**ISSN** 1749-3889 (print), 1749-3897 (online)
**International Journal of Nonlinear Science**
Vol.13(2012) No.2,pp.252-256

# Dynamic Analysis of Nonlinear Triopoly Game with Heterogeneous Players in Finance Invest Model

Hongxing Yao[1] *, Xiaohui Yang[2], Zhaoping Jiang[3]
[1, 2] Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, P.R. China
*(Received 20 September 2011 , accepted 01 January 2012)*

**Abstract:** Game of the duopoly production has been very common now. But with foreign companies enter the market, which makes the investment in the state of tension set. In the three oligopolistic market, the investment businesses compete in order to win the largest market share. But because of the market information is not fully established, the market will produce the three different decision-makers, namely: naive, limited rationality, selective. Then the stability of power systems is analyzed, and then we use the numerical Matlab Simulation to analyze. For a limited region in chaos, we apply delayed feedback to control chaos successfully, and making the system stable finally.

**Keywords:** triopoly game; investment; Nash equilibrium points; stability; chaos control

## 1 Introduction

Under the condition of incomplete information,because each investors have different information, their decisions will not the same when they decide what to do next step. In recent years, many scholars have studied the Cournot modle. For example, H.X. Yao and Chengyao Wu studied a class of Financial cournot duoply modle, and they used the linear control to control the chaos phenomenon successfully. And duopoly model has been used in many fields. For example, in 2005, H.X. Yao, Xu Feng applied the duopoly modle in the competitive advertising, and used the limited rational methods to analyze it. And E.M.E labbasy etc studied and analyzed a class of nonlinear triopoly game with heterogeneous players. In addition, Elsadan Agiza in this paper puts forward three different kinds of investors, they are na?ve investors, limited rationality investors and selective investors, and analyze the stability of the system. J. Zhang, Q. Da and Y. Wang [6], etc. have applied Agiza and Elsadan's method to study the heterogeneous nonlinear duopoly game model, and verify the stability of the model. Since China joined the WTO in 2001, due to the introduction of foreign businessmen, competition is becoming more intense. The investors concern the investment rate of return on capital providers, and the choice of investment decisions will be novelty. But in the course of the game, chaos phenomena still occur. In response to these chaotic phenomena, we can take appropriate control method to control these harmful phenomena of chaos [7]. In the past few years, there have been many control methods to control non-linear model, such as adaptive control method [8], the system variable pulse feedback control method [9]. But these are mostly used for the application of physical phenomena. Because there is a range of some parameters, and most system is discrete system, the choice of control method should be very cautious. For example, Pyragas [10] proposed the delayed feedback method in the economic system, and the method can be well applied.

This paper studies dynamic analysis of nonlinear triopoly game with heterogeneous players three heterogeneous investors are: naive investors, rational investors and selective investors. And then analyze the effect which the change of the parameters to the whole system's stability. Finally, we apply the delay feedback method to control the chaotic phenomena, and put forward the chaotic control method under the significance of the investment.

*Corresponding author. *E-mail address*: freehxyao@ujs.edu.cn


IJNS.2012.04.15/601

# 2 The model

n investment market, there are three investors, they invest the same products. Every investor strategy space on the market is the choice of customers' investment amount. The investment decision happened in the discrete cycle $t = 0, 1, 2...$;$x(t), y(t), z(t)$ represent the investment of at the $t$ stage. Customers share of the $t$ stage $U$ include the three investors' investment, and it is depended on the function $U = g(Q)$ ,$Q(t) = x(t) + y(t) + z(t)$

Obviously the total customer market share is about the amount of investment function, but with the saturated growth of the market ,the speed becomes slowly. Suppose at a stage ,the market can hold the biggest money $k$ , represent the money utilization rate of the investor $x, y, z$ .Then the total customers market share function is

$$U = g(Q) = a - b[K - (\beta_1 x + \beta_2 y + \beta_3 z)^2] \tag{1}$$

The parameter $a, b, \beta_1, \beta_2$ is the positive constant, and $a$represent the biggest customer share that the market can hold.$b$ means the customers share loss coefficient because of lack of funds or overflow.$\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}$ express the change of customers share that brought by tiny disturbance of $x, y, z$.

$$\begin{cases} \frac{\partial U}{\partial x} = 2b\beta 1[K - (\beta_1 x(t) + \beta_2 y(t) + \beta_3 z(t))] \\ \frac{\partial U}{\partial y} = 2b\beta_2[K - (\beta_1 x(t) + \beta_2 y(t) + \beta_3 z(t))] \\ \frac{\partial U}{\partial z} = 2b\beta_3[K - (\beta_1 x(t) + \beta_2 y(t) + \beta_3 z(t))] \end{cases} \tag{2}$$

Then $Ux = x\frac{\partial U}{\partial x}, Uy = y\frac{\partial U}{\partial y}, U_z = z\frac{\partial U}{\partial z}$,they represent the customer market share of investor $x, y, z$ at the $t$ stage.

Because manufactures don't have complete market information in real life, their behaviors are also different. But their same aim is to make their profit maximization, and then their investment can get the most benefits. Now let us consider three different kinds of investment decisions.

The first is the limited rational decider $A$, the $t + 1$ period has the following form:

$$x(t+1) = x(t) + \alpha x(t)\frac{\partial Ux}{\partial x} \tag{3}$$

The second is the selective decider $B$, then we will get the next decision of the decider $B$ as following:

$$y(t+1) = (1-\nu)y(t) + \nu f(y) \tag{4}$$

$f(y)$ is the solusion of

$$\frac{\partial U_y}{\partial y} = 2b\beta_2 K - 4b{\beta_2}^2 y(t) - 2b\beta_1\beta_2 x(t) - 2b\beta_1\beta_3 z(t) = 0$$

The third is the naive decider $C$, we can apply the following equation, and obtain the next decision:

$$\frac{\partial U_z}{\partial z} = 2b\beta_3 K - 4b{\beta_3}^2 z(t) - 2b\beta_1\beta_2 x(t) - 2b\beta_1\beta_2 y(t) = 0$$

Then:

$$z(t+1) = \frac{k - (\beta_1 x(t) + \beta_2 y(t))}{2\beta_3} \tag{5}$$

Combined with(3)-(5),we can obtain the three deciders'decision function is

$$\begin{cases} x(t+1) = x(t) + \alpha x(t)[2b\beta_1 k - 4b{\beta_1}^2 x(t) - 2b\beta 1\beta_2 y(t) - 2b\beta 1\beta_3 z(t)] \\ y(t+1) = (1-\nu)y(t) + \nu\frac{k-(\beta_1 x\mathrm{t}+\beta_3 z(t))}{2\beta_2} \\ z(t+1) = \frac{k-(\beta_1 x(t)+\beta_2 y(t))}{2\beta_3} \end{cases} \tag{6}$$

# 3 Qualitative behavior of the model

## 3.1 The equilibrium point

In the system (6), let $x(t+1) = x(t), y(t+1) = y(t), z(t+1) = z(t)$ and then we can obtain its two fixed points equations:

$$\begin{cases} x(t)[2b\beta_1 k - 4b{\beta_1}^2 x(t) - 2b\beta 1\beta_2 y(t) - 2b\beta 1\beta_3 z(t)] = 0 \\ \beta_1 x(t) + 2\beta 2y(t) + \beta_3 z(t) = k \\ \beta_1 x(t) + \beta 2y(t) + 2\beta_3 z(t) = k \end{cases} \tag{7}$$

We can get the two equilibrium point is: $E_1 = (0, \frac{K}{3\beta_2}, \frac{K}{3\beta_3})$,$E_2 = (\frac{K}{4\beta_1}, \frac{K}{4\beta_2}, \frac{K}{4\beta_3})$. $E_1$ is bounded equilibrium, and $E_2$ is Nash equilibrium. The following we discuss local stability of the two points.

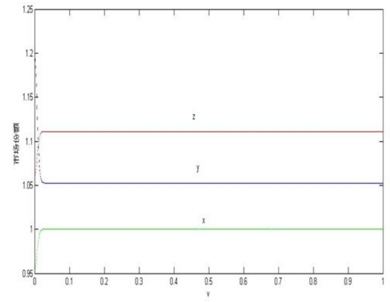

Figure 1: Bifurcate figure when $\alpha = 0.25$

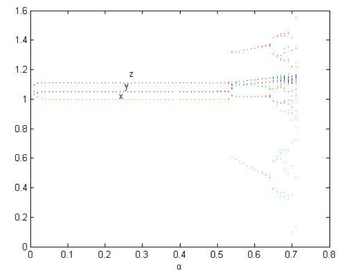

Figure 2: Bifurcate figure when $\nu = 0.1$

### 3.2 Stability analysis

**Theorem 1** *Boundary point $E_1$ is saddle points of system (6).*

**Proof.** We put point $E_1$ into Jocobian matrix, then we get:

$$J(E_1) = \begin{bmatrix} 1 + \frac{2}{3}\alpha\beta_1 k & 0 & 0 \\ \frac{-\nu\beta_1}{2\beta_2} & 1-\nu & \frac{-\nu\beta_3}{2\beta_2} \\ \frac{-\beta_1}{2\beta_3} & \frac{-\beta_2}{2\beta_3} & 0 \end{bmatrix}$$

And then we obtain the two characteristic values at the Boundary point $E_1 = (0, \frac{K}{3\beta_2}, \frac{K}{3\beta_3})$:$\lambda_1 = 1 + \frac{2}{3}\alpha\beta_1 k$,$\lambda_{2,3} = \frac{1}{2} - \frac{1}{2}\nu \pm \frac{1}{2}\sqrt{\nu^2+1}$.$\alpha, \beta_1, k$ are all positive parameters, and $\nu \in (0, 1)$.Obviously $|\lambda_1| > 1$,$|\lambda_{2,3}| < 1$.So Boundary point $E_1$ is saddle points of system (6). ■

**Theorem 2** *When meet the conditions* $\begin{cases} 3 + A_1 - A_2 - 3A_3 > 0 \\ 1 - A_2 + A_3(A_1 - A_3) > 0 \\ 1 - A_1 + A_2 - A_3 > 0 \end{cases}$ *,point $E_2$ is the local stability.*

**Proof.** The Jocobian matrix at point$E_2 = (\frac{K}{4\beta_1}, \frac{K}{4\beta_2}, \frac{K}{4\beta_3})$ of system6is $J(E_2) = \begin{bmatrix} 1 - b\alpha\beta_1 k & -\frac{1}{2}b\alpha\beta_2 k & -\frac{1}{2}b\alpha\beta_3 k \\ \frac{-\nu\beta_1}{2\beta_2} & 1-\nu & \frac{-\nu\beta_3}{2\beta_2} \\ \frac{-\beta_1}{2\beta_3} & \frac{-\beta_2}{2\beta_3} & 0 \end{bmatrix}$
From above, we get the eigenvalue of $E_2$ satisfy the following equation $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$.and $A_1 = b\alpha\beta_1 k + \nu - 2$, $A_2 = \frac{3}{4}b\alpha\beta_1 k\nu - \frac{5}{4}b\alpha\beta_1 k - \frac{5}{4}\nu + 1$, $A_3 = -\frac{3}{8}b\alpha\beta_1 k(\nu - 1) + \frac{1}{4}\nu$.To make all the eigenvalues of the $J(E_2)$,$|\lambda| < 1$,it needs satisfy the Jury condition: $\begin{cases} 3 + A_1 - A_2 - 3A_3 > 0 \\ 1 - A_2 + A_3(A_1 - A_3) > 0 \\ 1 - A_1 + A_2 - A_3 > 0 \end{cases}$ ■

# 4 The numerical simulation and analysis

Before the system get the stable point, the manufacturers will make a dynamic game. Manufacturers will continue to self-improvement investment system. And according to the last period's profitability the manufacturers will change the next period's investment.

In order to research the equilibrium of the local stability features, we use the following parameters values $k = 4, \beta_1 = 1, \beta_2 = 0.95, \beta_3 = 0.9, b = 0.92$.Reference figure 1, we can see, for $\nu \in (0, 1)$ when the value is small, the system is Stable at Nash equilibrium point. But with the increase of $\alpha$,the system also can appear bifurcate phenomenon. So we can get the speed on the edge of the rational has important influence on the whole system.

Now we analyze the bifurcate figure of the system with the change of weight $\nu$.Reference figure 1, we can see, at interval0,0.5,the system is stable. However, with the increase of $\alpha$, system gradually appear more than double cycle and bifurcation phenomena cycle. And when $\alpha$ increase to a certain value, system appears chaos phenomena.

Figure2 and figure3 show the bifurcate figure when $\nu = 0.1, \nu = 0.4$. All these figures show that Nash equilibrium point is local stable for smaller $\alpha$.And we also get that Chaotic phenomenon appear more early with increase of $\nu$. Figure 4 is the figure when $\nu = 0.8$.we can get that the whole system is stable at Nash equilibrium point for $\alpha \in (0.1)$.

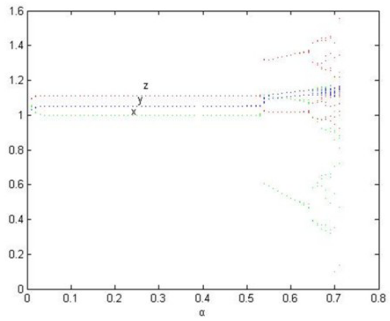


Figure 3: Bifurcate figure when $\nu = 0.4$

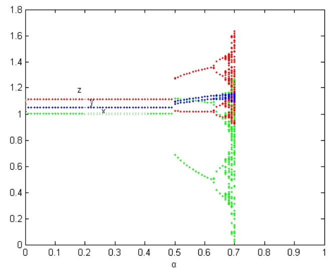


Figure 4: Bifurcate figure when $\nu = 0.8$

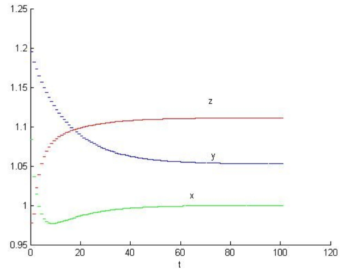


Figure 5: Dynamic figure with the change of $\omega$

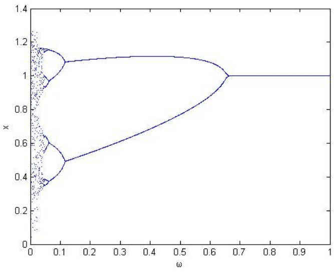


Figure 6: Three investors' investment share when $\omega = 0.9$

# 5 Chaos control

From the above we can see the chaos phenomena in the numerical simulation. In order to prevent the system into chaos, we can apply time delay methods to control.

$$\mu(t) = \omega(x(t+1-T) - x(t+1)), t > T$$

$T$ is the delay time,$\omega$ is control factor.

We can get the parameter Value $(k, \alpha, v, \beta_1, \beta_2, \beta_3, b) = (4, 0.7, 0.1, 0.9, 0.9, 0.9, 0.92)$, When the system appear chaos phenomena. Reference figure 1, we set $T = 1$,then the control system can be expressed as following:

$$\begin{cases} x(t+1) = x(t) + \frac{\alpha x(t)}{\omega+1}[2b\beta_1 k - 4b{\beta_1}^2 x(t) - 2b\beta 1\beta_2 y(t) - 2b\beta 1\beta_3 z(t)] \\ y(t+1) = (1-\nu)y(t) + \nu\frac{k-(\beta_1 x+\beta_3 z)}{2\beta_2} \\ z(t+1) = \frac{k-(\beta_1 x+\beta_2 y)}{2\beta_3} \end{cases}$$

Then the Jocobian matrix is:

$$J(x,y,z) = \begin{bmatrix} 1 + \frac{2b\alpha\beta_1 - 2b\alpha\beta_1\beta_2 y - 2b\alpha\beta_1\beta_3 z - 8b\alpha{\beta_1}^2 x}{\omega+1} & \frac{-2b\alpha\beta_1\beta_2 x}{\omega+1} & \frac{-2b\alpha\beta_1\beta_3 x}{\omega+1} \\ \frac{-\nu\beta_1}{2\beta_2} & 1-\nu & \frac{-\nu\beta_3}{2\beta_2} \\ \frac{-\beta_1}{2\beta_3} & \frac{-\beta_2}{2\beta_3} & 0 \end{bmatrix}$$

Then we put the Nash equilibrium and Chaotic parameter into above, the Jocobian matrix is changed as

$$J(x,y,z) = \begin{pmatrix} \frac{\omega-5.77488}{\omega+1} & \frac{-1.36059}{\omega+1} & \frac{-1.28898}{\omega+1} \\ -0.05263 & 0.9 & -0.04737 \\ -0.55556 & -0.52778 & 0 \end{pmatrix}$$

According to the Jury conditions, when and only when $\omega > 0.64682$all the eigenvalues of the system are less than 1.And this means that the system is stable around Nash equilibrium points.

We can see from figure 5, when $\omega(x(0), y(0), z(0)) = (1.1, 1.2, 0.8)$,let $\omega > 0.64682$,the system is controlled to be stable from chaos.

Figure 5 shows that the system's stable figure when $\omega = 0.8$,and $(x(0), y(0), z(0)) = (1.1, 1.2, 0.8)$.we can get that the system converge to Nash equilibrium point.

# 6 Conclusion

This paper mainly studies triopoly game with heterogeneous players with incomplete information in the market. Then we analyze the stability of the system. And then we apply numerical simulation to analyze the important parameters. For example, limited rational investor can use the change of adjustment speed $\alpha$ to bring effect on the stability of the system. Selective investor can use the change of adjustment weigh $\alpha$ to bring effect on the stability of the system. However, the system in some areas will still get chaos phenomena. The system will appear bifurcation even the chaotic state with adjustment speed increase $\alpha$. In order to solve this problem, we apply the method of delay feedback to control the system effectively. Finally, the system is controlled to be stable at Nash equilibrium point.

# Acknowledgments

The paper is supported by the National Nature Science Foundation of China (No. 70871056)and Jiangsu province ”three talent peak” project.

# References

[1] H.X.Yao, Chengyao Wu. Analysis and Control of Chaos in a Financial Cournot Model[J]. *Systems Engineering Theory & Practice*,5(2007):55-62.

[2] Hongxing Yao, Feng Xu. Complex dynamics analysis for a duopoly advertising model with bounded rationality[J]. *Systems Engineering Theory & Practice*, 12(2005): 32-37.

[3] Tingfang Zhang, H.X.Yao. Feedback control and techniques and analysis of Chen's chaotic system[J]. *Systems Engineering Theory & Practice.*, 8(2005):97-102.

[4] Peiqing Tong. Aptive control of chaos[J]. *Journal of Physics*,2(1995):169-176.

[5] Yang T,Yang L B,Yang C M. Theory of control of chaos using sample data[J] *Phys Lett A*,246,(1997):248-288.

[6] K.Pyrages. Continuous control of chaos by self-controllingfeedback. *Phy,Lett.*,170(1992):421-428.

[7] E.M.Elabbasy,H.N.Agiza. Analysis of nonlinear triopoly game with heterogeneous players. *Computer and Mathematics with Applications*,57(2009):488-499.

[8] H.N.Agiza,A.A.Elsadany. Nonlinear dynamics in the Cournot duopoly game with heterogeneous players. *,Physica A*,320(2003):512-524.

[9] H.N.Agiza,A.A.Elsadany. Chaotic dynamics in nonlinear duopoly game with heterogeneous players. *Appl.Math comput.*,149(2004):843-860.

[10] J.Zhang,Q.Daand,Y.Wang. Analysis of nonlinear duopoly game with heterogeneous players *Econ.Model* ,24(2007):138-148.