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On the Revolving Ferrofluid Flow Due to Rotating Disk

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Abstract: This paper deals with the theoretical investigation of the effect of rotation on ferrofluid flow due to rotating disk by solving boundary layer equations. Here, we have solved the coupled nonlinear differential equations by power series approximations. Expressions for the components of velocity and pressure profile are obtained in cylindrical co-ordinate system by considering the z-axis as the axis of rotation. It is observed that there is significant increment in the thickness of the boundary layer over rotating disk in comparison to the ordinary case of viscous fluid flow without rotation. The results for all above variables are obtained numerically and discussed graphically.

Keywords: Axi-symmetric; rotating disk; boundary layer; ferrofluid; magnetic-field

1 Introduction

Ferrofluids are stable suspensions of colloidal ferromagnetic particles of the order of 10nm in suitable non-magnetic carrier liquids. These colloidal particles are coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, since last five decades, the investigation on them has been fascinating the researchers and engineers, vigorously. One of the many fascinating features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice-versa [1, 2]. Sealing of the rotating shafts is the most known application of the magnetic fluids. Ferrofluids are widely used in sealing of hard disk drives, rotating x-ray tubes under engineering applications.

The major application of ferrofluid in electrical field is the controlling of heat in loudspeakers. Control on heating makes the life of sound speakers longer and increases the acoustical power without any change in its geometrical shape. Magnetic fluids are used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery. Therefore, ferrofluids play an important role in the field of bio-medical science also. Besides, there are rotationally symmetric flows of the incompressible ferrofluids in the field of fluid mechanics, having all three velocity components viz. radial, tangential and vertical, in space, different from zero. In such type of flows, the variables are independent of angular coordinates and the angular velocity is uniform at large distance from the rotating disk. We consider this type of flow for an incompressible ferrofluid when the rotating disk is subjected to the magnetic field $(H_r,\ 0,\ H_z)$. Verma [3, 4, 5] has solved the research problems on paramagnetic Couette flow, helical flow with heat and flow through a porous annulus, respectively.

Rosensweig [6] has given an authoritative introduction to the research on magnetic liquids in his monograph and studied the effect of magnetization resulting in interesting information. The pioneering study of ordinary viscous fluid flow due to the infinite rotating disk was carried by Karman [7]. He introduced the famous transformation, which reduces the governing partial differential equations into ordinary differential equations. Karman rotating disk problem is extended to the case of flow started impulsively from rest, and also the steady state is solved to a higher degree of accuracy than previously done by a simple analytical method which neglects the resembling difficulties in Cochran's [8] well known solution. Cochran obtained asymptotic solutions for the steady hydrodynamic problem formulated by Karman. Benton [9] improved Cochran's solutions and also, solved the unsteady case. Attia [10] studied the unsteady state in the presence of an applied uniform magnetic field. The steady flow of ordinary viscous fluid due to the rotating disk with uniform suction was studied by Mithal [11]. Attia and Aboul-Hasan [12] discussed about flow due to an infinite disk rotating in the presence of an axial uniform magnetic field by taking Hall effect into consideration.

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In general, magnetization is a function of magnetic field, temperature and density of the fluid. This leads to convection of ferrofluid in the presence of the magnetic field gradient. Sunil *et al.* [13] studied the effect of rotation on thermosolutal convection in a ferromagnetic fluid considering a horizontal layer of an incompressible ferromagnetic fluid. Venkatasubramanian and Kaloni [14] investigated the effect of rotation on the thermo-convective instability of a horizontal layer of ferrofluid heated from below in the presence of uniform vertical magnetic field. Das Gupta and Gupta [15] examined the onset convection in a horizontal layer of ferromagnetic fluid heated from below and rotating about a vertical axis in the presence of a uniform magnetic field. Chauhan and Agrawal [16] studied the MHD flow in a parallel-disk channel partially filled with a porous medium in a rotating system including Hall current. Ram *et al.* [17] solved the non-linear partial differential equations using power series approximations and discussed the effect of magnetic field-dependent viscosity on velocity components and pressure profile. Further, the effect of porosity on velocity components and pressure profile has been studied by Ram *et al.* [18]. Turkyilmazoglu [19] obtained analytical expressions for the solution of steady, laminar, incompressible, viscous fluid of the boundary layer flow due to a rotating disk in the presence of a uniform suction or injection and homotopy analysis method was employed to obtain the exact solutions.Sajid [20] has employed HAM to investigate the slip effects on the two problems of viscous incompressible flows which corresponds to the planar and axi-symmetric stretching of the sheet.

In this problem, we have discussed the solution of the steady laminar flow of an incompressible viscous electrically non conducting ferrofluid over a rotating disk in the presence of a uniform magnetic field and expressions for components of velocity and pressure profile considering various types of fluid responses are obtained in cylindrical co-ordinate system with z-axis as the axis of rotation. It is found that there is a variation in the boundary layer displacement thickness as compared to the ordinary viscous flow case. We also have given the expression for total volume flowing outwards the axis taken over a cylinder of radius R around the z axis using the description given on page 229 in Schlichting [21]. The present study can serve as a theoretical support for experimental investigations. This problem, to the best of our knowledge, has not been investigated yet.

2 Mathematical formulation and solution

The flow in an isotropic medium is considered to be steady $\left(\frac{\partial}{\partial t}=0\right)$ and Axi-symmetric $\left(\frac{\partial}{\partial \theta}=0\right)$ excluding the thermal effects. The fluid and the ferrous particles have the same velocity. The fluid and disk are assumed to be electrically non-conducting. The whole system is rotating with angular velocity $\mathbf{\Omega}=(0,\,0,\,\Omega)$ along the vertical axis, which is taken as z-axis. One additional simplification is assumed in momentum equation that the viscosity is independent of magnetic-field intensity. Basic governing equations of ferrofluid due to Finlayson [22] are as follows: The continuity equation for an incompressible ferrofluid:

$$\nabla\cdot\mathbf{V}=\mathbf{0} \tag{1}$$

The momentum equation for an incompressible ferromagnetic fluid with constant viscosity in a rotating frame of reference:

$$\rho\left[\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}\right]=-\nabla p'+\mu_0\,(\mathbf{M}.\nabla)\mathbf{H}+\mu_f\nabla^2\mathbf{V}+2\rho(\mathbf{\Omega}\times\mathbf{V})+\frac{\rho}{\mathbf{2}}\nabla|\mathbf{\Omega}\times\mathbf{r}|^{\mathbf{2}} \tag{2}$$

The effect of rotation includes two terms: Centrifugal force $-(\rho/2)\ grad|\mathbf{\Omega}\times\mathbf{r}|^{\mathbf{2}}$ and Coriolis acceleration $2\rho(\mathbf{\Omega}\times\mathbf{V})$. In (2), $p'-(\rho/2)\nabla|\mathbf{\Omega}\times\mathbf{r}|^{\mathbf{2}}=p$ is the reduced pressure, where p' stands for fluid pressure.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents:

$$\nabla\times\mathbf{H}=\mathbf{0};\qquad \nabla\cdot(\mathbf{H}+\mathbf{4}\pi\mathbf{M})=\mathbf{0} \tag{3}$$

Assumptions

$$\mathbf{M}=\chi\mathbf{H};\quad \chi=\tfrac{\mu_f}{\mu_0}-1;\quad \mathbf{M}\times\mathbf{H}=\mathbf{0} \tag{4}$$

Attentions have been drawn to the fact that the external magnetic field interacts with magnetic particles, not only through forces but also through couples. The velocity component v_z is less as compared to v_r and v_θ. The equation of motion and equation of continuity reduce to

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu_0}{\rho}|\mathbf{M}|\frac{\partial}{\partial r}|\mathbf{H}|+\nu\left[\frac{\partial^2 v_r}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)+\frac{\partial^2 v_r}{\partial z^2}\right]+2\Omega v_\theta = v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r} \tag{5}$$

$$\nu\left[\frac{\partial^2 v_\theta}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right]-2\Omega v_r = v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r} \tag{6}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu_0}{\rho}|\mathbf{M}|\frac{\partial}{\partial z}|\mathbf{H}|+\nu\left[\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right]= v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z} \tag{7}$$

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0 \tag{8}$$

The approximate initial and boundary conditions for the flow due to rotation of an infinitely long disk $(z=0)$ with constant angular velocity ω are given by

$$\left.\begin{array}{l} at\ \ z=0;\ v_r=0,\ v_\theta=r\omega,\ v_z=0 \\ at\ \ z\to\infty;\ \ v_r=0,\ v_\theta=0 \end{array}\right\} \tag{9}$$

Here, v_z does not vanish for $z\to\infty$ but tends to a finite negative value. Considering boundary layer approximation $-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu_0}{\rho}|\mathbf{M}|\frac{\partial}{\partial r}|\mathbf{H}|=-r\omega^2$ as used by Ram *et al.*[17] and very less variation of magnetic-field along z-direction and using transformations, $v_r=r\omega E(\alpha), v_\theta=r\omega F(\alpha), v_z=\sqrt{\nu\omega}G(\alpha)$; where $\alpha=z\sqrt{\frac{\omega}{\nu}}$; from equations (5)-(8), we get a system of non-linear coupled differential equations in E, F, G and P, as follows:

$$E''-GE'-E^2+F^2+2F-1=0 \tag{10}$$

$$F''-GF'-2EF-2E=0 \tag{11}$$

$$P'-G''+GG'=0 \tag{12}$$

$$G'+2E=0 \tag{13}$$

Boundary conditions (9) become

$$\left.\begin{array}{l} E(0)=G(0)=F(0)-1=P(0)-P_0=0 \\ E(\infty)=F(\infty)=0 \end{array}\right\} \tag{14}$$

G must tend to a finite limit, say $-c$, as $\alpha\to\infty$, i.e. $G(\infty)=-c, (c>0)$. The values of E, F, G and P are compared graphically with their corresponding values in classical case. Cochran indicated that formal asymptotic expansions (for large α) of the system of equations (10)-(13)are the power series in $exp(-c\alpha)$, i.e.

$$E(\alpha)\approx\sum_{i=1}^{\infty}A_i\ exp(ic\alpha) \tag{15}$$

$$F(\alpha)\approx\sum_{i=1}^{\infty}B_i\ exp(ic\alpha) \tag{16}$$

$$G(\alpha)\approx G(\infty)+\sum_{i=1}^{\infty}C_i\ exp(ic\alpha) \tag{17}$$

$$H(\alpha)\approx\sum_{i=1}^{\infty}D_i\ exp(ic\alpha) \tag{18}$$

Integrating the equations (10) and (11) between 0 and ∞ , and using the relations

$$\int_0^\infty GE'\,d\alpha=[GE]_0^\infty-\int_0^\infty G'E\,d\alpha=2\int_0^\infty E^2\,d\alpha$$

$$\int_0^\infty GF'\,d\alpha=[GF]_0^\infty-\int_0^\infty G'F\,d\alpha=2\int_0^\infty EF\,d\alpha$$

Together with $E'(\infty)=0$ and $F'(\infty)=0$, we get

$$-E'(0) = \int_0^\infty (3E^2 - F^2 - 2F + 1)\, d\alpha$$

$$-F'(0) = \int_0^\infty (4EF + 2E)\, d\alpha$$

Using the supposition $E'(0) = a$ and $F'(0) = b$ in equations (10) - (14), we get the following additional boundary conditions for the approximate solution:

$$\left.\begin{array}{lll} E''(0) = -2, & E'''(0) = -4b; & \\ F''(0) = 0, & F'''(0) = 4a; & \\ G'(0) = 0, & G''(0) = -2\,a, & G'''(0) = 4; \\ P'(0) = -2a, & P''(0) = 4, & P'''(0) = 8b. \end{array}\right\} \qquad (19)$$

3 Results and discussion

First four coefficients in each equations (15) - (18), calculated with the help of (14) and (19) are as follows:

$$A_1 = \left(-\tfrac{2b}{3c^3} - \tfrac{3}{c^2} + \tfrac{13a}{3c}\right), A_2 = \left(\tfrac{2b}{c^3} + \tfrac{8}{c^2} - \tfrac{19a}{2c}\right); \ A_3 = \left(-\tfrac{2b}{c^3} - \tfrac{7}{c^2} + \tfrac{7a}{c}\right), \ A_4 = \left(\tfrac{2b}{3c^3} + \tfrac{2}{c^2} - \tfrac{11a}{6c}\right);$$

$$B_1 = \left(\tfrac{2a}{3c^3} + \tfrac{13b}{3c} + 4\right), \quad B_2 = \left(-\tfrac{2a}{c^3} - \tfrac{19b}{2c} - 6\right); \ B_3 = \left(\tfrac{2a}{c^3} + \tfrac{7b}{c} + 4\right), \quad B_4 = \left(-\tfrac{2a}{3c^3} - \tfrac{11b}{6c} - 1\right);$$

$$C_1 = \left(\tfrac{2}{3c^3} - \tfrac{3a}{c^2} + 4c\right), \quad C_2 = \left(-\tfrac{2}{c^3} + \tfrac{8a}{c^2} - 6c\right); \ C_3 = \left(\tfrac{2}{c^3} - \tfrac{7a}{c^2} + 4c\right), \ C_4 = \left(-\tfrac{2}{3c^3} + \tfrac{2a}{c^2} - c\right);$$

$$D_1 = \left(\tfrac{4b}{3c^3} + \tfrac{6}{c^2} - \tfrac{26a}{3c}\right), \ D_2 = \left(-\tfrac{4b}{c^3} - \tfrac{16}{c^2} + \tfrac{19a}{c}\right); D_3 = \left(\tfrac{4b}{c^3} + \tfrac{14}{c^2} - \tfrac{14a}{c}\right), \ D_4 = \left(-\tfrac{4b}{3c^3} - \tfrac{4}{c^2} + \tfrac{11a}{3c}\right).$$

Using the values $a = 0.54$, $b = -0.62$ and $c = 0.886$ from Cochran [8], we calculate the numerical values of the coefficients A_1, A_2, A_3, A_4; B_1, B_2, B_3, B_4; C_1, C_2, C_3, C_4; D_1, D_2, D_3 and D_4; and draw the graphs of velocity components and asymptotic pressure with the dimensionless parameter α. The present numerical results are presented in table 1 and give very good approximate solution of the above system of non-linear differential equations.

Table 1: The steady state velocity field and pressure as functions of α

α	E	F	G	$P(\alpha)-P_0$	E'	F'
0	0	1	0	0	0.54886	-0.62000
0.4	0.08894	0.76572	-0.05778	-0.17909	-0.00241	-0.53308
0.8	0.05266	0.57609	-0.17309	-0.10562	-0.13267	-0.41783
1.2	0.00351	0.42861	-0.30498	-0.00710	-0.10197	-0.32301
1.6	-0.02627	0.31524	-0.43171	0.05253	-0.04825	-0.24658
2.0	-0.03740	0.22935	-0.54162	0.07480	-0.01078	-0.18521
4.0	-0.01482	0.04218	-0.81773	0.02965	0.01132	-0.03673
6.0	-0.00282	0.00727	-0.87408	0.00563	0.00244	-0.00643
8.0	-0.00049	0.00124	-0.88397	0.00098	0.00043	-0.00110
12.0	-1.4E-05	3.6E-05	-0.88594	2.8E-05	1.3E-05	-3.2E-05
∞	0	0	-0.886	0	0	0

The boundary layer displacement thickness is calculated as:

$$d = \frac{1}{r\omega}\int_{z=0}^\infty v_\theta\, dz = \int_{\alpha=0}^\infty F(\alpha)\, d\alpha = 1.3456145$$

Total volume flowing outward the z-axis,

$$\begin{aligned} Q &= 2\pi R\int_0^\infty v_r dz = 2\pi R\int_0^\infty r\omega E(\alpha)\sqrt{\nu/\omega} d\alpha \\ &= -\pi R^2\sqrt{\omega\nu}\, G(\infty) = 2.786094\ R^2\sqrt{\omega\nu} = 2.786094\ R^2\nu\frac{\alpha}{z} \end{aligned}$$

Hence from above, the total volume flowing outward the z-axis is proportional to the dimensionless parameter α.

The fluid is taken to rotate at a large distance from the wall, the angle becomes

$$\tan\varphi_0 = -\left(\frac{\partial v_r}{\partial z}\Big/\frac{\partial v_\theta}{\partial \theta}\right) = -\frac{E'(0)}{F'(0)} = \frac{0.54}{0.62} = 0.870967 \;\Rightarrow\; \varphi_0 = 41^0$$

Particular Case: If we remove the effect of both, the centrifugal force $-(\rho/2)\, grad|\mathbf{\Omega}\times\mathbf{r}|^{\mathbf{2}}$ and Coriolis acceleration $2\rho(\mathbf{\Omega}\times\mathbf{V})$ and boundary layer approximation $-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu_0}{\rho}|\mathbf{M}|\frac{\partial}{\partial r}|\mathbf{H}| = -r\omega^2$, the problem reduces to Cochran's case [8] of ordinary viscous fluid flow without rotation.

The problem considered here involves a number of parameters, on the basis of which, a wide range of numerical results have been derived. Of these results, a small section is presented here for brevity. The numerical results for the velocity profiles, commonly known as radial, tangential, vertical (axial) velocities, are shown in figures 1.1, 1.2, 1.3 respectively. Also, we have calculated the angle between the wall and the revolving ferrofluid, which is 41^0.

In figure 1.1, the curve E_2 represents the effects of rotation on the radial velocity in case of ferrofluid flow due to rotating disk. Whereas, E_1 indicates the radial velocity profile of the Cochran's study of ordinary viscous fluid flow. Due to rotation, the radial velocity reaches its maximum value near the surface of the disk with magnitude 0.08894 at $\alpha = 0.4$, whereas, in Cochran's case, the maximum value 0.181 of radial velocity is attained at comparatively distant point $\alpha = 0.9$. Here, it is noticed that the radial velocity E_2 has very less peak value in comparison to E_1 because of thickening of the fluid layer due to the rotation of the whole system. Thus, the effect of rotation is more pronounced than the force of magnetization in the sense of fluid thickening. Also it is quite interesting to see that in case of rotation before converging to zero, the radial velocity once becomes negative.

Figure 1.2, is the graphical comparison of the tangential velocity of ferrofluid flow with rotation (F_2) to that of Cochran's ordinary viscous fluid flow (F_1). In our case, if we increase the value of α, the tangential velocity F_2 decreases continuously and tends to zero for large value of α. It is observed from the table 1, the value of tangential velocity is 0.49762 at $\alpha = 1$, whereas for the ordinary viscous fluid, the tangential velocity is 0.46800 for the same value of α. Therefore, at $\alpha = 1$, there is an approximate increment of 6.7% in the value of tangential velocity in comparison to that of Cochran's value. Also from the figure, it is clear that F_1 converges to zero little faster than F_2, however, both the curves have similar trends.

Figure 1.3 shows the axial velocity profile, which is zero in the beginning and tends to a finite value -0.886 for $\alpha = 14.8$ onwards. When we increase the value of α, it decreases continuously in the negative region. Our axial velocity value is -0.23858 at $\alpha = 1$, whereas for Cochran, the axial velocity is -0.266 for the same value of α. Meaning thereby, for the same value of α, the axial velocity component acquires larger value than the value in ordinary case.

The pressure profile $P(\alpha) - P_0$ with the initial pressure, P_0 at $\alpha = 0$ is shown in figure 2. Here, pressure goes to negative region near the surface of the disk, and at $\alpha = 0.4$, it goes to maximum negative value -0.17909. Onwards to $\alpha = 0.4$, pressure starts increasing with increasing the value of α, and at $\alpha = 1.3$, it enters in the positive region and attains maximum value 0.07667 at $\alpha = 2.2$. Finally $P(\alpha) - P_0$ converges to zero i.e. $P(\alpha)$ converges to P_0.

Comparing figures 1.1 and 2, we conclude that when radial velocity increases, the pressure of the ferrofluid decreases and when radial velocity decreases, ferrofluid pressure increases i.e. they are converse in convergence behaviour. Also, the tangential velocity diminishes slower than axial velocity component. The change in the curve of radial velocity is faster due to the effect of external magnetic field resulting in reducing the time required for velocity profile to reach their convergence level. In figures 3 and 4, derivatives of radial and tangential components of velocities are shown.

In nut shell, the rotation of the disk along with revolution of the ferrofluid results in an increased displacement thickness 1.34562 more than that of Benton [9].

4 Conclusion

From these results, we conclude that magnetization force i.e., $\mu_0(\mathbf{M}\cdot\nabla)\mathbf{H}$ reduces the pressure. Also, it has been observed that magnetic field intensity increases the radial velocity; whereas, the fluid rotation has reverse effect. The effect of rotation is more pronounced than the force of magnetization due to which the radial velocity takes very less peak value in comparison to ordinary viscous fluid flow case. Due to the rotation, the retardation of the radial velocity increases the thickness of the magnetic fluid layer.

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Nomenclature

V	Velocity of ferrofluid (m/s)
p'	Fluid pressure $(kg\,/m\,s^2)$
$\mathbf{M}$	Magnetization (A/m)
$\mathbf{H}$	Magnetic field intensity (A/m)
r	Radial direction (m)
z	Axial direction (m)
v_r	Radial velocity (m/s)
v_θ	Tangential velocity (rad/s)
v_z	Axial velocity (m/s)
E	Dimensionless component of radial velocity
F	Dimensionless component of tangential velocity
G	Dimensionless component of axial velocity
P	Karman's dimensionless pressure
P_0	Initial pressure (absolute value)
d	Thickness of the ferrofluid layer (m)
Q	Total volume flowing outward the z-axis (m^3)
Greek Symbols	
Ω	Angular velocity of whole system (rad/s)
χ	Magnetic susceptibility
θ	Tangential direction (rad)
ω	Angular velocity of the disk (rad/s)
∇	Gradient operator (m^{-1})
ρ	Fluid density (kg/m^3)
μ_0	Magnetic permeability of free space (H/m)
μ_f	Reference viscosity of fluid (kg/ms)
χ	Magnetic susceptibility
ν	Kinematic viscosity (m^2/s)
α	Karman's parameter (dimensionless)
φ_0	Angle of rotation $(\deg)$

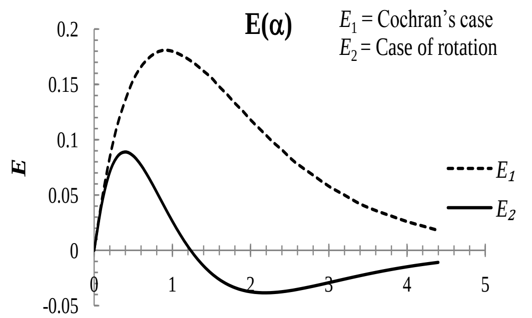

Figure 1.1: Effect of rotation on radial velocity profile

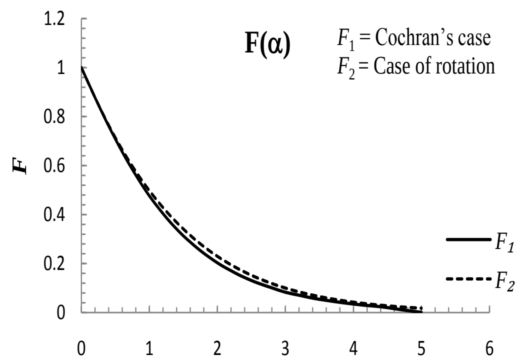

Figure 1.2: Effect of rotation on tangential velocity profile

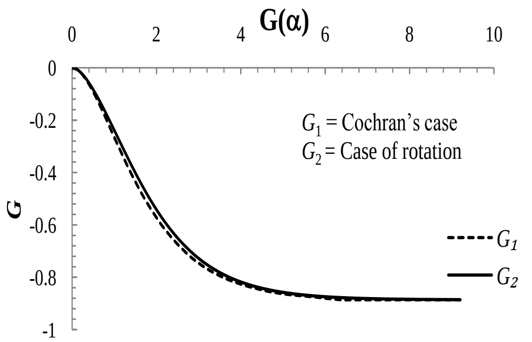

Figure 1.3: Effect of rotation on axial velocity profile

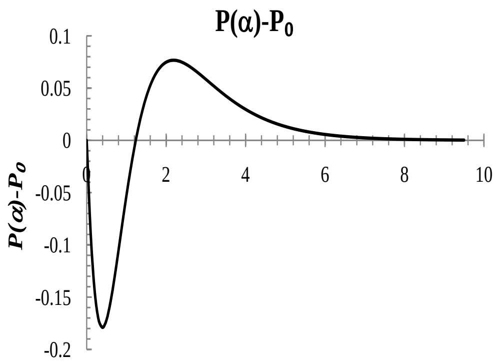

Figure 2: Effect of rotation on pressure profile

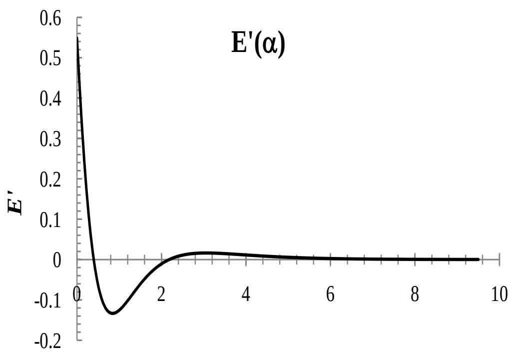

Figure 3: Derivative of radial velocity

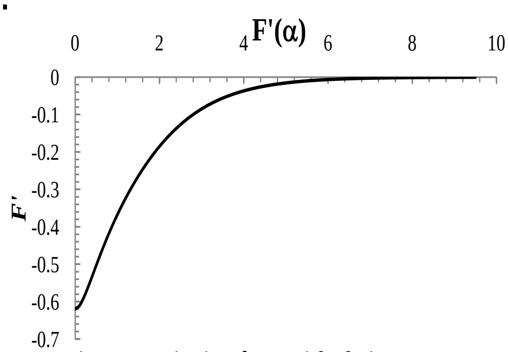

Figure 4: Derivative of tangenial velocity