A Novel Differential Equation Model of Energy Price: the Case of Jiangsu Province

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Abstract: Based on the complex relations among energy price, energy demand-supply and net imports, this paper establishes a novel differential equation model of energy price in Jiangsu Province, the stability of equilibrium point of the equation is analysed, the condition of local stability of energy equilibrium price for Jiangsu Province is given. At last the related conclusions are reached.

Keywords: energy resources demand-supply; energy price; net import; equilibrium price; stability; differential equation model

1 Introduction

Energy is a kind of natural resource that can be utilized to obtain power by mankind. It is an important basic resource for human survival, economical development and social progress. It is crucial to national economy and security. It also plays an important role both in the social and economic development and the structure of modernization. As the best means of regulation in market economy, price can truly and sensitively reflect the supply and demand relations of energy market. It is also an important lever in guiding and promoting the rational development, effective utilization and optimal allocation of energy resources.

Many scholars at home and at abroad carry out plenty of research work on energy supply, demand, energy price by applying scientific qualitative and quantitative method. For example, at abroad, Ryan W. McCarthy introduces an effective and credible method to assess reliability in energy supply systems in terms of adequacy and security by qualitative consideration [1]. Khalifa H. Ghali etc. carry out an integrative analysis for energy supply and demand of Canada [2] and so on. At home Yangang Zhang and Maoling Li analyse the supply and consumption of energy in Chinese by EMD on multi-scale based on the statistics of the total supply and demand of energy sources, establish the dynamics model of energy production and consumption to reveal its dynamic characteristic, carry out the numerical simulation and put forward the theory of virtual energy [3]. Lixin Tian etc. first introduce nonlinear chaos dynamics theory into energy demand-supply economic system, and score great successes in model of energy demand-supply and energy price and its analysis [4-8]. Based on the researches above, the main question is that net import hasn’t been taken into account in the study of the energy price changes.

As we know, Jiangsu is a province with huge consumption yet a little energy production, its energy mostly comes from import. Therefore it is very important for Jiangsu Province to maintain the energy equilibrium price basic stability. According to the relations among energy price, energy supply-demand and net import, we can establish differential equation model of energy price in Jiangsu Province. Thus the solution to the energy balance price stability problem is transformed into the stability of the equilibrium point of the energy price differential equations. Based on the complex relations among energy price, energy supply-demand and net imports, this paper establishes a new differential equation model of energy price in Jiangsu Province, the stability of equilibrium point of the equation is analysed, the condition of local stability of energy equilibrium price for Jiangsu Province is given. At last the related conclusions are reached.

2 Establishment of model

According to [5-6], in terms of the change rules of the interaction between supply and demand in relationship, energy supply-demand, energy net import and energy price of market are all the functions of time. We use $S(t)$, $D(t)$, $I(t)$, $P(t)$, $Q(t)$...
to represent quantity supplied of energy, quantity demanded of energy, net quantity imported of energy (import minus export), energy price, quantity stocked of energy (not including net import) respectively. Assume the rising rate of price \( \frac{dP(t)}{dt} \) is proportional to the gap between quantity stocked \( Q(t) \) and the determined quantity stocked \( \bar{Q}(t) \) plus net quantity imported, then

\[
\frac{dP(t)}{dt} = u(\bar{Q}(t) - Q(t)) + rI(t)
\]

(1)

where \( u > 0, r > 0 \), and they are all constants, their size only determine adjustment velocity of energy price. \( Q(t) \) is equivalent to the initial energy stock \( Q_0 \) plus total volume of gap between current energy supply and energy demand, then

\[
Q(t) = Q_0 + \int_0^t (S(\tau) - D(\tau))d\tau
\]

(2)

Suppose that the relation between energy demand and energy price isn’t linear. When energy price doesn’t surpass the threshold that purchasing power can bear, energy demand stimulates the increase of energy price, and energy price is positively correlated with energy demand, but when the increasing energy price surpasses the threshold, purchasing power can decrease with the increase of energy price, both are negatively correlated, yet \( \frac{dP(t)}{dt} \) promotes this relation, then

\[
D(t) = D_0 - \beta P(t) - \psi(P(t))\frac{dP(t)}{dt}
\]

(3)

where \( D_0 > 0, \beta > 0, \beta \) means marginal energy demand quantity. \( \psi(P(t)) \) is function of energy price \( P(t) \), its size reflects the dependence of energy demand on the increase rate of energy price. For simplicity of research, let \( \psi(P(t)) \) be linear function of \( P(t) \), then

\[
\psi(P(t)) = bP(t) + c
\]

(4)

where \( b, c \) are constants.

Energy supply is energy quantity that energy producing enterprise is willing to sell and can be sold under the given energy price level, thus

\[
S(t) = S_0 + \alpha P(t)
\]

(5)

where \( S_0 \geq 0, \alpha > 0, \alpha \) denotes marginal energy supply quantity[7].

Energy net import quantity is denoted by \( I(t) \). Suppose that \( I(t) \) is linear function of \( P(t) \), thus

\[
I(t) = n - mP(t)
\]

(6)

where \( m > 0, n > 0 \).

According to [6-7], combining (1), (2), (3), (4), (5), (6), we can obtain the following energy price differential equation model:

\[
\frac{d^2P(t)}{dt^2} = -u(\alpha + \beta)P(t) - [u(bP(t) + c) + rm]\frac{dP(t)}{dt} + u(D_0 - S_0)
\]

(7)

### 3 Analysis of closed orbit and stability of equilibrium point

According to [8], in (7), let \( P(t) = x(t) \), \( \frac{dP(t)}{dt} = y(t) \), so \( \frac{d^2P(t)}{dt^2} = \frac{dx(t)}{dt^2} = \frac{dy(t)}{dt} \), thus

\[
\begin{aligned}
\frac{dx(t)}{dt} &= y(t) \\
\frac{dy(t)}{dt} &= -u(\alpha + \beta)x(t) - [u(bx(t) + c) + rm]y(t) + u(D_0 - S_0)
\end{aligned}
\]

(8)

When \( D_0 > S_0 \), system (8) has only equilibrium point \( (\frac{D_0 - S_0}{\alpha + \beta}, \bar{P}) \). Energy price while energy demand quantity is equivalent to energy supply quantity at any time is energy equilibrium price, so \( \bar{P} = \frac{D_0 - S_0}{\alpha + \beta} \), it is energy equilibrium price.

Let \( x(t) = X(t) + \bar{P}, y(t) = Y(t) \), then system (8) becomes:

\[
\begin{aligned}
\frac{dX(t)}{dt} &= Y(t) \\
\frac{dY(t)}{dt} &= -u(\alpha + \beta)X(t) - u[b(X(t) + \bar{P}) + c]Y(t) - rmY(t)
\end{aligned}
\]

(9)

According to [9], equation system (9) is class I quadratic plane differential equation system, according to theory 12.5 of [6], when \( \sigma \neq 0 \), equation system (8) has not closed orbit and odd closed orbit.
Characteristic equation of equation system (9) at equilibrium point (0, 0) is

\[
\begin{vmatrix}
\lambda & -1 \\
u(\alpha + \beta) & \lambda + u(b\bar{p} + c) + rm
\end{vmatrix} = 0
\]

Simplify above

\[\lambda^2 + \lambda[u(b\bar{p} + c) + rm] + u(\alpha + \beta) = 0 \tag{11}\]

For simplicity of writing, let

\[
\sigma = u(b\bar{p} + c) + rm, \quad \Delta = u(\alpha + \beta)
\]

Due to \(u > 0, \alpha > 0, \beta > 0\), so \(\Delta > 0\).

Thus the above reduced characteristic equation (11) can be written as

\[\lambda^2 + \sigma \lambda + \Delta = 0 \tag{12}\]

Its characteristic roots are

\[
\lambda_{1,2} = -\frac{\sigma \pm \sqrt{\sigma^2 - 4\Delta}}{2}
\]

The following studies the characteristic roots in three cases which are different real roots, double roots, complex roots:

1. \(\sigma^2 - 4\Delta > 0\)
   Due to \(\Delta > 0\), so two roots are all real roots.
   According to [10], when \(\sigma < 0\), two roots are same positive, equilibrium point is instable node; when \(\sigma > 0\), two roots are same negative, equilibrium point is stable node.

2. \(\sigma^2 - 4\Delta = 0\)
   According to [10], when \(\sigma < 0\), characteristic roots are positive double root, equilibrium point is instable critical node; when \(\sigma > 0\), characteristic roots are negative double root, equilibrium point is asymptotically stable degenerate node.

3. \(\sigma^2 - 4\Delta < 0\)
   According to [10], when \(\sigma > 0\), characteristic roots are complex root and their real part are negative, equilibrium point is asymptotically stable focus; when \(\sigma < 0\), characteristic roots are complex root and their real part are positive, equilibrium point is instable focus; when \(\sigma = 0\), real part of characteristic roots are zero, thus equilibrium point is the center of the linearized equation of system (9) at the equilibrium point. At this time the category and stability of equilibrium point (0, 0) for equation system (9) can’t be determined by its linear part, but observe that vector field of the determined equation system (9) \(X(x, y), Y(x, y) = (y, -\sigma x - ubxy)\) is symmetrical by the axis of \(y\), \(X(-x, y) = X(x, y), Y(-x, y) = -Y(x, y)\). Therefore, equilibrium point of equation system (9) is center and stable.

Now analyse and explain the above equilibrium point: according to \(\bar{p} = \frac{D_0 - S_0}{\alpha + \beta}\), we get \(\frac{\partial p}{\partial D_0} = \frac{1}{\alpha + \beta} > 0\), so equilibrium price \(\bar{p}\) increases with the increase of \(D_0\). By similarly analysing, we know the relation of equilibrium price \(\bar{p}\) with \(S_0, \alpha, \beta, b\).

Now fix the value of parameter \(u, \alpha, \beta, b, D_0, S_0, m\), observe the impact of the changing parameter \(r\) on equilibrium price: let the initial value of \(r\) satisfy \(\sigma > 0\) and \(\sigma^2 - 4\Delta > 0\), thus equilibrium price is global asymptotically stable node; reducing the value of \(r\) till \(\sigma > 0\) and \(\sigma^2 - 4\Delta = 0\), at this time equilibrium price is asymptotically stable degenerate node; gradually reducing the value of \(r\) till \(\sigma > 0\) and \(\sigma^2 - 4\Delta < 0\), at this time equilibrium price is asymptotically stable focus; gradually reducing the value of \(r\) till \(\sigma = 0\) and \(\sigma^2 - 4\Delta < 0\), equilibrium price is center, at this time equilibrium price has a sign of periodically oscillating, but equilibrium price is stable; gradually reducing the value of \(r\) till \(\sigma < 0\) and \(\sigma^2 - 4\Delta < 0\), equilibrium price is instable focus, at this time energy price has a sign of oscillating and being increasing till infinity. Similarly analysing, we conclude the impact of every changing parameter on equilibrium price \(\bar{p}\).

4 Conclusions
This paper applies the stability theory of differential equation to conduct a dynamical analysis of the established difference equation model and proves the stability of system equilibrium price. The condition of local stability existence of energy price for Jiangsu and the impact of every parameter on equilibrium price are given. The model reveals the actual change law of energy supply and energy demand for Jiangsu within a certain range, and has a certain guiding significance to the practice. It requests people to strengthen macroscopic readjustment and control mechanism to solve the loss of market sensitivity and make up for market failures. This requires the price formation mechanism that sensitively reflect energy market change and the integration government-fixed price with government-guided price. Based on gradually introducing energy price mechanism, supply and demand mechanism, competition mechanism under the principle of fair

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and benefit, balance production and consumption, we should prevent the energy producing enterprises to manipulate the "monopoly price" and "monopoly profit" and eliminate the negative impact on the economy running smoothly.

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