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The Energy of Strength-λ- Possion Processes

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Abstract: In this paper,we study strength-λ- Possion processes using wavelet transform.we study its some properties and energy.

Keywords: strength-λ- Possion processes;wavelet;energy

1 Introduction

The stochastic system is very importment in many aspacts. strength-λ- Possion processes is a sort of importment stochastic processes,and it is a class of useful stochastic processes in practies,its study is very value.

We will take wavelet and use them in a series expansion of signal or function. Wavelet has its energy concentrated in time to give a tool for the analysis of transient,nonstationary,or time-varying phenomena.It still has the oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation.We take wavelet and use them in a series expansion of signals or functions much the same way a Fourier series the wave or sinusoid to represent a signal or function.In order to use the idea of multiresolution ,we will start by defining the scaling function and then define the wavelet in terms of it.

As with all transform methods such as approach hopes to achieve that the computation is faster in the new system of coordinates than in the original domain,wavelet based algorithms exhibit a number of new and important properties .Recently some persons have studied wavelet problems of stochastic process or stochastic system ([1]-[14]).

2 Definitions

Definition 1 *Let* $\{\ N(t), t \geq 0\ \}$*is count processesif*

$$P[N(t+\Delta t) - N(t) \geq 2] = o(\Delta t)$$

then we call N(t) as strength-λ- Possion processes.

We know

$$E[N(t)] = \lambda t$$

$$R_N(u,v) = E[N(u)N(v)] = \lambda^2 uv + \lambda \min(u,v) \tag{1}$$

Definition 2 *Let* $\{x(t), t \in R\ \}$ *is stochastic processes,then its wavelet transform is*

$$WX(s,x) = \frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt \tag{2}$$

where,$\psi$is mather wavelet.

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Definition 3 *Haar wavelet is*

$$\psi(x)=\begin{cases}1,0\le x<0.5\\-1,0.5\le x<1\\0,otherwise\end{cases}\tag{3}$$

Wavelet transform of N(t)

$$WN(s,x)=\frac{1}{s}\int_R N(t)\psi(\frac{x-t}{s})dt\tag{4}$$

Then

$$WN(s,x+\tau)=\frac{1}{s}\int_R N(t_1)\psi(\frac{x+\tau-t_1}{s})dt_1\tag{5}$$

3 Energy

Then

$$\begin{aligned}R(\tau)&=E[WN(s,x)WN(s,x+\tau)]\\&=\frac{1}{s^2}\iint_{R^2}E[N(t)N(t_1)]\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1\\&=\frac{1}{s^2}\iint_{R^2}[\lambda^2tt_1+\lambda\min(t,t_1)]\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1\\&=\frac{1}{s^2}\iint_{R^2}\lambda^2tt_1\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1+\frac{1}{s^2}\iint_{R^2}\lambda\min(t,t_1)\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1\\&=I1+I2\end{aligned}$$

where

$$I1=\frac{1}{s^2}\iint_{R^2}\lambda^2tt_1\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1$$

$$I2=\frac{1}{s^2}\iint_{R^2}\lambda\min(t,t_1)\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1$$

We may let $t_1\le t$
Then

$$I2=\frac{1}{s^2}\iint_{R^2}\lambda t_{1\psi}(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1$$

Because (3),we have

$$\psi(\frac{x-t}{s})=\begin{cases}1,x-\frac{s}{2}\le t<x\\-1,x-s\le t\le x-\frac{s}{2}\end{cases}\tag{6}$$

$$\psi(\frac{x+\tau-t}{s})=\begin{cases}1,x+\tau-\frac{s}{2}\le t<x+\tau\\-1,x+\tau-s\le t<x+\tau-\frac{s}{2}\end{cases}\tag{7}$$

Then ,we have

$$\begin{aligned}I1&=\frac{\lambda^2}{s^2}[\iint_{R^2}tt_1\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1]\\&=\frac{\lambda^2}{s^2}[\int_{x-s/2}^{x}tdt\int_{x+\tau-s/2}^{x+\tau}t_1dt_1-\int_{x-s/2}^{x}tdt\int_{x+\tau-s}^{x+\tau-s/2}t_1dt_1]\\&-\frac{\lambda^2}{s^2}[\int_{x-s}^{x-\frac{s}{2}}tdt\int_{x+\tau-s/2}^{x-\frac{s}{2}}t_1dt_1+\int_{x-s}^{x-s/2}tdt\int_{x+\tau-s}^{x+\tau-s/2}t_1dt_1]\\&=\frac{\lambda^2}{s^2}\{\ \frac{1}{4}[x-\frac{s}{4}][s^2/2-2\tau(x-\frac{s}{2})+\tau^2+s(x+\tau)-\frac{3}{4}s^2]\ \}\end{aligned}$$

$$\begin{aligned}
I2 &= \frac{1}{s^2}\iint_{R^2}\lambda t_1\psi(\frac{x-t}{s})\psi(\frac{x+\tau-t_1}{s})dtdt_1\\
&= \frac{1}{s^2}\lambda[\int_{x-s/2}^{x}dt\int_{x+\tau-s/2}^{x+\tau}t_1dt_1-\int_{x-s/2}^{x}dt\int_{x+\tau-s}^{x+\tau-s/2}t_1dt_1]\\
&-\frac{\lambda}{s^2}[\int_{x-s}^{x-\frac{s}{2}}tdt\int_{x+\tau-s/2}^{x-\frac{s}{2}}t_1dt_1+\int_{x-s}^{x-s/2}tdt\int_{x+\tau-s}^{x+\tau-s/2}t_1dt_1]\\
&= \frac{\lambda}{4s}[\frac{s}{4}-x-\tau-2\tau(x-\frac{s}{2})-\tau^2]
\end{aligned}$$

Then

$$\begin{aligned}
R(\tau) &= I1+I2\\
&= \frac{\lambda^2}{s}\left\{\frac{1}{4}[x-\frac{s}{4}][s^2/2-2\tau(x-\frac{s}{2})+\tau^2+s(x+\tau)-\frac{3}{4}s^2]\right\}\\
&+\frac{\lambda}{4s}[\frac{s}{4}-x-\tau-2\tau(x-\frac{s}{2})-\tau^2]
\end{aligned} \tag{8}$$

Then

$$R'(\tau)=\frac{dR}{d\tau}=\frac{\lambda^2}{2s}[(x-s/4)\tau(s/2-x)]+2\tau+s\tau]+\frac{\lambda}{4s}[-1-2(x-s/2)-2s] \tag{9}$$

$$R''(\tau)=\frac{\lambda^2}{2s}[(x-s/4)(s/2-x)+s+2] \tag{10}$$

$$R'''(\tau)=R^{(4)}(\tau)=0 \tag{11}$$

Then

$$R(0)=\frac{\lambda^2}{s}\left\{\frac{1}{4}[x-\frac{s}{4}][s^2/2+sx-\frac{3}{4}s^2]\right\}+\frac{\lambda}{4s}[\frac{s}{4}-x] \tag{12}$$

$$R''(0)=\frac{\lambda^2}{2s}[(x-s/4)(s/2-x)+s+2] \tag{13}$$

Then,use (12)-(13) ,we can obtain the zero density degree of WN(s , x) is

$$\sqrt{\left|\frac{R''(0)}{\pi^2R(0)}\right|}$$

$$=\frac{\lambda}{2\pi s}[(x-s/4)(s/2-x)+s+2]/\ \frac{\lambda}{s}\left\{\frac{1}{4}[x-\frac{s}{4}][s^2/2+sx-\frac{3}{4}s^2]+\frac{1}{4s}[\frac{s}{4}-x]\right\}\Big|^{\frac{1}{2}}$$

Use (11),we know
The averge density degree of WN(s,x) is zero:

$$\sqrt{\left|\frac{R^{(4)}(0)}{\pi^2R^{(2)}(0)}\right|}=0$$

We have all

$$E[WN(s,x)]=\frac{1}{s}\int_R\lambda t\psi(\frac{x-t}{s})dt=\frac{\lambda}{s}[\int_{x-s/2}^{x}tdt-\int_{x-s}^{x-s/2}tdt]=\frac{sx}{4}$$

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