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Numerical Study on Wall Temperature and Surface Heat Flux Natural Convection Equations Arising in Porous Media by Rational Legendre Collocation Approach

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Abstract: In this paper, a new powerful approach, called rational Legendre collocation method (RLC) is used to obtain the solution for nonlinear ordinary deferential equations that often appear in boundary layers problems arising in heat transfer. These kinds of the equations contain infinity boundary condition. The main objective is to reduce the solution of the problem to a solution of a system of algebraic equations, which do not require linearization and imposing the asymptotic condition transforming and physically unrealistic assumptions. Numerical results are compared with those of other methods, showing that the collocation method leads to more accurate results.

Keywords: collocation method; nonlinear ODE; porous media; rational Legendre functions; heat and mass transfer

1 Introduction

Natural convective heat transfer in porous media has received considerable attention during the past few decades. This interest can be attributed due to its wide range of applications in ceramic processing, nuclear reactor cooling system, crude oil drilling, chemical reactor design, ground water pollution and filtration processes. External natural convection in a porous medium adjacent to heated bodies was analyzed by Nield and Bejan [1], Merkin [2, 3], Minkowycz and Cheng [4–6], Pop and Cheng [7, 8], Ingham and Pop [9]. In all of these analyses, it was assumed that boundary layer approximations are applicable and the coupled set of governing equations were solved by numerical methods.

In this paper, the same approximations are applied to the problem of natural convection about an inverted heated cone embedded in a porous medium of infinite extent. No similarity solution exists for the truncated cone, but for the case of full cone, if the prescribed wall temperature or surface heat flux is a power function of distance from the vertex of the inverted cone similarity solutions exist [1, 7, 10], a great deal of information is available on heat and fluid flow about such cones as reviewed by Refs. [11, 12].

Bejan and Khair [13] used Darcy's law to study the vertical natural convective flows driven by temperature and concentration gradients. Nakayama and Hossain [14] applied the integral method to obtain the heat and mass transfer by free convection from a vertical surface with constant wall temperature and concentration. Yih [15] examined the coupled heat and mass transfer by free convection over a truncated cone in porous media for variable wall temperature and concentration or variable heat and mass fluxes and [16] applied the uniform transpiration effect on coupled heat and mass transfer in mixed convection about inclined surfaces in porous media for the entire regime. Cheng [17] used an integral approach to study the heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and [18] studied the Soret and Dufour effects on the boundary layer flow due to natural convection heat and mass transfer over a vertical cone in a porous medium saturated with Newtonian fluids with constant wall temperature. The problem of steady laminar hydromagnetic heat transfer over a vertical plate embedded in a uniform porous medium is studied in [19–21]. [22] presented, a linear stability analysis to trace the time evolution of an infinitesimal on the base flow of an electrically conducting fluid in a channel filled with a saturated porous medium. Natural convective mass transfer from upward-pointing vertical cones, embedded in saturated porous media, has been studied using the limiting diffusion

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[23]. The natural convection along an isothermal wavy cone embedded in a fluid-saturated porous medium are presented in [24, 25]. Lai and Kulacki [26] studied the natural convection boundary layer flow along a vertical surface with constant heat and mass flux including the effect of wall injection. In [10, 27] fluid flow and heat transfer of vertical full cone embedded in porous media have been solved by homotopy analysis method.

Guo et al. [35] introduced a new set of rational Legendre functions which are mutually orthogonal in $L^2(0, +\infty)$. They applied a spectral scheme using the rational Legendre functions for solving the Korteweg-de Vries equation on the half line.

Parand et al. [36–42] applied spectral method to solve nonlinear ordinary differential equations on semi-infinite intervals. Their approach was based on rational tau and collocation method.

In this paper a collocation technique based on rational Legendre functions is applied to solve present nonlinear differential equations, on semi-infinite domain.

2 Problem formulation

Consider an inverted cone with semi–angle γ and take axes in the manner indicated in Fig. 1(a). If the thermal boundary layer is sufficiently thin in comparison with the local radius, the boundary layer equations for natural convection of a Darcian fluid about a cone over the heated frustum $x = x_0$ In terms of the stream function ψ are:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

$$u = \frac{\rho_\infty \beta K g \cos\gamma(T - T_\infty)}{\mu}$$

$$\frac{1}{r}\left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial y^2}, \quad x_0 \le x < \infty \tag{2}$$

For a thin boundary layer for the local radius we have approximately $r = x\sin(\gamma)$. We suppose the temperature is the power function of distance from the vertex of the inverted cone. Accordingly, the boundary conditions at infinity are:

$$y \to \infty \quad u = 0, \quad T = T_\infty \tag{3}$$

and at the wall are

$$y = 0: \quad v = 0$$

the third condition, in the case of prescribed wall temperature, is [1, 7]:

$$T = T_w = T_\infty + A(x - x_0)^\lambda \quad y = 0, x_0 \le x \le \infty \tag{4}$$

If the surface heat flux q_w is prescribed, Eq. (4) is replaced by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = A(x - x_0)^\lambda \quad y = 0, x_0 \le x \le \infty \tag{5}$$

For the case of a full cone $(x_0 = 0, Fig.1(b))$ a similarity solution exists [1, 7]. For the case of prescribed wall temperature with boundary condition given by Eq. (4), the similarity solution for the stream function ψ and T is

$$\begin{aligned} &\psi = \alpha r(Ra_x)^{1/2} f(\eta), \\ &T - T_\infty = (T_w - T_\infty)\theta(\eta), \\ &\eta = \frac{y}{x}(Ra_x)^{1/2}, \end{aligned} \tag{6}$$

where the local Rayleigh number for the case of prescribed wall temperature is

$$Ra_x = \frac{\rho_\infty g\beta K\cos(\gamma)(T_w - T_\infty)x}{\mu\alpha} \tag{7}$$

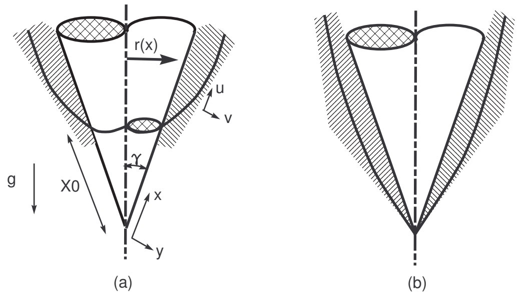

Figure 1: (a)Coordinate system for the boundary layer on a heated frustum of a cone, (b)full cone, $x_0 = 0$.

The dimensionless momentum and energy equations are, Ref. [1, 7]:

$$\begin{aligned} &f' = \theta \\ &\theta'' + \frac{\lambda+3}{2} f\theta' - \lambda f'\theta = 0, \end{aligned} \tag{8}$$

Subjected to boundary conditions as:

$$f(0) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0 \tag{9}$$

Finally, from Equations (8) and (9) we have [1, 7, 10]:

$$\begin{cases} ODE. \quad f''' + \frac{\lambda+3}{2} f f'' - \lambda (f')^2 = 0, \\ B.C. \quad f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0, \end{cases} \tag{10}$$

In the case of prescribed surface heat flux, we let:

$$\begin{aligned} &\psi = \alpha r (Ra_x)^{1/3} f(\eta), \\ &T - T_\infty = \frac{q_w x}{k} (Ra_x)^{-\frac{1}{3}} \theta(\eta), \\ &\eta = \frac{y}{x} Ra_x^{1/3}, \end{aligned} \tag{11}$$

where

$$Ra_x = \frac{\rho_\infty g \beta K \cos(\gamma) q_w x^2}{\mu \alpha k} \tag{12}$$

The governing equations become [7]

$$\begin{aligned} &f' = \theta \\ &\theta'' + \frac{\lambda+5}{2} f\theta' - \frac{2\lambda+1}{3} f'\theta = 0, \end{aligned} \tag{13}$$

Subjected to boundary conditions as:

$$f(0) = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0 \tag{14}$$

Finally from Equations (13) and (14) we have [7, 27]:

$$\begin{cases} ODE. \quad f''' + \frac{\lambda+5}{2} f f'' - \frac{2\lambda+1}{3} (f')^2 = 0, \\ B.C. \quad f(0) = 0, \quad f''(0) = -1, \quad f'(\infty) = 0, \end{cases} \tag{15}$$

It is of interest to obtain the value of the local Nusselt number which is defined as:

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} \tag{16}$$

Where q_w for the case of prescribed wall temperature can be computed from:

$$q_w = -k(\frac{\partial T}{\partial y})_{y=0} \tag{17}$$

From Eqs. (16), (17), (6) and (7) it follows that the local Nusselt number is given by:

$$Nu_x = \begin{cases} {Ra_x}^{1/2}[-\theta'(0)], & \text{prescribed wall temperature} \\ {Ra_x}^{1/3}[-\theta(0)]. & \text{prescribed surface heat flux} \end{cases} \tag{18}$$

3 Rational Legendre interpolation

In this section, first we introduce rational Legendre functions and express some of their basic properties. Then the approximation of a function is presented, using Gauss integration and rational Legendre-Gauss points.

3.1 Rational Legendre functions

The well-known Legendre polynomials are orthogonal in the interval $[-1, 1]$ with respect to the weight function $\rho(y) = 1$ and can be determined with the help of the following recurrence formula:

$$\begin{aligned} &P_0(y) = 1, \qquad P_1(y) = y, \\ &P_{n+1}(y) = \left(\frac{2n+1}{n+1}\right) y P_n(y) - \left(\frac{n}{n+1}\right) P_{n-1}(y), \qquad n \geq 1. \end{aligned} \tag{19}$$

The new basis functions, denoted by $R_n(x)$, are defined as follows

$$R_n(x) = P_n\left(\frac{x-L}{x+L}\right), \tag{20}$$

where the constant parameter L sets the length scale of the mapping. Boyd [43] offered guidelines for optimizing the map parameter L for rational Chebyshev functions, which is also useful for rational Legendre functions.

$R_n(x)$ is the nth eigenfunction of the singular Sturm-Liouville problem

$$\frac{(x+L)^2}{L}\Big(xR_n'(x)\Big)' + n(n+1)R_n(x) = 0, \qquad x \in [0, \infty), \quad n = 0, 1, 2, ... \tag{21}$$

and by Eq. (19) satisfies in the following recurrence relation:

$$\begin{aligned} &R_0(x) = 1, \qquad R_1(x) = \frac{x-L}{x+L}, \\ &R_{n+1}(x) = \left(\frac{2n+1}{n+1}\right)\left(\frac{x-L}{x+L}\right) R_n(x) - \left(\frac{n}{n+1}\right) R_{n-1}(x), \qquad n \geq 1. \end{aligned} \tag{22}$$

3.2 Function approximation

Let $w(x) = \frac{2L}{(x+L)^2}$ denotes a non-negative, integrable, real-valued function over the interval $\Lambda = [0, \infty)$. We define

$$L^2_w(\Lambda) = \Big\{ v : \Lambda \to \mathbb{R} \;\Big|\; v \text{ is measurable and } \|v\|_w < \infty \Big\}, \tag{23}$$

where

$$\|v\|_w = \left(\int_0^\infty |\, v(x)\,|^2 \, w(x)\mathrm{d}x\right)^{\frac{1}{2}}, \tag{24}$$

is the norm induced by the inner product of the space $L^2_w(\Lambda)$,

$$\langle u, v \rangle_w = \int_0^\infty u(x)v(x)w(x)\mathrm{d}x. \tag{25}$$

Thus $\Big\{R_n(x)\Big\}_{n\geq 0}$ denotes a system which is mutually orthogonal under (25), i.e.,

$$\langle R_n, R_m\rangle_w = \frac{2}{2n+1}\delta_{nm}, \tag{26}$$

where δ_{nm} is the Kronecker delta function. This system is complete in $L^2_w(\Lambda)$. For any function $u \in L^2_w(\Lambda)$ the following expansion holds

$$u(x) = \sum_{k=0}^{+\infty} a_k R_k(x), \tag{27}$$

with

$$a_k = \frac{\langle u, R_k\rangle_w}{\| R_k\|^2_w}. \tag{28}$$

The a_ks are the discrete expansion coefficients associated with the family $\{R_k(x)\}$.

3.3 Rational Legendre interpolation approximation

Canuto et al. [44] and Gottlieb et al. [45] introduced Gauss integration. Further, Guo et al. [35] introduced rational Legendre-Gauss points. Let

$$\mathfrak{R}_N = span\Big\{R_0, R_1, ..., R_N\Big\}, \tag{29}$$

and $y_j,\ j = 0, 1, \dots, N$, be the $N+1$ roots of the polynomial $P_{N+1}(x)$. These points are known as Legendre-Gauss points. We define

$$x_j = L\frac{1+y_j}{1-y_j}, \quad j = 0, 1, \dots, N, \tag{30}$$

which are called as rational Legendre-Gauss nodes. In fact, these points are zeros of the function $R_{N+1}(x)$. Using Gauss integration we have:

$$\begin{aligned}\int_0^\infty u(x)w(x)\mathrm{d}x &= \int_{-1}^1 u\left(L\frac{1+y}{1-y}\right)\rho(y)\mathrm{d}y\\ &= \sum_{j=0}^{N} u(x_j)w_j \quad \forall u \in \mathfrak{R}_{2N},\end{aligned} \tag{31}$$

where

$$w_j = \frac{2L}{(x_j+L)^2 x_j [R'_{N+1}(x_j)]^2}, \quad j = 0, ..., N, \tag{32}$$

are the corresponding weights with the $N+1$ rational Legendre-Gauss nodes.

The interpolating function of a smooth function u on a semi-infinite interval is denoted by $P_N u$. It is an element of $\mathfrak{R}_N$ and is defined as

$$P_N u(x) = \sum_{k=0}^{N} a_k R_k(x). \tag{33}$$

$P_N u$ is the orthogonal projection of u upon $\mathfrak{R}_N$ with respect to the inner product (25) and the norm (24). Thus by the orthogonality of rational Legendre functions we have

$$\langle P_N u - u, R_i\rangle_w = 0, \qquad \forall R_i \in \mathfrak{R}_N. \tag{34}$$

To obtain the order of convergence of rational Legendre approximation, at first we define the space

$$H^r_{w,A}(\Lambda) = \{v : v \text{ is measurable and } \|v\|_{r,w,A} < \infty\}, \tag{35}$$

where the norm is induced by

$$\|v\|_{r,w,A} = \left(\sum_{k=0}^{r}\left\|(x+1)^{\frac{r}{2}+k}\frac{\mathrm{d}^k}{\mathrm{d}x^k}v\right\|^2_w\right)^{\frac{1}{2}}, \tag{36}$$

and A is the Sturm-Liouville operator as follows:

$$Av(x) = -w^{-1}(x)\frac{\mathrm{d}}{\mathrm{d}x}\left(x\frac{\mathrm{d}}{\mathrm{d}x}v(x)\right). \tag{37}$$

We have the following theorem for the convergence:

Theorem 1 *For any $v \in H^r_{w,A}(\Lambda)$ and $r \geq 0$,*

$$\|P_N v - v\|_w \leq cN^{-r}\|v\|_{r,w,A}. \tag{38}$$

A complete proof of the theorem and discussion on convergence is given in [35].

To apply a collocation approach, we consider the residual $Res(x)$ when the expansion is substituted into the governing equation. It requires that a_k's be selected so that the boundary conditions are satisfied, but make the residual zero at as many (suitable chosen) spatial points as possible.

4 Numerical results

First, consider equation that prescribed wall temperature case that is expressed by Eq.(10). In the first step of our analysis, we apply P_N operator on the function $f(\eta)$ as follows:

$$P_N f(\eta) = \sum_{k=0}^{N} a_k R_k(\eta) \tag{39}$$

Then, we construct the residual function by substituting $f(\eta)$ by $P_N f(\eta)$ in the model Eq.(10):

$$\begin{aligned} Res(\eta) \quad &= \quad \frac{d^3}{d\eta^3}P_N f(\eta) + \frac{\lambda+3}{2}P_N f(\eta)\left(\frac{d^2}{d\eta^2}P_N f(\eta)\right) \\ &\quad -\lambda\left(\frac{d}{d\eta}P_N f(\eta)\right)^2, \end{aligned} \tag{40}$$

The equations for obtaining the coefficients a_ks come from equalizing $Res(\eta)$ to zero at rational Legendre-Gauss points $(x_j, \quad j = 1, 2, ..., N-1)$ plus two boundary conditions:

$$\begin{cases} Res(x_j) = 0, \quad j = 1, 2, ..., N-1, \\ P_N f(0) = 0, \\ \frac{d}{d\eta}P_N f(\eta)\Big|_{\eta=0} = 1. \end{cases} \tag{41}$$

The 3th boundary condition already is satisfied. Solving the set of equations we have the approximating function $P_N f(\eta)$.

Table 1 shows good agreement between RLC method, homotopy analysis and Runge-Kutta method for $f''(0)$ or $\theta'(0)$ with various λ.

The results for $f'(\eta)$ have been shown in Table 2 with two selected $\lambda = 0$ and $\lambda = 1/2$ and comparison has been made between the Runge-Kutta's solution and the presented numerical solution. Absolute errors show that RLC gives us approximate solution with a high degree of accuracy with a small N.

The resulting graph of Eq. (10) for $N = 10$ is shown in Fig.2.

Now, we restart the previous procedure for the case of prescribed wall heat flux with the thermal boundary condition given by Eq.(15).

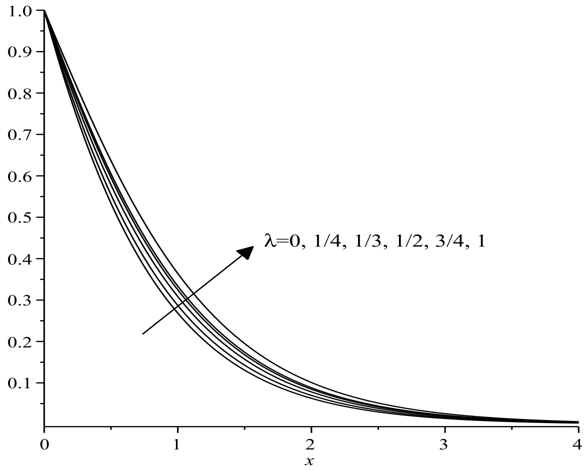

Figure 2: RLC approximation of $f'(\eta)$ for different values $\lambda = 0,\ 1/4,\ 1/3,\ 1/2,\ 3/4$ and 1 in the case of prescribed wall temperature

Table 1: A comparison of methods in [7, 10] and the present method with the values for $f''(0)$ in the case of prescribed wall temperature

	Runge-Kutta	RLC method				Other methods	
λ	Solution[10]	N	L	RLC	Error	HAM[10]	Ref. [7]
0	-0.76854	10	1.4675	-0.76855	0.00001	-0.77363	-0.769
$1/4$	-0.88498	10	1.453	-0.88499	0.00001	-0.88890	$-$
$1/3$	-0.92101	10	1.448	-0.92102	0.00001	-0.92433	-0.921
$1/2$	-0.98956	10	1.434	-0.98957	0.00001	-0.99382	-0.992
$3/4$	-1.08518	10	1.415	-1.08519	0.00001	-1.08840	$-$
1	-1.17372	10	1.397	-1.17372	0.00000	-1.17686	$-$

Table 2: Comparison between RLC solution and Runge-Kutta solution for $f'(\eta)$ with $\lambda = 0$ and $\lambda = 1/2$ with $N = 10$ in the case of prescribed wall temperature

	$\lambda = 0, f'(\eta)$			$\lambda = 1/2, f'(\eta)$		
η	RLC Solution	Runge-Kutta Solution	Absolute Error	RLC Solution	Runge-Kutta Solution	Absolute Error
0	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
0.2	0.8478	0.8478	0.0000	0.8129	0.8130	0.0001
0.4	0.7035	0.7036	0.0001	0.6499	0.6500	0.0001
0.6	0.5732	0.5733	0.0001	0.5124	0.5124	0.0000
0.8	0.4599	0.4599	0.0000	0.3994	0.3994	0.0000
1	0.3641	0.3643	0.0002	0.3084	0.3084	0.0000
1.2	0.2853	0.2855	0.0002	0.2364	0.2364	0.0000
1.4	0.2217	0.2218	0.0001	0.1802	0.1802	0.0000
1.6	0.1713	0.1713	0.0000	0.1368	0.1367	0.0000
1.8	0.1317	0.1315	0.0002	0.1035	0.1034	0.0001
2	0.1009	0.1007	0.0002	0.0782	0.0780	0.0002

Table 3: A comparison of methods in [27] and the present method with the values for $f'(0)$ in the case of prescribed surface heat flux

	Runge-Kutta	RLC method				Other methods
λ	Solution[27]	N	L	RLC	Error	HAM[27]
0	0.94760	10	1.98	0.94765	0.00005	0.94783
$1/4$	0.91130	10	2	0.91133	0.00003	0.91119
$1/3$	0.90030	10	2	0.90032	0.00002	0.90103
$1/2$	0.87980	10	2	0.87975	0.00005	0.87964
$3/4$	0.85220	10	2.02	0.85216	0.00004	0.85242
1	0.82760	10	2.02	0.82757	0.00003	0.82726

Same as the first step which was followed we apply P_N operator on the function $f(\eta)$, Then, we construct the residual function by substituting $f(\eta)$ by $P_N f(\eta)$ in the model Eq.(15):

$$\begin{aligned} Res(\eta) &= \frac{d^3}{d\eta^3} P_N f(\eta) + \frac{\lambda+5}{2} P_N f(\eta) \left(\frac{d^2}{d\eta^2} P_N f(\eta) \right) \\ &- \frac{2\lambda+1}{3} \left(\frac{d}{d\eta} P_N f(\eta) \right)^2, \end{aligned} \tag{42}$$

The equations for obtaining the coefficients a_ks come from equalizing $Res(\eta)$ to zero at rational Legendre-Gauss points ($x_j, \quad j = 1, 2, ..., N-1$) plus two boundary conditions:

$$\begin{cases} Res(x_j) = 0, \quad j = 1, 2, ..., N-1, \\ P_N f(0) = 0, \\ \frac{d^2}{d\eta^2} P_N f(\eta) \Big|_{\eta=0} = -1. \end{cases} \tag{43}$$

The 3th boundary condition already has been satisfied. Solving the set of equations we have the approximating function $P_N f(\eta)$.

Table 3 shows good agreement between RLC method, homotopy analysis and Runge-Kutta method for $f'(0)$ or $\theta(0)$ with various λ.

The results for $f'(\eta)$ have been shown in Table 4 with two selected $\lambda = 0$ and $\lambda = 1/2$ and comparison have been made between the Runge-Kutta's solution and the presented numerical solution. Absolute errors show that RLC gives us approximate solution with a high degree of accuracy with a small N.

The resulting graph of Eq. (15) for $N = 10$ is shown in Fig. 3.

Table 4: Comparison between RLC solution and Runge-Kutta solution for $f'(\eta)$ with $\lambda = 1/4$ and $\lambda = 3/4$ with $N = 10$ in the case of prescribed surface heat flux

	$\lambda = 1/4, f'(\eta)$			$\lambda = 3/4, f'(\eta)$		
η	RLC Solution	Runge-Kutta Solution	Absolute Error	RLC Solution	Runge-Kutta Solution	Absolute Error
0	0.911336	0.911295	0.00004	0.852162	0.852193	0.00003
0.1	0.813649	0.813604	0.00006	0.755349	0.755377	0.00003
0.2	0.721378	0.721351	0.00002	0.665407	0.665448	0.00004
0.3	0.635508	0.635531	0.00003	0.582895	0.582985	0.00011
0.4	0.556638	0.556661	0.00003	0.508036	0.508141	0.00011
0.5	0.485026	0.484997	0.00003	0.440786	0.440849	0.00016
0.6	0.420667	0.420587	0.00008	0.380895	0.380907	0.00001
0.7	0.363353	0.363276	0.00008	0.327971	0.327973	0.00000
0.8	0.312722	0.312677	0.00005	0.281523	0.281536	0.00001
0.9	0.268308	0.268264	0.00004	0.241002	0.241013	0.00001
1	0.229585	0.229508	0.00008	0.205834	0.205832	0.00000
1.1	0.195999	0.195878	0.00014	0.175450	0.175434	0.00002
1.2	0.166999	0.166847	0.00015	0.149298	0.149275	0.00002
1.3	0.142054	0.141837	0.00022	0.126861	0.126821	0.00004
1.4	0.120665	0.120362	0.00030	0.107663	0.107596	0.00007
1.5	0.102374	0.102025	0.00030	0.091276	0.091196	0.00080

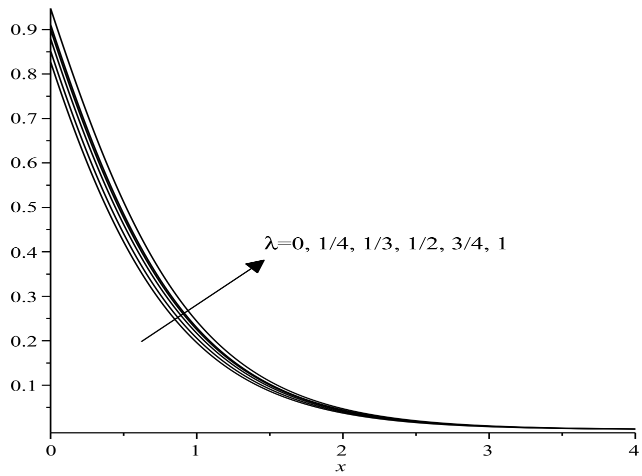

Figure 3: RLC approximation of $f'(\eta)$ for different values $\lambda = 0,\ 1/4,\ 1/3,\ 1/2,\ 3/4$ and 1 in the case of prescribed surface heat flux

5 Conclusion

A numerical solution based on orthogonal functions has been presented for three order nonlinear differential equations arising from similarity solution of inverted cone embedded in porous medium. The problem of natural convection about a cone embedded in a porous medium at local Rayleigh numbers is analyzed based on the boundary layer approximation and Darcy's law. Local similarity solutions are obtained for a full cone with the prescribed wall temperature or surface heat flux being a power function of distance from the vertex of the inverted cone. The obtained approximate solution by RLC provides us with calculating Nusselt numbers.

Most numerical methods reported in the literature thus far are based on transforming maps of the physical domain $[0,+\infty)$ to the finite domains, shooting methods or finite-difference methods obtained by first truncating the semi-infinite physical domain of the problem to a finite domain at an unknown finite boundary, which is determined as a part of the solution by imposing an "asymptotic boundary condition" at this boundary.

The method presented in this paper used a set of orthogonal rational Legendre functions and solved the problems on the whole domain without requiring small parameters, truncating it to a finite domain, imposing the asymptotic condition transforming and transforming domain of the problem and physically unrealistic assumptions.

These orthogonal functions are proposed to provide an effective but simple way to improve the convergence of the solution by collocation method. The validity of the method is based on the assumption that it converges by increasing the number of collocation points. Through the comparisons made among the numerical solutions of Sohouli et al. [10, 27], Cheng et al. [7] and the current work, it has been shown that the present work has provided an acceptable approach for this equation; also it was confirmed by the theorem that this approach has an exponentially convergence rate. In total, an important concern of spectral methods is the choice of basis functions; the basis functions have three properties: easy computation, rapid convergence and completeness, which means that any solution can be represented to arbitrarily high accuracy by taking the truncation N sufficiently large.

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