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# A Simple Dissortative Bipartite Network Model

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**Abstract:** In this paper, we propose and study a simple bipartite network model. Significantly, we prove that the degree distribution of one kind nodes obey power-law form with adjustable exponents. And the other obeys exponential distribution. Furthermore, we study the degree-degree correlation of the model by calculating the mixing coefficient and find that the network model is dissortative. Numerical simulations results are given to verify the theoretical results.

**Keywords:** complex networks; bipartite networks; degree distribution; power-law distribution; exponential distribution

## 1 Introduction

Complex networks are researched widely in resent years. However, the bipartite networks don't get enough attention. The bipartite networks are common in our life, such as scientists-papers networks [1-3] which can be modeled by two types of vertices, scientists and papers, where links always run between nodes of unlike type. Such networks are called bipartite networks. Bipartite networks appear in many social structures, for instance in affiliation network. The two types of vertices then represent the individuals and groups, while the links between them indicate group membership [4]. Some models have been proposed to capture the main properties met in practice. Watts et. al [5] provide highly clusterized networks but the obtained degree distribution is still Poisson-shaped. Guillaume et. al [6] gave a static model showed that all complex networks had similar underlying bipartite structure, and explored that their basic properties could be viewed as consequences of bipartite structure. Many other attempts have been made to produce networks having all the properties we cited [7-9].Most above models show that both kinds of nodes have same degree distribution. The purpose of this paper is to set a simple bipartite network model which has different degree distribution of two types nodes. We will show that he degree distribution of one kind nodes obey power-law form with adjustable exponents. And the other obeys exponential distribution. The judgment of the degree-degree correlation of our bipartite networks is given. And the numerical simulation results are well agreed with theoretical analytic results.

## 2 The model

The bipartite graph is triple $H = (M, N, E)$, where $M$ and $N$ are two disjoint sets of vertices, respectively the top and bottom vertices, and $E \subseteq M \times N$ is the set of edges. The difference with classical graphs is that every edge goes from $M$ to $N$ . The model starts from a initial bipartite network: there exist $m_0$ nodes in set $M$, $n_0$ nodes in set $N$ and $a(\subseteq E)$ sides. Given a top vertex $i$ in $M$ and a bottom vertex $j$ in $N$ , we denote $k_i$ the degree of $i$and $J_j$ the degree of $j$ in the bipartite network. Then,$a = \sum k_i = \sum J_j ,(k_i, J_j \geq 1, i = 1, 2, \cdots , m_0, j = 1, 2, \cdots , n_0)$.

From the given initial network, the model evolves as following three rules at every time step$t$:

(1) Add a new node to $M$. The new node is linked to $n$ old nodes in$N$ by random probability $\frac{1}{n_0+t-1}$ . (The old nodes are added into system before the time step . )

(2) Put a new node into $N$. The new node is linked to $m$ old nodes in $M$ by preferential probability$K_i/\sum\limits_{i=1}^{m_0+t-1} K_i$.

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# 3 Analysis of degree distribution of the bipartite network

We can get the change rate of degree for node $i(\in M)$ by the generating process of the networks:

$$\frac{\partial k_i}{\partial t} = m\frac{k_i}{\sum_{i=1}^{m_0+t-1} k_i} = \frac{mk_i}{a+(t-1)(m+n+b)} \approx \frac{mk_i}{t(m+n)} \tag{1}$$

where $t \gg m, n, a.i = 1, 2, ...t$.
We can also get the initial degree of node $i$ satisfying $k_i(t_i) = n$ , where $t_i$ represents the time that node $i$ is added into$M$ . We obtain Eq. (2) by solving Eq. (1):

$$k_i(t) = n(\frac{t}{t_i})^{\frac{m}{m+n}} \tag{2}$$

Let$k_i(t) < k$ , then$t_i > t(\frac{n}{k})^{\frac{m+n}{m}}$ . Thus we can denote the probability $p(k_i(t) < k)$as$p(t_i > t(\frac{n}{k})^{\frac{m+n}{m}})$.
Assuming that nodes are put into system with the same time interval, so$t_i$ have constant probability $p(t_i) = \frac{1}{m_0+t}$.we get

$$p(k_{\mathrm{i}}(t) < k) = p(t_i > t(\frac{n}{k})^{\frac{m+n}{m}}) = 1 - \frac{t}{m_0+t}(\frac{n}{k})^{\frac{m+n}{m}} \tag{3}$$

So the top degree distribution function satisfies

$$p(k_{\mathrm{i}}(t) < k) = p(t_i > t(\frac{n}{k})^{\frac{m+n}{m}}) = 1 - \frac{t}{m_0+t}(\frac{n}{k})^{\frac{m+n}{m}} \tag{4}$$

as $t \to \infty$. From Eq. (4), We can know that the top degree distribution accords with power law distribution, and the scale exponent $\gamma_M = 2 + \frac{n}{m}$ can be changed by adjusting initial parameters $m$,$n$.
Similar to the above operation, we get$J_j(t) \approx n\ln\frac{t}{t_j} + m$ and degree distribution$q(J)$ of bottom nodes satisfy exponential distribution

$$q(J) \approx e^{\frac{m-J}{n}} \tag{5}$$

From the above degree distribution expressions, we get that the two disjoint sets of vertices have different degree distribution, nodes in set $M$ obey power law distribution and nodes in set $N$ obey exponential distribution. This is consistent with our simulated result (Fig.1). In Fig.1(a), the scale exponent of $M$ satisfy $r_{simulation} = 3.0993 \approx r_{analysis}$.

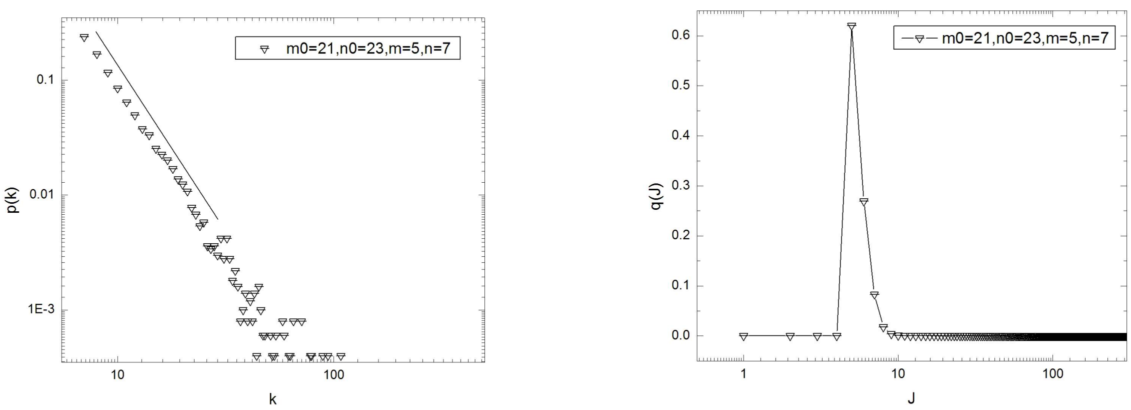


Figure 1: (a) Degree distribution figure of set $M$. (b) Degree distribution figure of set $N$ when parameters set as $m_0 = 21, n_0 = 23, m = 5, n = 7$

# 4 Degree-degree correlation judgments of the bipartite network model

We research degree-degree correlation by calculating the mixing coefficient [10]. The values of mixing coefficients of different set are all negative, where $I_M(g) = -0.5575, I_N(g) = -0.5764$ when our network produced by the initial parameters set as $m_0 = 21, n_0 = 23, m = 5, n = 7$. Such results means the bipartite networks model is dissortative.

Another judgment of degree correlation is average degree $< k_{nn}(k) >$ of the nearest neighbors of a randomly picked vertex with degree $k$. In one-mode graphs, the average degree $< k_{nn}(k) >$ of nearest neighbors of vertices of degree $k$ increases monotonically, then the network is assortative mixing, i.e. node degrees in network are positive correlation. Accordingly, $< k_{nn}(k) >$ decreases monotonically, then the network is dissortative mixing, i.e. node degrees are negative correlation. These results are also applied to describe the degree correlations of bipartite network.

In bipartite network, for any nodes $i(\in M)$ whose degree is $k$ ,

$$k_{nn}(k) = \frac{1}{k} \sum_{j \in V(i)} J_j \tag{6}$$

where $V(i) \subseteq N$ is nearest neighbors of node $i$ , $k \in [k_{\min}, k_{\max}]$ .
And

$$< k_{nn}(k) >= \frac{\sum k_{nn}(k)}{N_k} \tag{7}$$

Where $N_k$ means the number of nodes that degrees are $k$.
Similarly, for any nodes $j(\in N)$ whose degree is $J$ ,

$$J_{nn}(J) = \frac{1}{J} \sum_{i \in V(j)} k_i \tag{8}$$

where $V(j) \subseteq M$ is nearest neighbors of node $j$,$J \in [J_{\min}, J_{\max}]$.
And

$$< J_{nn}(J) >= \frac{\sum J_{nn}(J)}{N_J} \tag{9}$$

, where $N_J$ means the number of nodes that degrees are $J$.

The simulation details are given in fig. 2 by the average degree of nearest neighbors of vertices [11]. Both average degree of nearest neighbors of vertices is decrease monotonically, i.e. the network is dissortative. This is accord with our analysis results.

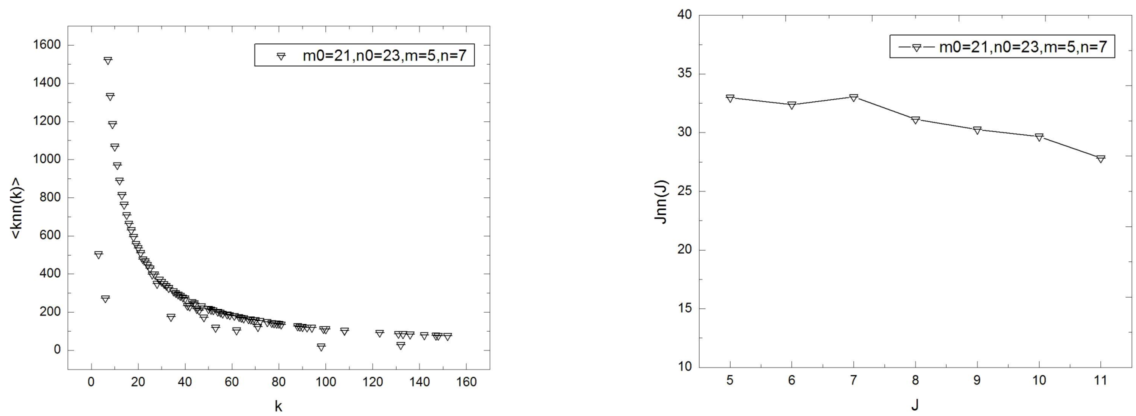


Figure 2: (a) Average degree$k_{nn}(k)$ of the nearest neighbors of vertex with degree $k$ in top set $M$ . (b) Average degree $J_{nn}(J)$ of the nearest neighbors of vertex with degree $J$ in bottom set $N$.

# 5 Conclusion

In this paper we propose a simple bipartite network that have different degree distribution function. We know that one kind of nodes obey power-law distribution and the other obey exponential distribution. Furthermore, we studied the prosperity of degree-degree correlation by calculated the mixing coefficient and find that the network model is dissortative. We also give the simulation results in our paper. The simulation results are well accord with our analytical results.

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