ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.13(2012) No.2,pp.194-199

The Proposal of a New Niche-fitness Model Based on Type-2 Fuzzy Niche Concept and Its Application

Tong Zhao[1,2], Tengfei Zha[3], Yimin Li[3] *
[1] Department of Mathematics,East China Normal University,Shanghai,200241,China
[2]Nantong shipping college,Nantong,Jiangsu,226010,China
[3]Faculty of Science, Jiangsu University, Zhenjiang, 212013, PR China
(Received 23 June 2011 , accepted 15 January 2012)

Abstract: The species' niche-fitness is one of the important measurements to assess the species' ecological adaptive capacity to environment. It depends on the difference between the species' optimum environmental requirement and the actual situation of its immediate environment. Because the species' living state in the immediate environment and their optimum environment requirement can be respectively reflected by the species' practical niche and its optimum niche, so in this paper, we expand the definition of niche-fitness into the summary of the similarity between the species' practical niche and the optimum niche on all ecological factors axis considering the weight of different ecological factor. A new model is established on the base of the type-2 fuzzy niche concept and the fuzzy sets' similarity calculation. The article carries out an in-depth analysis by using crop population as its study object and nonlinear regression analysis as its method in the simulation part. It is found that the fitness values obtained from the new model have wide and well-proportioned value distribution and are obviously nonlinear correlative to grain yield. As a result, the new model is more dynamic, reasonable and fitter for the reality compared with previous models.

Keywords:fuzzy niche; optimum niche; similarity;importance of ecological factors; niche-fitness

1 Introduction

Species' niche-fitness is regarded as the characterization of ecological adaptive capacity to environment. Its value depends on the difference between the species' optimum environmental requirement and the actual situation of its immediate environment. It is one of the most important measurements which are closely related to the niche concept. As to Hutchinson expression, if each of measurable environmental features was marked as a coordinate in n-dimensional space, then the niche could be defined as a sector in the n-dimensional space in which fitness of individual organism was positive[1]. That is to say, the fitness at different points in n-dimensional space demonstrating niche is different. The nearer it approaches the region's border, the lower the fitness will be.

Since the 1960s, researches on niche-fitness theories have been primarily focused on the utility of species on their environmental resource, but the study on this respect are much less quantitative. So Li Zizhen[2] defined the niche-fitness of species in its niche region as the closeness degree between optimum niche point and actual resource state. He proposed new formulas to calculate the absolute difference of resource points and the niche fitness of species in the paper. Although the result obtained from this model have wider distribution range and contain a selectable parameter, the limitation of the model lead to the inaccuracy and distortion of the niche-fitness.

So on the base of previous research, we give a new fuzzy expression of optimum niche on the base of fuzzy niche definition in Li Yimin's article[3-5] and redefine the niche-fitness with the introduction of the similarity between different types of niche and the consideration of the weight of various ecological factors. We propose new formulas to calculate and the analysis of the example shows that the result of the new model is more rational, more readily applicable and also close to the practice more.

*Corresponding author. *E-mail address*: llym@ujs.edu.cn

IJNS.2012.04.15/593

2 The new model of the niche-fitness

Choose a kind of species population as a research object and a group of experimental area to be the model's prediction sample to assess how suitable each experimental area fits the species' inhabitability.

The first-phase preparation is to determine the important environmental variables x_k $(k = 1, 2, \cdots, m)$ in the species' growing process and obtain the environmental condition $\vec{X} = (x_1, x_2, \cdots, x_m)$. Consequently, $\vec{X}_i = (x_{i1}, x_{i2}, \cdots, x_{im})$ denote species' immediate environment condition in the ith experimental area.

Next, we turn our thoughts to the establishment of a new model which can measure the niche-fitness of species to a habitat properly. This work is proceeding according to four steps.

The first step is to measure the niche occupied by the species in each ecological factor axis and the realized niche corresponding to the index value of the ecological variable in each experimental area on the base of the exiting fuzzy definition in Li Yimin's article[3-5].

Let the x_k-axis be the kth ecological factor axis for the species, $\tilde{A}_k$ is the Type-2 fuzzy set of the resource curve whose upper MF and lower MF[6] are respectively denoted by $\bar{\mu}_{\tilde{A}_k}(x)$ and $\underline{\mu}_{\tilde{A}_k}(x)$. The niche of the species on the axis is defined as the footprint of uncertainty (FOU) of $\tilde{A}_k$ on the axis.

When the species inhabit in the ith experimental area, it will occupy the niche related to the point x_{ik} as its realized niche on the kth ecological factor axis, namely, $R_{ik} = \left[\bar{\mu}_{\tilde{A}_k}(x_{ik}), \underline{\mu}_{\tilde{A}_k}(x_{ik})\right]$. So R_{ik} is a Type-1 internal fuzzy number[7] and it also can be presented as $R_{ik} = \int_{u\in[0,1]} U_{x_{ik}}(u)/u, u \in J_{x_{ik}} \subseteq [0,1]$, where $U_{x_{ik}}(u)$ is the M-F. If the realized niche on all ecological factors in the ith experimental area are obtained, a cluster will be created as $\Lambda_i = [R_{i1}, R_{i2}, \cdots, R_{ik} \cdots, R_{im}]$.

The second step is proceeded to give the fuzzy illustration of the optimum niche of the species on an ecological factor axis on the base of Li Yimin's niche definition[4].

Definition 1 *Suppose$\tilde{A}$ is the type-2 fuzzy set of the resource curve whose FOU[6] on $x-$axis is the niche of the species corresponding to a certain ecological factor, and a numerical range $[p, q]$ contains all values of variable x which lead to the optimum environmental tolerance and a riotous growth state of species. The optimum niche of species on axis is defined as the part of the niche corresponding to the numerical range of $[p, q]$.*

Following the above definition, suppose $[p_k, q_k]$ is the set of optimum values ecological variablex_k can have. The optimum niche on this axis is a type-2 fuzzy set $\tilde{H}_k$ in $[p_k, q_k]$. Its membership grade of any point $x_k \in [p_k, q_k]$ in $\tilde{H}_k$ can be present as $\mu_{\tilde{H}_k}(x) = \int\limits_{u\in[0,1]} U_{x_k}(u)/u$, $p_k \leq x_k \leq q_k$, $u \in J_{x_k} \subseteq [0,1]$[6]. So the cluster of the optimum niche of species on each ecological factor axis is $\Lambda_i^* = \left[\tilde{H}_1, \tilde{H}_2, \cdots \tilde{H}_k, \cdots, \tilde{H}_m\right]$. It will be used as a golden standard to reflect the species' optimum environment requirement to the habitat.

In the third step, we will propose a similarity formula to measure the closeness of the species' realized niche which species actually occupies on one ecological factor axis in a certain environment and the optimum niche of species corresponding to the same ecological factor. That is to say, the important task in this stage is to introduce a new method to respectively measure the similarity between R_{ik} $(i = 1, 2, \cdots, n)$ and $\tilde{H}_k$. This work is started with the definition of the FOU[6]. According to the definition of the FOU, $\tilde{H}_k$ can be regarded as the union of all primary membership related to the ecological variable $x_k \in [p_k, q_k]$, namely, $\tilde{H}_k = \bigcup\limits_{x_k\in(p_k,q_k)} V_{x_k}$. When variable$x_k$ is fixed at a certain value within the range of $[p_k, q_k]$, V_{x_k} can be noted as $\left[\bar{\mu}_{\tilde{A}_k}(x_k),\ \mu_{\tilde{A}_k} x_k\right]$, which is also the expression of the realized niche of species related to a certain point x. Consequently, looked at this from another angle, the optimum niche can be regarded as the union of the realized niche of the species related to all points in $[p_k, q_k]$.

Here, $V_{x_k} = \int_{u\in[0,1]} U_{x_k}(u)/u, u \in J_{x_k} \subseteq [0,1]$, and $U_{x_k}(u)$ is the MF of the type-1 fuzzy set V_{x_k}. x_k is regarded as a dynamic parameter and the presence of the MF varies with the different value of x_k.

Proceeding from this point, we fix our research on the ith experimental area and define the similarity between R_{ik} and $\tilde{H}_k$ as follows:

Definition 2 *The similarity between the similarity between R_{ik} and $\tilde{H}_k$ is the average similarity of the R_{ik} and all representation of V_{x_k} under the condition of $p_k \leq x_k \leq q_k$.*

Specifically, when ecological variable x_k is a discrete variable in the range of $[p_k, q_k]$, the values of x_k can be noted as $x_k^{(1)}, x_k^{(2)}, \cdots, x_k^{(t)}$, so

$$\tau(R_{x_{ik}}, \tilde{H}_k) = \frac{1}{t}\left[\tau(R_{x_{ik}}, V_{x_k^{(1)}}) + \tau(R_{x_{ik}}, V_{x_k^{(2)}}) + \cdots + \tau(R_{x_{ik}}, V_{x_k^{(t)}})\right]; \tag{1}$$

when ecological variable x_k is a continuous variable within the range of $[p_k, q_k]$,

$$\tau(R_{x_{ik}}, \tilde{H}_k) = \frac{1}{q_k - p_k} \int_{p_k}^{q_k} \tau(R_{x_{ik}}, V_{x_k}) dx \tag{2}$$

Here, $\tau(R_{x_{ik}}, V_{x_k})$ denote the similarity of two type-1 fuzzy set. There are several existing similarity formulas to be adopted in the calculation [8].

The final step is embarked on the formula of the niche-fitness of species as follows:

$$f_i = \sum_{k=1}^{m} a_k \tau(R_{x_{ik}}, \tilde{H}_k), \tag{3}$$

where a_k is the weight value corresponding to the kth ecological factor.

3 Simulation of the new model in the crop growth:

All data used in modeling are from the field experiments recorded in the article of Li Zizhen[9]. These experiments were carried out on horizontal terraces in the semi-arid area of Mid-Gansu Province. The drought-resistant and improved variety of the spring wheat, Dingxi 24, is selected as the experimental crop. Every experiment is done with timed observation at fixed positions. The quantitative indexes of environmental factor such as sunlight, temperature, soil water, soil nutrition are measured in each experimental region. The results are shown in the Tables 1.

Table 1 the observed index of ecological factors in the experiments

NO	The observed value of the experimental region						Yield
	$x_1\,(^\circ C)$	$x_2\,(^\circ C)$	$x_3\,(\%)$	$x_4\,(\%)$	$x_5(mg \cdot kg^{-1})$	$x_6(mg \cdot kg^{-1})$	$Y(g \cdot m^{-2})$
1	18.76	19.12	12.55	13.22	55.07	15.95	207.1
2	18.93	19.56	13.47	13.38	66.68	16.53	270.0
3	19.10	19.80	13.67	14.13	84.95	19.13	357.1
4	20.33	20.26	14.39	15.24	95.65	22.89	441.7
5	19.36	19.15	10.34	9.67	57.55	15.66	172.9
6	18.91	19.33	10.72	9.86	67.99	16.81	227.0
7	19.12	19.59	11.99	11.34	83.65	20.00	274.9
8	19.39	20.03	11.81	11.36	92.78	22.31	316.7
9	18.37	18.63	8.31	7.43	57.68	14.50	136.7
10	18.64	19.10	7.96	7.63	66.81	16.24	163.5
11	18.66	18.87	7.76	8.35	79.73	17.97	177.0
12	17.67	17.73	7.78	7.65	86.13	18.84	169.2
13	18.86	19.15	10.52	9.88	62.77	15.65	181.8
14	18.91	19.77	13.10	11.35	62.90	17.10	229.0
15	20.82	21.17	12.71	12.08	54.07	17.39	290.3

Illustration: a) the main variable in the table:x_1= the medial temperature in soil of 0-30 cm; x_2= the medial temperature in soil of 0-50 cm; x_3= the rate of water content in the soil layer of 0-30 cm; x_4= the rate of water content in the soil layer of 30-60 cm;x_5= the amount of quick-acting nitrogen manure in the soil; x_6= the amount of quick-acting phosphate manure in the soil

b) the data in the table are mean values of three samples

Let us use the new model to measure the niche-fitness of the species in experimental regions.

(1). The relative MF[10] related to $x_k(k = 1, 2, 3, 4, 5, 6)$are shown as follows:

$$\mu_{\tilde{A}_1}(x_1) = \exp\left[-\frac{1}{2}\left(\frac{x_1 - m_1}{5}\right)^2\right], \quad m_1 \in [21.35,\ 22.65], \quad x_1 \in (16.0,\ 28.0); \tag{4}$$

$$\mu_{\tilde{A}_2}(x_2) = \exp\left[-\frac{1}{2}\left(\frac{x_2 - m_2}{5.1}\right)^2\right], \quad m_2 \in [21.05,\ 23.15], \quad x_2 \in (16.5,\ 28.5); \tag{5}$$

$$\mu_{\tilde{A}_3}(x_3) = \exp\left[-\frac{1}{2}\left(\frac{x_3 - m_3}{10}\right)^2\right], \quad m_3 \in [15.20,\ 17.60], \quad x_3 \in (6.40,\ 26.40\,); \tag{6}$$

$$\mu_{\tilde{A}_4}(x_4) = \exp\left[-\frac{1}{2}\left(\frac{x_4 - m_4}{10.40}\right)^2\right], \quad m_4 \in [15.60,\ 18.00], \quad x_4 \in (6.80,\ 26.80\,); \tag{7}$$

$$\mu_{\tilde{A}_5}(x_5) = \exp\left[-\frac{1}{2}\left(\frac{x_5 - m_5}{55.63}\right)^2\right], \quad m_5 \in [97.70,\ 113.70], \quad x_5 \in (50.07,\ 161.33\,); \tag{8}$$

$$\mu_{\tilde{A}_6}(x_6) = \exp\left[-\frac{1}{2}\left(\frac{x_6 - m_6}{14.40}\right)^2\right], \quad m_6 \in [23.60,\ 25.60], \quad x_6 \in (10.20,\ 39.00\,); \tag{9}$$

(2). Take the first experimental region as the example to show how to calculate by the method.
The index values of all ecological factors can be presented as a cluster:

$$X_i = [x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}] = [18.76,\ 19.12,\ 12.55,\ 13.22,\ 55.07,\ 15.95]$$

The upper and lower MF of variable x_1 are obtained as

$$\bar{\mu}_{\tilde{A}_1}(x_1) = \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{x_1 - 21.35}{5}\right)^2\right] & x_1 < 21.35 \\ 1 & 21.35 \le x_1 \le 22.65 \\ \exp\left[-\frac{1}{2}\left(\frac{x_1 - 22.65}{5}\right)^2\right] & x_1 > 22.65 \end{cases} \tag{10}$$

$$\underline{\mu}_{\tilde{A}_1}(x_1) = \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{x_1 - 22.65}{5}\right)^2\right] & x_1 \le 22.00 \\ \exp\left[-\frac{1}{2}\left(\frac{x_1 - 21.35}{5}\right)^2\right] & x_1 > 22.00 \end{cases} \tag{11}$$

As previously stated, the realized niche of species related to the point x_{11} is noted as $R_{11} = \left[\bar{\mu}_{\tilde{A}_1}(x_{11}), \underline{\mu}_{\tilde{A}_1}(x_{11})\right]$ and the MF is

$$U_{x_{11}}(u) = \exp\left[-\left(\frac{u - 0.8068}{0.2712}\right)^2\right]. \tag{12}$$

The optimum niche is $\tilde{H}_1 = \bigcup_{x_1 \in [21.35, 22.65]} V_{x_1}$, and the MF of V_{x_1} can be denoted as

$$U_{x_1}(u) = \exp\left[-\left(\frac{u - a_1}{\sigma_1}\right)^2\right]. \tag{13}$$

So according to formula (2), we get

$$\tau(R_{x_{11}}, \tilde{H}_1) = \frac{1}{22.65 - 21.35}\int_{21.35}^{22.65} \tau(R_{x_{11}}, V_{x_1})dx. \tag{14}$$

Here,

$$\tau(R_{x_{11}}, V_{x_1}) = (R_{x_{11}} \circ\ V_{x_1}) \wedge (R_{x_{11}} \odot\ V_{x_1}), \tag{15}$$

where$R_{x_{11}} \circ\ V_{x_1}$ and $R_{x_{11}} \odot\ V_{x_1}$ are respectively the inner product and the outer product of the two fuzzy sets[8]. And the calculated result related to the point x_{11} is $\tau(R_{x_{11}}, \tilde{H}_1) = 0.7026$.

Through the calculation, we obtain the similarity value of species corresponding to all factors and the cluster of the similarity values is presented as follows:

$$\Gamma_1 = [0.7026,\ 0.9356,\ 0.8901,\ 0.9072,\ 0.4926,\ 0.3056].$$

In accordance with the research on the spring wheat's growth habit, we determine the weight values of the different ecological factor and the niche-fitness can be calculated according to the formula(3):

$$f_1 = 0.7026 \times\ 0.1667\ + 0.9356 \times\ 0.1429\ \ + 0.8901 \times 0.1806$$
$$+\ 0.9072 \times 0.1289\ \ + 0.4926 \times 0.2024 +\ 0.3056 \times 0.1785$$

The final calculated result is $f_1 = 0.6827$.

(3) In order to operate the calculation efficiently, we use software Maple to repeat above steps by programming. The results of the calculation are read out as:

$$f_1 = 0.6827,\ f_2 = 0.7567,\ f_3 = 0.8691,\ f_4 = 0.9736,\ f_5 = 0.6294,$$
$$f_6 = 0.6874, f_7 = 0.8401, f_8 = 0.8918, f_9 = 0.4953, f_{10} = 0.5603,$$
$$f_{11} = 0.6309, f_{12} = 0.5967, f_{13} = 0.6410, f_{14} = 0.7336, f_{15} = 0.7537.$$

4 Comparison and conclusion:

We adopt nonlinear regression $y = ae^{bx}$ to analyze the correlation of the niche-fitness value and the crop yield, and we obtain the relationship as follows:

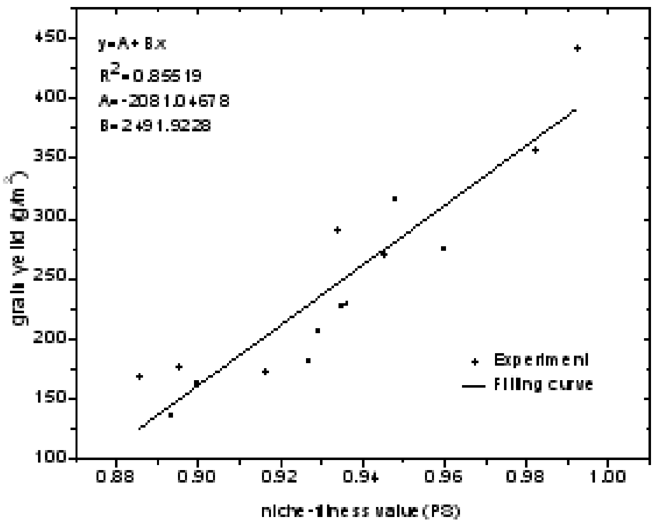

Figure 1: The relationship between niche-fitness(f_i) and grain yield of spring wheat

The relationship between the niche-fitness calculated by Li Zizhen's model and crop yield and the relationship between the niche-fitness calculated by Feinsinger's percentage similarity formula[11] are respectively shown as follows:

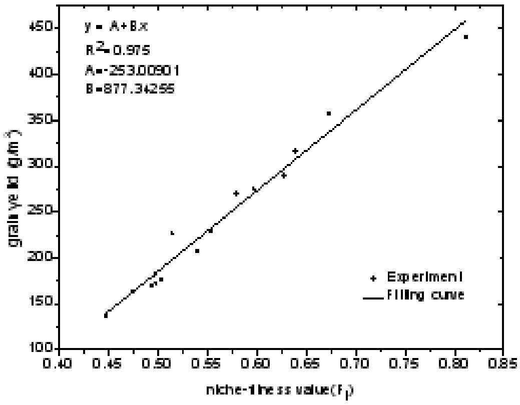

Fig.2 The relationship between niche-fitness(F_i) and grain yield of spring wheat

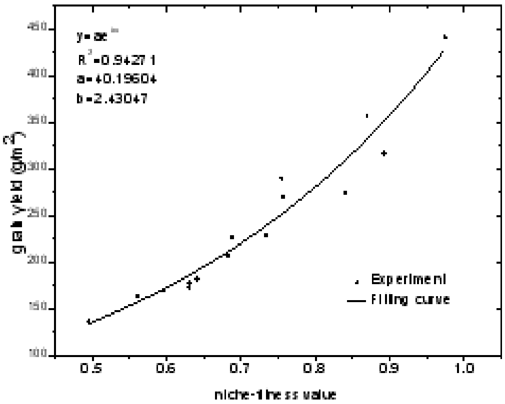

Fig. 3 The relationship between niche-fitness(PS) and grain yield of spring wheat

Through the comparison of these images, we find that the fitness values obtained from the new model have wide and well-proportioned value distribution. The first image shows that the niche-fitness values have obvious nonlinear correlation to grain yields, which is better than the linear relationship presented by other two images. Because the larger the niche-fitness value is, the more the amount of increase in production will be. As a result, the new model is more dynamic, reasonable and fitter for the reality compared with previous models.

Acknowledgements

Works supported by national science fundation(11072090) and project of advanced talents of Jiangsu University(10JDG093).

References

[1] G.E. Hutchinson, Concluding remarks. Cold Spring Harbor Symposium. *Quant. Biol.*, 22(1957):415-427.
[2] Z.Z. Li and H. Lin. The niche-fitness model of crop population and its application[J]. *Ecological Modelling*, 104(2)(1997): 199-203.
[3] Y.M. Li, X.H. Sun , C.Y. Gong. Niche width and niche overlap: a method based on type-2 fuzzy set. *Ecol. Res.*, 21(2006):713–722.
[4] Y.M. Li , X.H. Sun. Modelling dynamic niche and community model by type-2 fuzzy set. *Ecological modeling*, 211(2008):375–382.
[5] Y.M. Li , J. Hua. Type-2 fuzzy mathematical modeling and analysis of the dynamical behaviors of complex ecosystems[J]. *Simulation Modelling Practice and Theory*, 16(2008):1379–1391.
[6] J.M. Mendel. Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions[M]. Prentice-Hall, Upper-Saddle River (2001).
[7] I. Albrecht. Fuzzy rules and fuzzy functions: A combination of logic and arithmetic operations for fuzzy numbers. *Fuzzy sets and system*, 99(1998): 49-56.
[8] L.Y. Han , P.Z. Wang. Application of fuzzy mathematic. Capital university of economics and business press: Beijing(1998).
[9] W.L. Li, Z.Z. Li , W.D. Li.Effect of the niche-fitness at different water supply and fertilization on yield of spring wheat in farmland of semi-arid areas. *Agricultural Water Management*, 67(1)(2004):1-13
[10] S.Y. Chen.The determination of relative membership degree(in Chinese). *Explor Nat*. 2(1993):25-25.
[11] Feinsinger P., Spear E.E., Poole R.W., A simplemeasure of niche breadth. *Ecology*,62(1981): 27-32.