Study on the Evolutionary Game of Financial Institutions and Listed Companies

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Abstract: First, Take the financial institutions and listed companies as background. In an evolutionary game theoryEstablished a new dynamic system replication. And analysis the Evolutionary game, Study the equilibrium point, at last shows the effect of Evolutionary path when the pays changed and Different values of initial conditions. Get fit with the reality of evolutionary stable strategy, Provides valuable reference about Financial Institutions and Listed Companies.

Keywords: Evolutionary game; financial institution; Listed Company; replication dynamic

1 Introduction

In 2008, the global financial crisis to the influence of real economy spread constantly, the domestic and foreign development environment changes to the real economy especially listed company has brought great impact .While countries respond to the crisis "package" to financial institutions have been more investment opportunities. Financial institutions such as Banks to provide funds of listed companies that necessary, extra purchasing power, the listed company may be more big development space and financial institutions can also have more income. The reality of the social economic system, there is a lot of uncertainty, and the rational decision-making is limited, so, in financial institutions and listed companies with the background of evolutionary game theory about their contacts process, research strategy, to the listed company to survive the crisis, go out the development dilemma, improve the competition power, for financial institutions to bring about greater returns to have the important meaning.

The evolutionary game theory fusion game theory and evolution thought, belongs to the game theory and the research of evolution economics category, in recent years, the game theory and the evolution of new development. Evolutionary game theory first produced in biology, for the study of biological in a different environment and evolution of the activities of the process. J.Maynard-Smith and R.Selten, combined with the game theory and dynamic evolutionary theory, put forward the evolutionary game theory (evolutionary game theory) [1]. In 1992, the evolution of the development of the game theory in the international conference held at Cornell University, officially confirmed the evolutionary game theory in economics academic position, since then, this theory in economics, and has won the rapid development and wide application. More and more economists use evolutionary game theory to analysis, such as social system change (Axelrod and Hamilton, 1981). Stock market development direction (Conlisk, 1980), On the choice of brands for consumers (Katz and Shapiro, 1985), Social learning process(Fudenberg,1995), and The custom of society (Young, 1993 ,1998)and the fields of the related problems. At present, a gradual transition evolutionary game model to economic and social fields used for simulation and analysis the real economic, social problems.

In recent years, evolutionary game theory in the home also lifted the application in every field of research hot flashes, Yi Yu Yin, Shaw, Cheng Zhao Han article such as using military respectively evolutionary game method was used to study the credit market, duopoly market, independent innovation behavior, cooperation of the developing opportunities, a series of problems [2, 3]. Cui Hao, Chen Xiao, Jian and Zhang Dao Wu in the evolutionary game theory, this paper analyzes the limited rational stakeholders in the common management structure to participate in the enterprise ownership allocation and reach the Nash equilibrium evolutionary game process [4]. Noble and pure, Cheng Zhao Han, Huang Xian, such as Wang Zhan Hua respectively with evolutionary game theory analyzes the power generation enterprise in bidding for access to process the dynamic evolvement process of bidding strategies and electric power market for the evolution of the stable

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Table 1:

Financial Institutions	Listed Company	
A	B_1 strategy 1 (cooperation)	B_2 strategy 2 (defense)
A_1 strategy 1 (cooperation)	$(a_4 \quad b_4)$	$(a_1 \quad b_3)$
A_2 strategy 2 (defense)	$(a_3 \quad b_1)$	$(a_2 \quad b_2)$

strategy [5, 6]. In 2009 June Mei Jiang, Ma Guo Jian, Du Jian Guo evolutionary game theory with the study of the theory of the small and medium-sized enterprise credit guarantee path evolution rule, establish Banks and guarantee company pay matrix, Banks and guarantee company interaction analysis of the evolution process model, the numerical simulations show the different initial parameters of decision-making and initial conditions of the change to affect the results of evolution [9]. In 2009 December Wang De Xiang, Wang wei, from game theory analysis Angle to evolutionary game theory for starting point, through the Chinese banking financial companies to mixes industry the management change the dynamic process, and then explore and explain the reasons, and financial system that the process of choice is to participate in the process of main body long-term game, initial conditions on the choice of the financial system, the influence, when the initial conditions are not at the same time, financial institutions will be long-term evolution behavior in a distinct pattern.

2   Evolution model

We might as well the commercial institutions that a member of the listed company and a member of the information asymmetry in the two random "hunt deer" game. Hunt deer game model is based on the story Rousseau hunt deer brought out, is a direct link between the prisoner dilemma and coordination game the game (Ullmann-Margalit. 1977). Its meaning is when the hunters to hunt deer, can it hunted the deer, but if they work together to words, each of them harvest will better. Now, we to simple calculation, suppose we only have two strategies, strategy 1 (cooperation) and strategy 2 (cooperation), the commercial agency has two strategies: strategy 1 (cooperation) tend to the financial system of cooperation, and strategy 2 (cooperation) is relatively independent. Likewise, securities companies also have two strategies: strategy 1 (cooperation) said the commercial institutions and establishes the cooperation relations, strategy 2 (cooperation) is the commercial institutions and said don't establish cooperation relationship.

The p and $1 - p$ respectively of A financial institution is said in A game of A_1 and A_2 take the strategy of probability, and representative of the listed company B respectively the strategy and the probability of B_2 and B_1, including probability can also understand for group game change of strategy selection participants proportion, the table below:

While a_m, $b_m (m = 1, 2, 3, 4)$ representatives of the benefits of listed company and financial institutions, To choose a certain cost, select strategy 1 will be a higher income, if all select strategy 1, can get big profits. So naturally a_4 and b_4 is greater. And when financial institutions and appear on the market the company is selection strategy 2 because everybody is not very trust each other, it is difficult to get listed company with more money to development, can't play to the advantages of financial asset specificity, then financial institutions return will become very few, units of the lower profits, So that the income a_2 and b_2 is low. If financial institutions selection strategy 1, the listed company selection strategy 2, you can understand the financial institutions to actively listed company industry infiltration, can guarantee the company to provide enough money, at the same time, the listed company to extra reward, and earnings than two kinds of institutions selection strategy 2 listed company profits, and financial institutions that will bear great risk, suffered a great loss. Therefore income less than two classes choose strategy 2 when financial institutions of income a_2, the same b_1 are lower than two appear on the market the company is selection strategy 2 when X of the listed company of income b_2, so: $0 < a_1 < a_2 < a_3 < a_4, 0 < b_1 < b_2 < b_3 < b_4$.

The benefits of financial organizations for strategies 1:

$$E(A_1) = a_4 q + a_1(1 - q);$$

The benefits of strategies 2:

$$E(A_2) = a_3 q + a_2(1 - q);$$

The average of the benefits:

$$E(A) = p[a_4q + a_1(1-q)] + (1-p)[a_3q + a_2(1-q)];$$

Similarly, we can obtain the benefits of the listed company:

$$E(B_1) = b_4p + b_1(1-p);$$

$$E(B_2) = b_3p + b_2(1-p);$$

$$E(B) = q[b_4p + b_1(1-p)] + (1-q)[b_3p + b_2(1-p)];$$

The system can be used by the evolution of a two differential equation to describe the system composition. Suppose that a strategy is its growth rate and relative fitness and it is proportional to the average fitness; as long as a strategy as the fitness of the fitness of group high, so this strategy will be developed.

Therefore, can draw the commercial organizations and the listed company group strategy contacts evolution over time for the dynamic equation:

$$\begin{cases} \dfrac{dp}{dt} = p(1-p)[(a_4 - a_3 + a_2 - a_1)q - (a_2 - a_1)] \{p[a_4q + a_1(1-q)] + (1-p)[a_3q + a_2(1-q)]\} & (1) \\[2mm] \dfrac{dq}{dt} = q(1-q)[(b_4 - b_3 + b_2 - b_1)p - (b_2 - b_1)] \{q[b_4p + b_1(1-p)] + (1-q)[b_3p + b_2(1-p)]\} & (2) \end{cases}$$

Through the equation, we can study group and the commercial institutions listed company group communication group of the behavior strategy evolution. Set of Jacobin matrix of the equation for J, write $J = Det(J)$, the trace is $T_r = J_{11} + J_{22}$.

3   Analysis of the model

According to the Friedman's proposed methods, Jacobin matrix of the local stability analysis can obtain a description of the group by differential equation dynamic stability of the equilibrium. From the equation of partial differentiation, we can get:

$$J = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}$$

where:

$$\begin{aligned} J_{11} = & (1 - 2p)[(a_4 - a_3 + a_2 - a_1)q - (a_2 - a_1)] \{p[a_4q + a_1(1-q)] + (1-p)[a_3q + a_2(1-q)]\} \\ & + \{[a_4q - a_1(1-q) - [a_3q + a_2(1-q)]\} \{p(1-p)[(a_4 - a_3 + a_2 - a_1)q - (a_2 - a_1)]\} \end{aligned}$$

$$\begin{aligned} J_{12} = & p(1-p)(a_4 - a_3 + a_2 - a_1) \{p[a_4q + a_1(1-q)] + (1-p)[a_3q + a_2(1-q)]\} \\ & + [p(a_4 - a_1) + (1-p)(a_3 - a_2)]p(1-p)[(a_4 - a_3 + a_2 - a_1)q - (a_2 - a_1)] \end{aligned}$$

$$\begin{aligned} J_{21} = & q(1-q)(b_4 - b_3 + b_2 - b_1) \{q[b_4p + b_1(1-p)] + (1-q)[b_3p + b_2(1-p)]\} \\ & + [q(b_4 - b_3) + (1-q)(b_3 - b_2)]q[b_4p + b_1(1-p)] + (1-q)[b_3p + b_2(1-p)] \end{aligned}$$

$$\begin{aligned} J_{22} = & \{(1 - 2q)[(b_4 - b_3 + b_2 - b_1)p - (b_2 - b_1)]\} \{q[b_4p + b_1(1-p)] + (1-q)[b_3p + b_2(1-p)]\} \\ & + \{b_4p + b_1(1-p) - [b_3p + b_2(1-p)]\} q(1-q)[(b_4 - b_3 + b_2 - b_1)p - (b_2 - b_1)] \end{aligned}$$

Replicator dynamic equation (1) shows that only when $p = 0, 1$ or $q = (a_2 - a_1)/(a_4 - a_3 + a_2 - a_1)$, when group select strategy 1, the financial institutions of proportion of is stable. Equation (2) shows that when $q = 0, 1$ or $p = (b_2 - b_1)/(b_4 - b_3 + b_2 - b_1)$, when group select strategy 1, the listed companies of proportion of is stable. Therefore, we can get five not equilibrium point. Use Jacobin matrix local stability analysis to five local balances the stability analysis.

Table 2:

equilibrium point	$Det(J)$		T_r		result
$p=0, q=0$	$a_2 b_2 (a_1 - a_2)(b_1 - b_2)$	+	$a_2(a_1 - a_2) + b_2(b_1 - b_2)$	−	ESS
$p=0, q=1$	$a_3 b_1 (a_4 - a_3)(b_2 - b_1)$	+	$a_3(a_4 - a_3) + b_1(b_2 - b_1)$	+	disequilibrium
$p=1, q=0$	$a_1 b_3 (a_2 - a_1)(b_4 - b_3)$	+	$a_1(a_2 - a_1) + b_3(b_4 - b_3)$	−	disequilibrium
$p=1, q=1$	$a_4 b_4 (a_3 - a_4)(b_3 - b_4)$	+	$a_4(a_3 - a_4) + b_4(b_3 - b_4)$	−	ESS
$p = \frac{b_2 - b_1}{b_4 - b_3 + b_2}$ $q = \frac{a_2 - a_1}{a_4 - a_3 + a_2}$	Δ		0		Saddle point

$$\Delta = \frac{(b_2 - b_1)(b_4 - b_3)}{(b_4 - b_3 + b_2 - b_1)^3} \frac{(a_2 - a_1)(a_4 - a_3)}{(a_4 - a_3 + a_2 - a_1)^3} \cdot \alpha \cdot \beta$$

$$\alpha = [a_2(b_2 - b_1)(b_4 - b_3) + a_2(b_4 - b_3)(a_4 - a_3) + a_3(b_4 - b_3)(a_2 - a_1) + a_4(b_2 - b_1)(a_2 - a_1)]$$

$$\beta = [b_1(a_2 - a_1)(a_4 - b_3) + b_2(a_4 - a_3)(a_4 - b_3) + b_3(a_4 - a_3)(b_2 - b_1) + b_4(a_2 - a_1)(b_2 - b_1)]$$

From the table we know that the five in balance system has two is stable, which is ESS Corresponding to the financial institutions and listed companies are cooperation or not cooperation. In the system of the listed company of financial institutions and the dynamic evolution process as below:
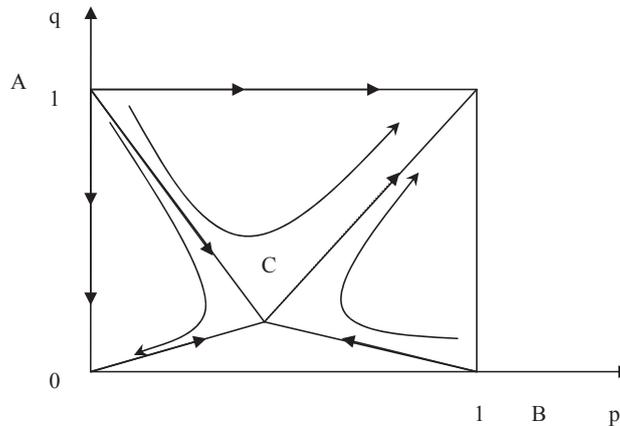

Figure 1: Financial institutions and the listed company dynamic process

From Figure 1, by two unstable points, A (0,1), B (1,0) and saddle point C, The unit can be thought of as a critical line. In the initial state in line to the bottom of the area, System will converge to (0,0) . All of the financial institutions and the listed company to take without cooperation group strategy, In the initial state over the top of the line in the area, System will converge to (1,1). That all financial institutions and appear on the market the company is using the strategy of cooperation.

We know that a saddle point $p = (b_2 - b_1)/(b_4 - b_3 + b_2 - b_1), q = (a_2 - a_1)/(a_4 - a_3 + a_2 - a_1)$, When the financial institutions and appear on the market the company is willing to cooperate, they yield is the relation between the unceasing increase, At this time at saddle to southwest direction line right area will be greater than to the left of the area, this time the convergence of the probability of cooperation is greater than the probability of cooperation, then so will the spontaneous evolution depends on cooperation and not the relative gains from cooperation. Financial institutions granted choose earnings relative relatively high mode for contacts, also is the financial institution in the cooperation and don't choose between cooperation, the income and the experience of the relevant system environment.

Table 3:

equilibrium	$Det(J)$		T_r		Result
$p = 0, q = 0$	$a_2b_2(a_1 - a_2)(b_1 - b_2)$	+	$a_2(a_1 - a_2) + b_2(b_1 - b_2)$	−	ESS
$p = 0, q = 1$	$a_3b_1(a_4 - a_3)(b_2 - b_1)$	0	$a_3(a_4 - a_3) + b_1(b_2 - b_1)$	+	Saddle point
$p = 1, q = 0$	$a_1b_3(a_2 - a_1)(b_4 - b_3)$	0	$a_1(a_2 - a_1) + b_3(b_4 - b_3)$	−	Saddle point
$p = 1, q = 1$	$a_4b_4(a_3 - a_4)(b_3 - b_4)$	+	$a_4(a_3 - a_4) + b_4(b_3 - b_4)$	+	unstable

When the initial conditions that is not the same, financial institutions will be long-term evolution behavior in a distinct pattern, one kind is financial institutions and the listed company's cooperation, additionally one kind is not cooperative mode; Through the above we know these two kinds of state are evolutionary stable state, financial institutions and listed company strategy are ESS.

The system environment of financial institutions and listed companies interact mode the influence of spontaneous evolution:

In the real world, because of each other of don't know, general is through the "edge credit and learn", through the "edge credit and learn" to get the other side of the information, that information to financial institutions and the listed company has very high value, in the initial state "edge credit and learn" function is not working. However, when the system arrangement not fully, at this time we will change to hunt deer game model of income structure. We might as well that, $a_2 = b_2 < 0, a_1 = b_1 = 0, a_2 < 0 < a_1 < a_4 < a_3, b_2 < 0 < b_1 < b_4 < b_3$.

From the Table 3, we know that the system of local equilibrium, only (0,0) is stable, (1,1) is unstable equilibrium. A, B are saddle points. That is not in the system became the dominant position of cooperation, From any of the initial conditions system will converge to (0,0). As below: We found an interesting phenomenon, this time the matrix of the income
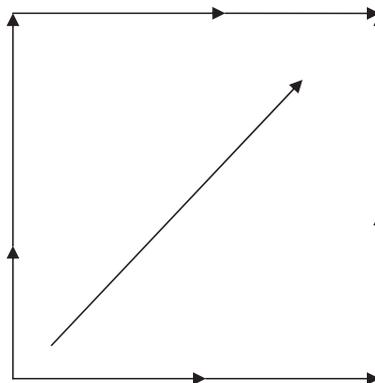

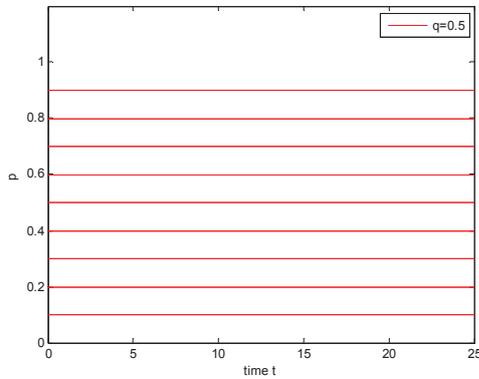Figure 2: Financial institutions and listed companies the dynamic process of communication

structure and the "prisoner's dilemma" model of the structure is very like income, so when the system arrangement is not reasonable, to hunt deer model will pay structure spontaneous evolved into the "prisoner's dilemma" model of income structure.
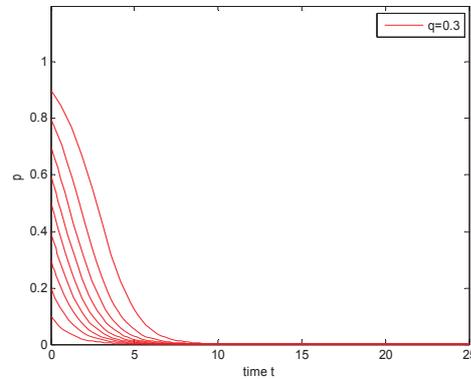
4 Simulation

Using the numerical experiment method below, choose a strategy of the initial analysis of the proportion of evolution and the influence of the results

1) When the system arrangement in reasonable, we assume that the financial structure and the payoff matrix of the listed company $\begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (1,1) \end{pmatrix}$, some of the proportion of the initial strategy of evolutionary change when the effect of the
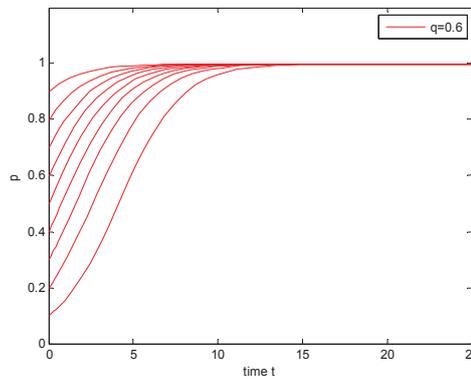
earnings matrix, when $q = 0.5$, as shown in Figure 3a shows, listed companies and financial institutions won't happen evolutionary game.

(a) $q = 0.5$ Financial institutions and listed companies the evolutionary path

(b) $q = 0.3$ Financial institutions and listed companies the evolutionary path

(c) $q = 0.6$ Financial institutions and listed companies the evolutionary path

Figure 3:

When $0 < q < 0.5$, We might as well hypothesis $q = 0.3$ we use digital simulation as shown in Figure 3b shows, and Figure 3a from different initial conditions as they start evolution to the balance state of evolution path absolutely wrong before stack, convergence to equilibrium time and financial institutions to choose the proportion of the initial has a direct relationship, As shown in Figure 3b shows, the proportion of the choice of the smaller, evolution of equilibrium state to time is shorter time more quickly, this kind of situation and fit with the actual situation, as p and q, We can say is the original meaning of cooperation between their degree, The smaller the said does not want to cooperate, Small so they cooperation between the end soon.

When $0.5 < q < 1$, we use digital simulation, Can see financial institutions for different initial value, and between the listed company's evolution path is not the same, absolutely not stack. When p is bigger, the possibility of cooperation between them is very big, so the faster speed, time is shorter.

2) Listed companies and financial institutions return matrix of change when to affect the results of evolution, if income increased, pay off matrix is $\begin{pmatrix} (8,8) & (1,3) \\ (3,1) & (1,1) \end{pmatrix}$ so only when $q = 1/7$, If constantly with the increase of income, in order to the pursuit of the interests, they will converge to the cooperation pattern, and financial companies and between the listed company evolutionary game dynamic process is consistent with the result of, line on the right of the area is more and more big, finally evolved into the state of cooperation.

5 Conclusion

As financial institutions and listed companies a background, establish a copy type dynamic model, and then analyze the local stability, and conclude that the system of two of the five balances is stable, evolution is stable strategy (ESS), corresponding to financial institutions and listed companies and not cooperation. Then use the method of numerical simulation shows the initial conditions, the income matrix for financial institutions and listed companies the influence of evolutionary game theory method, the results provide a reference value of reality, Although the evolutionary game in two game analysis was a great success, but in methods have some limitations, such as to ignore the participants and the quantity possible changes, and may be three or more evolution game, to the question, will in future study, through a complex network of the method of further research.

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