}\mid+\parallel u_{xt}\parallel)\parallel u_t\parallel\parallel u_x\parallel+2(\mid\omega_0\mid+\parallel u_x\parallel)\parallel u_t\parallel\parallel u_{xt}\parallel\\ &\le \delta\mid\omega_{0t}\mid^2+\delta\parallel u_{xt}\parallel^2+\frac{2}{\delta}\parallel u_t\parallel^2\parallel u_x\parallel^2+\frac{1}{\delta}(\mid\omega_0\mid^2+\parallel u_x\parallel^2)\parallel u_t\parallel^2+\delta\parallel u_{xt}\parallel^2\end{aligned} \tag{3.18}$$

it follows from (3.16) that

$$\frac{d}{dt}\int_0^1 u_t^2dx\le\frac{8\gamma^2}{k}(\omega_0^2+\omega_0^{10}+\omega_1^2+\omega_1^{10})+\frac{1}{\delta}(\mid\omega_0\mid^2+3\parallel u_x\parallel^2)\parallel u_t\parallel^2 \tag{3.19}$$

We then deduce using (3.6) that

$$\parallel u_t(t)\parallel^2\le C(\parallel u^0\parallel^2+\tilde{\eta}_0(0)^2+\tilde{\eta}_1(0)^2+\parallel u_t(0)\parallel^2+\frac{1}{\delta}\int_0^1(\mid\omega_0(s)\mid^2+3\parallel u_x(s)\parallel^2)\parallel u_t(s)\parallel^2\,ds \tag{3.20}$$

which, by (3.6) and Gronwall's inequality , implies that for $t\ge 0$

$$\parallel u_t(t)\parallel^2\le C(\tilde{\eta}_0(0)^2+\tilde{\eta}_1(0)^2+\parallel u^0\parallel_{H^2}^2+\parallel u^0\parallel_{H^1}^4)\times exp(C(\parallel u^0\parallel^2+\tilde{\eta}_0(0)^2+\tilde{\eta}_1(0)^2)) \tag{3.21}$$

To obtain the estimation of $\parallel u_{xx}(t)\parallel^2$, we set $y=u_{xx}$. Then by (2.11), we have

$$y_x=\frac{1}{\delta}(\varepsilon y-u_t-uu_x) \tag{3.22}$$

$$y(1)=k(\omega_1+\omega_1^9)+\eta_1(\omega_1+\omega_1^3)eqno(3.23)$$

Solving the equation, we obtain

$$u_{xx}=\frac{1}{\delta}\int_0^{1-x}[u_t(1-\xi,t)+u(1-\xi,t)u_x(1-\xi,t)]e^{\frac{\varepsilon}{\delta}(x-1+\xi)}d\xi+[k(\omega_1+\omega_1^9)+\eta_1(\omega_1+\omega_1^3)]e^{\frac{\varepsilon}{\delta}(x-1)} \tag{3.24}$$

It therefore follows that

$$\parallel u_{xx}\parallel^2\le C(\parallel u_t\parallel^2+\parallel u_x\parallel^2+\parallel u_x\parallel^4+\parallel u_x\parallel^6+\parallel u_x\parallel^{18}) \tag{3.25}$$

Further, since

$$\parallel u_{xxx}\parallel^2\le C(\parallel u_{xx}\parallel^2+\parallel u_t\parallel^2+\parallel u_x\parallel^4) \tag{3.26}$$

it follows from (3.5), (3.14)and (3.21) that

$$\parallel u(t)\parallel_{H^3}^2\le C(\sum_{i=0}^1(\tilde{\eta}_i(0)^2+\tilde{\eta}_i(0)^4+\tilde{\eta}_i(0)^8+\tilde{\eta}_i(0)^{12}+\tilde{\eta}_i(0)^{36})+\parallel u^0\parallel_{H^3}^2+\parallel u^0\parallel_{H^3}^4$$

$$+ \| u^0 \|_{H^3}^6 + \| u^0 \|_{H^3}^8 + \| u^0 \|_{H^3}^{18} + \| u^0 \|_{H^3}^{36} + \| u^0 \|_{H^3}^{54} + \| u^0 \|_{H^3}^{162}) \tag{3.27}$$

Now, we argue by contradiction to prove (2.15). Assume, on the contrary , that there exists a positive $\sigma_0 > 0$ and a sequence $\{t_n\}$ with

$$\lim_{n\to\infty} t_n = \infty$$

such that

$$\| u(t_n) \|_{H^s} \geq \sigma_0, \quad n = 1, 2, \cdots \tag{3.28}$$

By (3.27) and the imbedding theorem , there exists a subsequence $\{u(x, t_{n_i})\}$ such that $u(x, t_{n_i})$ converges to a function $\omega(x)$ in $H^s(0, 1)$ and also in $L^2(0, 1)$ as $i \to \infty$. Since by (2.13) we know that $u(x, t_{n_i})$ converges to 0 in $L^2(0, 1)$, we have $\omega = 0$, which contradicts (3.28). ■

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