ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.13(2012) No.2,pp.220-227

Existence Results for Non-Autonomous Semilinear Integro-Differential Systems

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(Received 29 June 2011, accepted 23 November 2011)

Abstract: In this paper, sufficient conditions are given for the existence of non-autonomous semilinear integro- differential equations with nonlocal conditions in an abstract space with the help of the fixed point theory.

Keywords: Mild solutions; nonlocal conditions; fixed point theorems; integro-differential equation

1 Introduction

In this paper, we discuss the non-autonomous semilinear integro-differential equations with nonlocal conditions of the form

$$\begin{aligned} x'(t) &= A(t)x(t) + f\Big(t, x(t), \int_0^t g(t,s,x(s))ds\Big) \\ x(0) &= \sum_{i=1}^{m} \nu_i\, x(t_i), \qquad t \in J = [0,T], \end{aligned} \tag{1.1}$$

where $T > 0$; $0 < t_1 < t_2 < t_3 < \cdots < t_m < T$, ν_i are real numbers and the family $\{A(t) : 0 \le t \le T\}$ of unbounded linear operators generates a linear evolution operator. Let X be a Banach space with the norm $\|\cdot\|$ and the functional $f : J \times X \times X \to X$; $g : \Delta \times X \to X$ are continuous functions. Here $\Delta = \{(t,s) : 0 \le s \le t \le T\}$. Let $E := C(J; X)$ be the Banach space of continuous functions $x : J \to X$, equipped with the norm,

$$\|x\|_E = \sup\{\|x(t)\| : t \in J\}.$$

Many mathematical formulation of physical phenomena contain integrodifferential equations, these equations arises in many fields like fluid dynamics, biological models and chemical kinetics. The interplay between applied sciences and mathematics leads to the development of initial or boundary value problems for nonlinear partial integrodifferential equations to model physical systems. Lin et al [1] studied the solutions of the semilinear integrodifferential equations with nonlocal Cauchy problem. The existence of mild solutions for nonautonomous integrodifferential equations with nonlocal conditions in a Banach space has been studied by Lin and Ezzinbi [2]. Pruss [3] studied the Volterra type equations through resolvent operators. The theory of integro-differential equations has been extensively studied in the literature see [4–11] and the references therein.

Recently, Under sufficient conditions, Boucherif [12, 13] studied a semilinear differential inclusions with nonlocal conditions through fixed point theory. Boucherif [13] gave a different form of nonlocal condition. This form reduces the Lipschitz condition on nonlocal initial condition. Further we refer [14–17] for semilinear differential equations and integro-differential equations.

Motivated from the above mentioned works [12–15], the main purpose of this paper is to study the existence results for the system (1.1) by means of fixed point theory. The paper is organized as follows: some preliminaries are presented in the section 2. In section 3, we investigate the existence of mild solutions for semilinear integro differential system

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IJNS.2012.04.15/597

using the Leray-Schauder alternative fixed point theorem with non-uniqueness. An interesting feature of this method is that this yields simultaneously the existence and maximal interval of existence. In section 4, we study the existence and uniqueness of mild solution for semilinear integro differential system by using Banach fixed point theorem. Finally, an example is presented in section 5 to show the applications of our abstract results.

2 Preliminaries

Before proceeding to main result, we shall set forth some preliminaries that will be used in our subsequent discussion. For the family $\{A(t) : 0 \leq t \leq T\}$ linear operators, we impose on the following restrictions:

(B_1) : The domain $D(A)$ of $\{A(t) : t \in [0, T]\}$ is dense in X and independent of t, $A(t)$ is a closed linear operator.

(B_2) : For each $t \in [0, T]$, the resolvent $R(\lambda, A(t))$ exists for all λ with $Re\lambda \leq 0$ and there exists $k > 0$ so that $\|R(\lambda, A(t))\| \leq \frac{k}{(|\lambda|+1)}$.

(B_3) : For any $t, s, u \in [0, T]$, there exists α $(0 < \alpha \leq 1)$ and $k_1 > 0$ so that $\|(A(t) - A(u))A^{-1}(s)\| \leq k_1|t - s|^{\alpha}$.

(B_4) : For each $t \in [0, T]$ and some $\lambda \in \rho(A(t))$, the resolvent set of $A(t)$, the resolvent $R(\lambda, A(t))$ is a compact operator.

Remark 2.1. *(B_2) implies that for each $t \in [0, T]$, $A(t)$ is the infinitesimal generator of an analytic semigroup and (B_4) insures that this semigroup is compact for $t > 0$.*

Then the family $\{A(t) : t \in [0, T]\}$ generates a unique linear evolution operator $T(t, s)$, $0 \leq s \leq t \leq T$, satisfying the following properties:

(i) : $T(t, s) \in \mathcal{L}(X)$, the space of bounded linear transformations on X, whenever $0 \leq s \leq t \leq T$ and for each $x \in X$, the mapping $(t, s) \to T(t, s)x$ is continuous.

(ii) : $T(t, s)T(s, u) = T(t, u)$ for $0 \leq u \leq s \leq t \leq T$;

(iii) : $T(t, t) = I$;

(iv) : $T(t, s)$ is a compact operator $t - s > 0$;

(v) : $\frac{\partial}{\partial t}T(t, s) = -A(t)T(t, s)$, for $s < t$;

(vi) : There is a constant $M \geq 1$ such that $\|T(t, s)\| \leq M$, $0 \leq s \leq t \leq T$;

(vii) : If $0 < h < 1$, $t - s > h$ and $0 < \gamma < 1$, then $\|T(t + h, s) - T(t, s)\| \leq \frac{C_3\, h^{\gamma}}{|t-s|^{\gamma}}$, for some $C_3 > 0$.

(viii) : If $w(t)$ is continuous on $[0, T]$, then the function $t \to \int_0^t T(t, s)w(s)ds$ is Hölder condition with any exponent $0 < \gamma < 1$.

The following proposition which will be used in section 3 is well known.

Proposition 2.1. *The family of operators $\{T(t, s) : t > s\}$ is continuous in t in the uniform operator topology uniformly for s.*

There exists a bounded operator B on $D(B) = X$ given by the formula $B = (I - \sum_{i=1}^{m} \gamma_i T(t_i, 0))^{-1}$. This is possible if, for instance $\sum_{i=1}^{m} |\gamma_i| < \frac{1}{M}$.

Definition 2.1. *A map $f : J \times X \times X \to X$ is said to be L^1-Carathèodory if:*

(i) $t \to f(t, x, y)$ *is measurable for each* $x, y \in X$.

(ii) $(x, y) \to f(t, x, y)$ *is continuous for almost all* $t \in J$.

(iii) for each positive integer $m > 0$, there exists $\alpha_m \in L^1(J : \Re^+)$ such that

$$\sup_{\|x\|\leq m}\left\|f\Big(t, x(t), \int_0^t g(t,s,x(s))ds\Big)\right\| \leq \alpha_m(t), \textit{ for} \qquad t \in J, \ a.e.$$

Definition 2.2. *$x \in E$ is a mild solution of equations (1.1) if the integral equation*

$$\begin{aligned} x(t) &= \sum_{i=1}^{m}\gamma_i T(t,0)B\int_0^{t_i} T(t_i,s)f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big)ds \\ &+ \int_0^t T(t,s)f\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau\Big)ds, \ \ t\in J, \end{aligned} \tag{2.1}$$

is satisfied.

3 Existence result I :

Our existence theorem is based on the following theorem.

Theorem 3.1. *Let S be a convex subset of a Banach space E and assume that $0 \in S$. Let $F : S \to S$ be a completely continuous operator and let*

$$U(F) = \{x \in S : x = \lambda Fx \ \ \textit{for some } 0 < \lambda < 1\}.$$

Then either is $U(F)$ unbounded or F has a fixed point.

In this section, we prove the existence theorem by using the following hypotheses:

(H_1) : There exists a continuous non-decreasing function for $\Omega : \Re^+ \to (0,\infty)$ and $p \in L^1(J;\Re^+)$ such that

$$\|f(t,x,y)\| \leq p(t)\Omega(\|x\| + \|y\|), \ \ t \in \ J; \ \ x,y \in X.$$

(H_2) : For each $(t,s) \in \Delta$, the functions $g(t,s,\cdot) : X \to X$ are continuous and for each $x \in X$ the functions $g(\cdot,\cdot,x) : \Delta \to X$ are strongly measurable.

(H_3) : There exits $q_1 \in L^1(J,\Re^+)$ such that

$$\left\|\int_0^t g(t,s,x)ds\right\| \ \ \leq \ \ q_1(t)\Omega(\|x\|), \ \ t\in\Delta, \ \ x \in X,$$

where $\Omega : [0,\infty) \to (0,\infty)$ is a continuous non-decreasing function.

Theorem 3.2. *If the hypotheses $(H_1) - (H_3)$ be hold. Then the system (1.1) has a mild solution $x(t)$, defined on J provided that following inequality is satisfied:*

$$\sup_{\rho\in[0,\infty)} \frac{\rho}{M\left\|\bar{M}\right\|_{L^1}\Omega(\rho)\Big[1 + M\|B\|\sum_{i=1}^{m}|\gamma_i|\Big]} > 1, \tag{3.1}$$

where $\bar{M}(t) = \max\{p(t), q_1(t)\}$.

Proof. Let T be an arbitrary number $0 < T < +\infty$ satisfying (3.1). It follows from (3.1) that there exists $\beta > 0$ such that

$$\frac{\beta}{M\left\|\bar{M}\right\|_{L^1}\Omega(\beta)\Big[1 + M\|B\|\sum_{i=1}^{m}|\gamma_i|\Big]} > 1 \tag{3.2}$$

Step-1: For $\lambda \in (0,1)$, consider the family of problems

$$\begin{aligned} x(t) &= \lambda \sum_{i=1}^{m} \gamma_i T(t,0)B \int_0^{t_i} T(t_i,s) f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) ds \\ &+ \lambda \int_0^t T(t,s) f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) ds, \ \ t \in J. \end{aligned} \tag{3.3}$$

Notice that if $x \in E$ is a solution of (3.3) for $\lambda = 1$, then x is a solution of (1.1).

Consider $U = \{x \in E; \|x\| < \beta\}$. Define $F : \bar{U} \to E$ by

$$\begin{aligned} Fx(t) &= \sum_{i=1}^{m} \gamma_i T(t,0)B \int_0^{t_i} T(t_i,s) f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) ds \\ &+ \int_0^t T(t,s) f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) ds, \ \ t \in J, \end{aligned}$$

we can easily show that F is continuous.

Step-2 : F maps bounded sets into bounded sets.

For, let $x \in B_\rho = \{v \in E : \|v\| \le \rho\}$, then $(H_1) - (H_3)$ implies that

$$\begin{aligned} \|Fx(t)\| &= \left\| \sum_{i=1}^{m} \gamma_i T(t,0)B \int_0^{t_i} T(t_i,s) f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) ds \right\| \\ &\quad + \left\| \int_0^t T(t,s) f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) ds \right\| \\ &\le M^2 \|B\| \sum_{i=1}^{m} |\gamma_i| \int_0^{t_i} \left\| f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) \right\| ds \\ &\quad + M \int_0^t \left\| f\Big(s, x(s), \int_0^s g(s,\tau,x(\tau))d\tau\Big) \right\| ds \\ &\le M^2 \|B\| \sum_{i=1}^{m} |\gamma_i| \int_0^{t_i} [p(s) + q_1(s)]\Omega(\|x(s)\|) ds \\ &\quad + M \int_0^t [p(s) + q_1(s)]\Omega(\|x(s)\|) ds, \\ &\le M^2 \|B\| \sum_{i=1}^{m} |\gamma_i| \int_0^{t_i} \bar{M}(s)\Omega(\rho) ds + M \int_0^t \bar{M}(s)\Omega(\rho) ds, \end{aligned}$$

so that,

$$\|Fx\|_E = \sup_{t \in J} \|Fx(t)\| \ \le \ M\Big[M \|B\| \sum_{i=1}^{m} |\gamma_i| + 1\Big] \left\|\bar{M}\right\|_{L^1} \Omega(\rho). \tag{3.4}$$

Step-3: $F(\bar{U})$ is a uniformly equicontinuous family of functions.

For, let $\tau_1 < \tau_2$ in J. Then

$$\begin{aligned}
&\|Fx(\tau_1) - Fx(\tau_2)\| \\
\leq\ & \|B\| \sum_{i=1}^{m} |\gamma_i| \int_0^{t_i} \|T(t_i, s)\| \|f\Big(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau\Big)\| ds \\
& \qquad \times \|T(\tau_1, 0) - T(\tau_2, 0)\| \\
+\ & \int_0^{\tau_1} \|T(\tau_1, s) - T(\tau_2, s)\| \|f\Big(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau\Big)\| ds \\
+\ & \int_{\tau_1}^{\tau_2} \|T(\tau_2, s)\| \|f\Big(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau\Big)\| ds. \\
\leq\ & M\|B\| \sum_{i=1}^{m} |\gamma_i| \, \|\alpha_\beta\|_{L^1} \|T(\tau_1, 0) - T(\tau_2, 0)\| \\
+\ & \|\alpha_\beta\|_{L^1} \max_{s\in J} \|T(\tau_1, s) - T(\tau_2, s)\| + M \int_{\tau_1}^{\tau_2} \alpha_\beta(s) ds. \\
\to\ & 0 \ \ as \ \ \tau_2 \to \tau_1.
\end{aligned}$$

As $\tau_2 \to \tau_1$ we get $\|T(\tau_1, 0) - T(\tau_2, 0)\| \to 0$, $\max_{s\in J} \|T(\tau_1, s) - T(\tau_2, s)\| \to 0$ and also $\int_{\tau_1}^{\tau_2} \alpha_\beta(s)ds \to 0$ as $\tau_2 \to \tau_1$. Because $\alpha_\beta \in L^1(J, \Re^+)$.

Step- 4: The set $\bar{U}(t) = \{Fx(t) : x \in \bar{U}\}$ is precompact in E.

For, let $t > 0$ and $0 < \epsilon < t$. For $x \in \bar{U}$ define

$$\begin{aligned}
F_\epsilon x(t) &= \sum_{i=1}^{m} \gamma_i T(t, 0) B \int_0^{t_i} T(t_i, s) f\Big(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau\Big) ds \\
&+ \int_0^{t-\epsilon} T(t, s) f\Big(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau\Big) ds.
\end{aligned}$$

Since $T(t, s)$ is compact for every $t > s$, the set

$$\bar{U}_\epsilon = \{F_\epsilon x(t) : x \in \bar{U}\}$$

is precompact in X. for every $\epsilon \in (0, t)$. Moreover for $x \in \bar{U}$ we have

$$\begin{aligned}
\|F_\epsilon x(t) - Fx(t)\| &\leq \int_{t-\epsilon}^{t} \left\| T(t, s) f\Big(s, x(s), \int_0^s g(s, \tau, x(\tau))d\tau\Big) \right\| ds \\
&\leq M \int_{t-\epsilon}^{t} \alpha_\beta(s) ds. \\
&\to 0 \quad \text{as} \quad \epsilon \to 0.
\end{aligned}$$

Since $\alpha_\beta(s) \in L^1$ and $meas([t - \epsilon, t]) < \epsilon$.

Step- 5: Next, by (H_1) and (H_3) all solutions of (3.3) satisfy

$$\|x\|_E \leq M[M\|B\| \sum_{i=1}^{m} |\gamma_i| + 1] \|\bar{M}\|_{L^1} \Omega(\|x\|_E).$$

Suppose, now that there exist $x \in \partial U$ and $\lambda \in (0, 1)$ such that $x \in \lambda Fx$. Then x satisfies (3.3) and $\|x\|_E = \beta$. It follows from (3.4) that

$$\beta \leq M\Big[M\|B\| \sum_{i=1}^{m} |\gamma_i| + 1\Big] \|\bar{M}\|_{L^1} \Omega(\beta).$$

This, obviously, contradicts the definition of β (see equation (3.2)). Moreover, the set U is bounded. Consequently, by Theorem 3.1, the operator F has a fixed point in E. Therefore, the system (1.1) has a mild solution. Thus the proof is completed. ■

4 Existence result II :

Concerning the existence and uniqueness of mild solution for the system (1.1), we need the following assumptions.

(H_f) : There exists a constant $\ell > 0$ such that

$$\|f(t,x_1,y_1) - f(t,x_2,y_2)\| \leq \ell\,(\,\|x_1 - x_2\| + \|y_1 - y_2\|\,),\ \ x_i, y_i \in X,\ i = 1,2.$$

(H_g) : There exists a constant $\ell_1 > 0$ such that

$$\|g(t,s,x_1) - g(t,s,x_2)\| \leq \ell_1\,(\,\|x_1 - x_2\|\,),\ \ x_i \in X,\ i = 1,2,\ t,s \in \Delta.$$

Theorem 4.1. *Let the hypotheses $(H_f) - (H_g)$ hold. If the following inequality*

$$\Lambda = M\Big[M\|B\|\sum_{i=1}^{m}|\gamma_i| + 1\Big]\ell\Big[1 + \ell_1 T\Big] < 1,$$

is satisfied, then the system (1.1) has a unique solution in E.

Proof. The operator F defined as in the proof of the Theorem 3.2. Now, we shall show that the operator F is a contraction. Let $x, y \in U$, then for each $t \in J$ we have

$$\begin{aligned}
\|Fx(t) - Fy(t)\| &= \|\sum_{i=1}^{m}\gamma_i T(t,0)B\int_0^{t_i} T(t_i,s)f\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau\Big)ds \\
&+ \int_0^t T(t,s)f\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau\Big)ds \\
&- \sum_{i=1}^{m}\gamma_i T(t,0)B\int_0^{t_i} T(t_i,s)f\Big(s,y(s),\int_0^s g(s,\tau,y(\tau))d\tau\Big)ds \\
&- \int_0^t T(t,s)f\Big(s,y(s),\int_0^s g(s,\tau,y(\tau))d\tau\Big)ds\| \\
&\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\|f\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau\Big) \\
&\quad -f\Big(s,y(s),\int_0^s g(s,\tau,y(\tau))d\tau\Big)\|ds \\
&\quad +M\int_0^t\|f\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau\Big) - f\Big(s,y(s),\int_0^s g(s,\tau,y(\tau))d\tau\Big)\|ds \\
&\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\ell\Big[\|x(s)-y(s)\| + \|\int_0^s g(s,\tau,x(\tau))d\tau \\
&\quad -\int_0^s g(s,\tau,y(\tau))d\tau\|\Big]ds \\
&\quad +M\int_0^t \ell\Big[\|x(s)-y(s)\| + \|\int_0^s g(s,\tau,x(\tau))d\tau \\
&\quad -\int_0^s g(s,\tau,y(\tau))d\tau\|\Big]ds \\
&\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\,\ell\int_0^{t_i}\Big[\|x(s)-y(s)\| + \ell_1 T\|x(s)-y(s)\|\Big]ds \\
&\quad +M\ell\int_0^t\Big[\|x(s)-y(s)\| + \ell_1 T\|x(s)-y(s)\|\Big]ds \\
&\leq M\Big[M\|B\|\sum_{i=1}^{m}|\gamma_i| + 1\Big]\ell\int_0^t\Big[\|x(s)-y(s)\| + \ell_1 T\|x(s)-y(s)\|\Big]ds \\
&\leq M\Big[M\|B\|\sum_{i=1}^{m}|\gamma_i| + 1\Big]\ell\Big[1 + \ell_1 T\Big]\int_0^t\|x(s)-y(s)\|ds.
\end{aligned}$$

Taking supremum over $t \in J$, we get,

$$\|Fx - Fy\|_E \leq M\Big[M\|B\|\sum_{i=1}^{m}|\gamma_i| + 1\Big]\ell\Big[1 + \ell_1 T\Big]\|x - y\|_E$$

Thus,

$$\|Fx - Fy\|_E \leq \Lambda\|x - y\|_E, \tag{4.1}$$

since $0 < \Lambda < 1$. This shows that operator F is a contraction. Uniqueness follows from $(H_f) - (H_g)$. Consequently, by (4.1), the operator F satisfies the assumptions of the Banach fixed point theorem. Therefore, in space U there is only one fixed point of F and this is the mild solution of the system (1.1). So, the proof of Theorem 4.1 is complete. ■

5 Example

In this section, we give an example of the partial integro-differential equation to illustrate the application of our main theorem

$$\begin{aligned} \frac{\partial v(t,u)}{\partial t} &= a(t,x)\frac{\partial^2 v(t,u)}{\partial u^2} + \mu\Big(t, v(t,u), \int_0^t \rho\big(t,s,v(s,u)\big)ds\Big), \\ v(t,0) &= v(t,\pi) = 0, \quad t \in J = [0,1],\ u \in I = [0,\pi], \\ v(0,u) &= \sum_{i=1}^{n}\alpha_i v(t_i,u),\ \ u \in I, \end{aligned} \tag{5.1}$$

where $\mu : J \times X \times X \to X$; $\rho : J \times J \times X \to X$ is continuous; $0 < t_i < b$; $\alpha_i \in \Re$ are prefixed numbers and the coefficient $a(t,x)$ is assumed to be continuous and uniformly holder continuous on t. Let $X = L^2[0,\pi]$. Define $A(t)$ an operator on X by $A(t)u = a(t,x)\frac{\partial^2 u}{\partial x^2}$ with the domain

$$D(A) = \Big\{v \in X \,\Big|\, v \text{ and } \frac{\partial v}{\partial x} \text{ are absolutely continuous}, \frac{\partial^2 v}{\partial x^2} \in X,\ v(0) = v(\pi) = 0\ \Big\}.$$

$\{A(t) :\ t \in [0,T]\}$ is shown to satisfy conditions (B_1) through (B_3), and the compactness condition for the resolvent is meet because it maps X to $D(A)$ holds (see [16]). The problem (5.1) can be modeled as the abstract integro-differential equation (1.1). Define now

$$f(t,x,y)u = \mu(t,x(u),y(u));\ \ g(t,s,x)u = \rho(t,s,x(u)).$$

The next result a consequence of Theorem 3.2.

Proposition 5.1. *Assume that the hypotheses* $(H_1) - (H_3)$ *hold. Then there exists a mild solution* v *of the system* (5.1) *provided*

$$\sup_{\rho\in[0,\infty)} \frac{\rho}{M\,\|\bar{M}\|_{L^1}\,\Omega(\rho)\Big[1 + M\|B\|\sum_{i=1}^{m}|\gamma_i|\,\Big]} > 1,$$

where $\bar{M}(t) = \max\{p(t), q_1(t)\}$, *is satisfied.*

The next result a consequence of Theorem 4.1.

Proposition 5.2. *Assume that the hypotheses* $(H_f) - (H_g)$ *hold. Then there exists an unique mild solution* v *of the system* (5.1) *provided*

$$M\Big[M\|B\|\sum_{i=1}^{m}|\gamma_i| + 1\Big]\ell\Big[1 + \ell_1 T\Big] < 1,$$

is satisfied.

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