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Linear Generalized outer Synchronization Between Two Different Complex Dynamical Networks with Noise Perturbation

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Abstract: This paper mainly studies the linear generalized outer synchronization between two delay-coupled complex networks, which have diverse node dynamics and different topological structures. Real systems are often subject to noise perturbation, so we take into account the stochastic effects in the networks, and the node dynamic need not satisfy the very strong and conservative uniformly Lipschitz condition. With a nonlinear control scheme, a sufficient synchronization criterion is derived. Furthermore, theoretical proofs are proposed on the basics of LaSalle-type invariance principle for stochastic differential equation. Numerical simulations are also presented to show the feasibility and effectiveness of the theoretical results.

Keywords: complex networks; linear generalized outer synchronization; noise perturbation; adaptive control

1 Introduction

In the past few years, the comprehensive studies of synchronization of dynamical networks have received great interest and notable attention in many research and application fields [1-3]. In literature, there are many widely studied synchronization patterns, such as complete synchronization [4], generalized synchronization [5], cluster synchronization [6], projective synchronization [7], and so on. Correspondingly, many theoretical methods and experimental techniques have been proposed for different type of chaos synchronization, such as nonlinear coupling techniques, feedback control methods, impulsive control methods, adaptive control methods, and so on [8-10].

Most previous researches on network analyze the synchronization in one network, which is regarded as inner synchronization. Different from the inner synchronization, another type of synchronization between two or more coupled networks called outer synchronization, which was firstly studied by Li et al.[11]. In [12], Wu, Zheng and Zhou proposed the concept of generalized outer synchronization (GOS), and synchronized two different complex networks by constructing a nonlinear controller. Li and Xue theoretically and numerically show that for balanced networks, outer synchronization of coupled networks can be achieved by using arbitrary coupling strength [13]. Wang, Ma, et al. firstly presented a new type of outer synchronization, i.e. mixed outer synchronization (MOS), which not only enables the COS and IOS but also can exhibit CS, AS, and AD simultaneously, a novel nonfragile linear state feedback controller is designed to realize the MOS [14]. Linear generalized outer synchronization (linear GOS) is a particular kind of GOS. The linear GOS problems between two complex networks are investigated by designing a nonlinear controller and adaptive update laws in [15].

Real systems are often subject to noise perturbations. Since noise is ubiquitous in natural and synthetic systems, to simulate more realistic complex networks, the influence of noise perturbation can not be ignored. Taken into the account the stochastic effects in the system model, the models are better described as stochastic differential equations. Several works have discussed different synchronization problems with noise perturbation. Xiao et al. studied the complete synchronization of two bi-directionally coupled piecewise linear chaotic systems when coupling strength is perturbed by common of different white noise [16]. In [17], outer synchronization between two non-identical networks with circumstance noise is investigated by designing a suitable adaptive controller. Time delay and noise perturbation was not considered in the investigation of linear GOS problems in [15]. We further explored the linear GOS behavior with time delay and noise perturbation, which is simulate more realistic complex networks. In our study, the two complex networks

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may differ in node dynamics, or topological structures, so the synchronization criterion derived can be applied to more general complex dynamical networks.

The rest of this paper is organized as follows. In Section 2, the model description and preliminaries are presented. Section 3 is devoted to deriving sufficient criteria for the linear GOS between two different complex dynamical networks with noise perturbation. In Section 4, numerical simulations are carried out to show the feasibility and effectiveness of the theoretical results. Conclusions are finally drawn in Section 5.

2 Network models and preliminaries

In this section, we will give the drive-response dynamical network model and a definition for linear GOS between two different complex dynamical networks, followed by an assumption and two lemmas which will be required in the subsequent study.

Some necessary notations that will be used throughout this paper are first introduced. $\|\xi\|$ denotes the 2-norm of the vector ξ, $I_n \in R^{n\times n}$ represents the identity matrix with dimension n, $\otimes$ denotes the Kronecker product of two matrices, $\lambda_{\max}(A)$ represents the maximum eigenvalue of a square matrix .

Consider a general complex dynamical network with delayed coupling consisting of N dynamical nodes with linear couplings, which is characterized by

$$dx_i(t) = [Ax_i(t) + f(x_i(t)) + \sum_{j=1}^{N} c_{ij}\Gamma x_j(t-\tau)]dt, i = 1, 2, ...N, \tag{1}$$

where, $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in R^n$ is the state vector of the ith node in the drive network, $A \in R^{n\times n}$ is a constant matrix, $f : R \times R^n \to R^n$ is a smooth nonlinear vector-valued function, $\tau \geq 0$ denotes the time delay of the network coupling, $\Gamma = diag\{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ is an inner coupling matrix determining the interaction of variables, $C = (c_{ij})_{N\times N}$ is the coupling configuration matrix representing the coupling strength and the topological structure of the network, in which c_{ij} is defined as follows: $c_{ij} = 1$ if there is a connection from node i to node j ($j \neq i$), otherwise, $c_{ij} = 0$. The diagonal elements of matrix C are defined as $c_{ii} = -\sum\limits_{j=1, j\neq i}^{N} c_{ij}, \;\; i = 1, 2, ...N$.

To realize the linear GOS between two coupled complex networks, we refer to model (1) as the drive network. Since in real systems (both natural and artificial), there are usually random perturbations, so the response network is given by the following equation

$$dy_i(t) = [By_i(t) + g(y_i(t)) + \sum_{j=1}^{N} d_{ij}\Gamma y_j(t-\tau) + u_i(t)]dt + \sigma(t, e_i(t), e_i(t-\tau))dw(t), \tag{2}$$

where, $y_i(t) = (y_{i1}(t), y_{i2}(t), \cdots, y_{in}(t))^T \in R^n$ is the state vector of the ith node in the response network, $B \in R^{n\times n}$ is a constant matrix, $g : R \times R^n \to R^n$ is a smooth nonlinear vector-valued function, $D = (d_{ij})_{N\times N}$ is the coupling configuration matrix, $\sigma(t, e_i(t), e_i(t-\tau)) \in R^n$ is the noise intensity function, $w(t)$ is one-dimensional Brown motion defined on a complete probability space, $u_i(t)$ is the adaptive controller designed for node i.

Definition 1 *Given a vector map $\phi : R^n \to R^n$, network (1) is said to achieve generalized outer synchronization with network (2) if* $\lim\limits_{t\to\infty} \|y_i(t) - \phi(x_i(t))\| = 0, \;\; i = 1, 2, ..., N$

For simplicity, in this paper, ϕ is selected in the linear form as $\phi(x) = Px + Q$, and P is a constant diagonal matrix, that is $P = diag(p_1, p_2, ..., p_n)$, Q is a constant matrix. For the predefined map $\phi(x) = Px + Q$, and P, we define the synchronization error signal as

$$e_i(t) = y_i(t) - Px_i(t) - Q, \quad i = 1, 2, ..., N, \tag{3}$$

where, $e_i(t) = (e_{i1}(t), e_{i2}(t), ..., e_{in}(t))^T \in R^n$, because $\sum\limits_{j=1}^{N} c_{ij}\Gamma Q = 0$, from (1) and (2), we can get the following error dynamical network

$$\begin{aligned} de_i(t) = &[By_i(t) + g(y_i(t)) + \sum_{j=1}^{N} d_{ij}\Gamma y_j(t-\tau) + u_i(t) - PAx_i(t) - Pf(x_i(t)) \\ &- \sum_{j=1}^{N} c_{ij}\Gamma P x_j(t-\tau)]dt + \sigma(t, e_i(t), e_i(t-\tau))dw(t) \end{aligned} \tag{4}$$

Remark 1 *From Definition 1 we can easily find that, the generalized outer synchronization reduces to complete outer synchronization when $P = diag(1, 1, ..., 1), Q = 0$; inverse outer synchronization when $P = diag(-1, -1, ..., -1), Q = 0$; and mixed outer synchronization when $P = diag(p_1, p_2, ..., p_n), p_l \in \{-1, 1, 0\}, Q = 0$.*

Furthermore, we give the following assumption and lemma, which plays an important role in the proof of the theorem.

Proposition 2 *The noise intensity function $\sigma(t, e_i(t), e_i(t-\tau))$ satisfies the local Lipschitz condition, the linear growth condition and*

$$trace(\sigma^T(t, e_i(t), e_i(t-\tau))\sigma(t, e_i(t), e_i(t-\tau))) \le e_i^T(t)G_1e_i(t) + e_i^T(t-\tau))G_2e_i(t-\tau),$$

where G_1 and G_2 are positively definite matrices.

Lemma 3 *For any vectors $x, y \in R^n$ and the positive definite matrix $Q \in R^{n\times n}$, the following matrix inequality holds $2x^Ty \le x^TQx + y^TQ^{-1}y$.*

Lemma 4 [18−19] *Consider an n-dimensional stochastic differential delay equation as follows*

$$dx(t) = f(t, x(t), x(t-\tau))dt + \sigma(t, x(t), x(t-\tau))dw(t), \tag{5}$$

where $w(t)$ is an m-dimensional Brown motion defined on the complete probability space $(\Omega, \mathcal{F}, P)$ with filtration $\{\mathcal{F}_t\}_{t\ge 0}$. Let $C^{2,1}(\mathrm{R^n} \times \mathrm{R_+};\mathrm{R_+})$ denote the family of all nonnegative functions $V(x,t)$ on $\mathrm{R^n} \times \mathrm{R_+}$, which are continuously twice differentiable in x and once differentiable in t. For each $V \in C^{2,1}(\mathrm{R^n} \times \mathrm{R_+};\mathrm{R_+})$. define an operator $\mathcal{L}V$ along with system (4) by

$$\mathcal{L}V = V_t + V_x f + \frac{1}{2}trace(\sigma^T V_{xx}\sigma), \tag{6}$$

where, $V_t = \partial V/\partial t, V_x = (\partial V/\partial x_1, \partial V/\partial x_2, ..., \partial V/\partial x_n), V_{xx} = (\partial^2 V/\partial x_i \partial x_j)_{n\times n}$. The LaSalle-type invariance principle for stochastic differential delay equation can be expressed as follows
(i) Assume that system (5) exists a unique solution $x(t,\phi)$ on $t \ge 0$ for any initial data $\phi \in C^{\mu}_{F_0}([-\tau, 0]; R^n)$, moreover, both $f(x,y,t)$ and $\sigma(x,y,t)$ are locally bounded in (x,y), and uniformly bounded in t, and satisfy the local Lipchitz condition and the linear growth condition,
(ii) Assume that there are functions $V \in C^{2,1}(\mathrm{R^n} \times \mathrm{R_+};\mathrm{R_+})$, $\gamma \in \mathrm{L}^1(R_+, R_+)$, and $\omega_1, \omega_2 \in \mathrm{C}(\mathrm{R^n};\mathrm{R_+})$, such that

$$\mathcal{L}V(x,\mathrm{y},\mathrm{t}) \le \gamma(\mathrm{t}) - \omega_1(x) + \omega_2(\mathrm{y}), \quad (x,\mathrm{y},\mathrm{t}) \in \mathrm{R^n} \times \mathrm{R^n} \times \mathrm{R_+},$$

$$\omega_1(x) > \omega_2(x), x \ne 0, and \lim_{\|x\|\to\infty} \inf_{0\le \mathrm{t}<\infty} V(x,t) = \infty.$$

Then, $\lim_{t\to\infty} x(t,\phi) = 0$ almost surely for every $\phi \in C^{\mu}_{F_0}([-\tau, 0]; R^n)$.

3 Linear GOS criteria

In this section, we will investigate the linear GOS between two different complex dynamical networks with noise perturbation based on the LaSalle-type invariance principle and the nonlinear control scheme. With the network model and the preliminaries given previously, we arrive at the following main theorem.

Theorem 5 *Suppose that Assumption holds, drive network (1) and response network (2) can realize linear GOS by applying the following nonlinear controller and update laws*

$$u_i(t) = -(BP - PA)x_i(t) - BQ - g(y_i(t)) + Pf(x_i(t)) + \sum_{j=1}^{N} m_{ij}\Gamma y_j(t-\tau) - g_i e_i(t) \tag{7}$$

$$\dot{g}_i = k_i e_i(t),\ \dot{m}_{ij} = -e_i^T(t)\Gamma y_j(t-\tau), \quad i = 1, 2, ..., N \tag{8}$$

Proof. The synchronization error between (1) and (2) can be written as

$$\begin{aligned} de_i(t) = {} & [Be_i(t) + \sum_{j=1}^{N} d_{ij}\Gamma y_j(t-\tau) + \sum_{j=1}^{N} m_{ij}\Gamma y_j(t-\tau) - \sum_{j=1}^{N} c_{ij}\Gamma P x_j(t-\tau) - g_i e_i(t)]dt \\ & + \sigma(t, e_i(t), e_i(t-\tau))dw(t) \end{aligned} \tag{9}$$

We construct the continuous two differential non-negative Lyapunov function as follows

$$V(t, e(t)) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(m_{ij} + d_{ij} - c_{ij})^2 + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_i}(g_i - \bar{g})^2 \tag{10}$$

Then, according to Lemma 2, the operator $\mathcal{L}V$ is given by

$$\begin{aligned} \mathcal{L}V(t) = {} & \sum_{i=1}^{N} e_i^T(t)[Be_i(t) + \sum_{j=1}^{N} d_{ij}\Gamma y_j(t-\tau) + \sum_{j=1}^{N} m_{ij}\Gamma y_j(t-\tau) - \sum_{j=1}^{N} c_{ij}\Gamma P x_j(t-\tau) - g_i e_i(t)] \\ & - \sum_{i=1}^{N}\sum_{j=1}^{N}(m_{ij} + d_{ij} - c_{ij})e_i^T(t)\Gamma y_j(t-\tau) + \sum_{i=1}^{N}(g_i - \bar{g})e_i^T(t)e_i(t) \\ & + \frac{1}{2}trace(\sigma^T(t, e_i(t), e_i(t-\tau))\sigma(t, e_i(t), e_i(t-\tau))) \\ = {} & \sum_{i=1}^{N} e_i^T(t)Be_i(t) - \bar{g}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\Gamma e_j(t-\tau) \\ & + \frac{1}{2}trace(\sigma^T(t, e_i(t), e_i(t-\tau))\sigma(t, e_i(t), e_i(t-\tau))) \\ \le {} & e^T(t)(\frac{B^T + B}{2})e(t) - \bar{g}e^T(t)e(t) + \frac{1}{2}e^T(t)FF^Te(t) + \frac{1}{2}e^T(t-\tau)e(t-\tau) + \frac{1}{2}e^T(t)G_1e(t) \\ & + \frac{1}{2}e^T(t-\tau)G_2e(t-\tau) \\ \le {} & -e^T(t)H_1e(t) + e^T(t-\tau)H_2e(t-\tau) \\ \le {} & -\omega_1(e(t)) + \omega_2(e(t-\tau)) \end{aligned}$$

where $H_1 = (\bar{g} - \lambda_{\max}(\frac{B+B^T}{2}))I - \frac{1}{2}FF^T - \frac{1}{2}G_1,\ H_2 = \frac{1}{2}I + \frac{1}{2}G_2,\ F = C \otimes \Gamma$.

Referring to the above calculations, it can be observed that for a suitable constant $\bar{g}$, the following inequality holds $\omega_1(e(t)) > \omega_2(e(t)), \quad \forall e(t) \neq 0$. Then, by applying the Lasalle-type invariance principle, we can obtain $\lim_{t\to\infty} e(t) = 0, \ a.s.$ The linear GOS between the drive network and the response network can be realized. This completes the proof. ■

Corollary 6 *If the drive network (1) and response network (2) have identical topological structures, then the two networks can achieve linear GOS by using the following adaptive controller*

$$u_i(t) = -(BP - PA)x_i(t) - BQ - g(y_i(t)) + Pf(x_i(t)) - g_i e_i(t) \tag{11}$$

$$\dot{g}_i = k_i e_i(t), \quad i = 1, 2, ..., N \tag{12}$$

Corollary 7 *If each node in network (1) and network (2) has identical dynamics, i.e.* $f = g, A = B$ *and* $AP = PA$, *then the two networks can realize linear GOS under the following adaptive controller*

$$u_i(t) = -AQ - f(y_i(t)) + Pf(x_i(t)) + \sum_{j=1}^{N} m_{ij}\Gamma y_j(t-\tau) - g_i e_i(t) \tag{13}$$

$$\dot{g}_i = k_i e_i(t),\ \dot{m}_{ij} = -e_i^T(t)\Gamma y_j(t-\tau), \quad i = 1, 2, ..., N \tag{14}$$

4 Numerical simulations

In this section, some numerical examples are provided to verify the effectiveness of the proposed synchronization criteria. The Lorenz chaotic system is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + f(x)$$

where $A = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{pmatrix}$, $\quad f(x) = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix}$.
The Rossler chaotic system is known as

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{pmatrix} = B \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + g(y)$$

where $B = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -5.7 \end{pmatrix}$, $\quad g(y) = \begin{pmatrix} 0 \\ 0 \\ y_1y_3 + 0.2 \end{pmatrix}$.

4.1 Linear GOS between two delayed networks with identical topological structures

In this subsection, we consider the case in which both networks (1) and (2) have the same configuration matrices. In the numerical simulation we choose the coupling configuration matrix and the noise intensity function as

$$\mathrm{C} = \begin{pmatrix} -3 & 0 & 1 & 1 & 1 \\ 0 & -2 & 1 & 0 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 0 & 1 & -2 & 0 \\ 1 & 1 & 1 & 0 & -3 \end{pmatrix}, \sigma(t, e_i(t), e_i(t-\tau)) = \begin{pmatrix} \sin(e_{i1}) & 0 & 0 \\ 0 & \cos(e_{i2}) & 0 \\ 0 & 0 & \cos(e_{i2}) \end{pmatrix}$$

Select $P = diag(1, -1, 1)$, $Q = [0\ 0\ 0]^T, \Gamma = diag(1, 1, 1)$, time delay $\tau = 0.5$, and errors $e_{i1} = y_{i1} - x_{i1}, e_{i2} = y_{i2} + x_{i2}, e_{i3} = y_{i3} - x_{i3}$, $i = 1, 2, ..., 5$. According to Corollary 1, we derive the control scheme by (11), (12), Linear GOS between network (1) and (2) can be achieved as shown in Fig.1 and Fig.2. Fig. 1 shows the trajectories of synchronization errors, it can be observed that all of the errors tend to zero, Fig. 2 plots the evolution of feedback strength g_i.

4.2 Linear GOS between two delayed networks with different topological structures

The second example consider the case that the drive network and the response network have different coupling configuration matrices. Select the configuration matrix C of the drive network as give in 4.1, and the configuration matrix for the response network is given by

$$\mathrm{D} = \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{pmatrix}$$

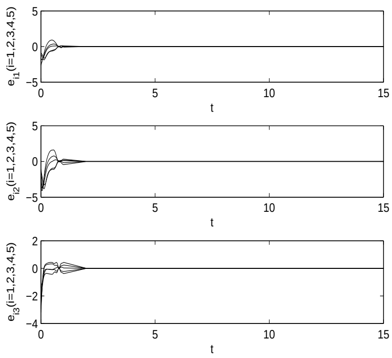

Figure 1: Evolution of the synchronization error with $C = D$.

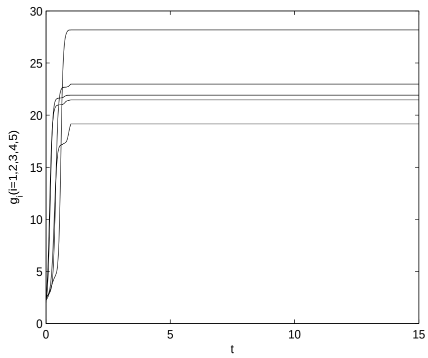

Figure 2: Evolution of feedback strength g_i $(i = 1, 2, ..., 5)$ with $C = D$.

Similarly, we choose $P = diag(1, -1, 1)$, $Q = [0\ 0\ 0]^T, \Gamma = diag(1, 1, 1)$, and the time delay $\tau = 0.5$, Fig. 3 shows the trajectories of synchronization errors, which shows the realization of synchronization between the two complex networks. Fig. 4 plots the evolution of feedback strength g_i.

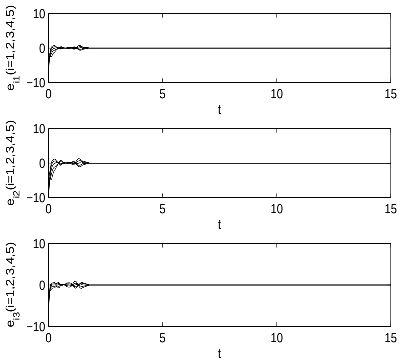

Figure 3: Evolution of the synchronization error with $C \neq D$.

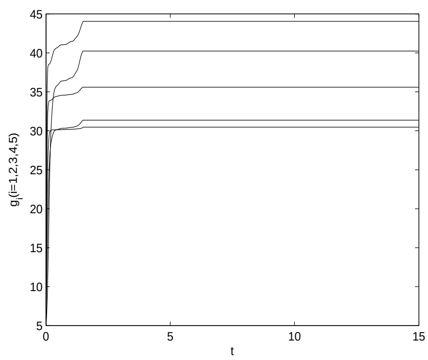

Figure 4: Evolution of feedback strength g_i $(i = 1, 2, ..., 5)$ with $C \neq D$.

5 Conclusions

In this paper, the linear GOS between two different delay-coupled complex networks is studied theoretically and numerically. Noise from circumstance is another factor which may affect the behavior of dynamics between coupled complex networks, so we studied the effect of noise on the linear GOS problem. Besides, the drive and response networks have distinct topologies and diverse node dynamics, which are more general and applicable. Based on the LaSalle-type invariance principle for stochastic differentical equation and the adaptive control scheme, the adaptive controller and update laws are designed. In this paper, the node dynamic need not satisfy the very strong and conservative uniformly Lipschitz condition. The feasibility and effectiveness of the presented synchronization criteria has been validated by the computer simulation.

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