Mean First-Passage Time on a Class of Treelike Fractal Networks

Meifeng Dai *, Dandan Chen, Yujuan Dong
Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University
Zhenjiang, Jiangsu, 212013, P. R. China
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Abstract: Since random walks on networks are very useful for us to estimate the efficiency of networks. It is difficult to determine the mean first-passage time between two nodes theoretically. In this paper, we address the calculation of the mean first passage time (MFPT) on a class of treelike fractal networks. We deduce an exact formula for the mean first passage time. Our research may be helpful for better understanding on random walks on the treelike fractals.

Keywords: mean first-passage time; treelike fractal network; receiving efficiency; random walk

1 Introduction

In recent ten years, dynamical processes on complex networks, especially random walks, have received a surge of attention, and already have infiltrated into an increasing number of other branches of science, such as computer [1], physics [2][3], biology [4], society [5], and so on. Trapping problem is to say that a single immobile trap is given on a certain place, absorbing all walkers visiting it and never leave it. Mean first-passage time (MFPT) is a related quantity in this problem. If the walker is regarded as an information sender, the MFPT also means the sending efficiency. Actually, the MFPT between node \( u \) and node \( v \) means the expected time for a walker starting from node \( u \) to first reach node \( v \). We also define the average receiving time (ART) as the average of MFPTs for a random walker to a given target hub node, averaged over all source points in the network. Since the random walker can be looked upon as an information messenger, the lower ART means the higher efficiency in transforming information.

The paper is organized as follows. In Section 2 a class of treelike fractal networks are introduced. In Section 3 the random walk (trapping problem) with trap or perfect absorber placed on the hub node is performed. In Section4, finally, is devoted to the global mean first-passage time (GMFPT) defined as the average of MFPTs over all node pairs. From the obtained analytical results, we give the scaling for both PMFPT and GMFPT and show that they both increase as a power-law function of the number of nodes, with the exponent larger than 1 but smaller than 2.

2 Construction and properties of the treelike fractals

The treelike fractal networks, denoted by \( T_g \) after \( g \geq 0 \) iterations, are constructed in the following way. Initially, \( T_0 \) is an edge connecting two nodes. For \( g \geq 1 \), \( T_g \) is obtained from \( T_{g-1} \) as shown in Figure 1: replace the edge by a path of three links long with the two end points of the path being the same end points of the original edge, then attach \( m \) new nodes to the two other internal nodes in the path, respectively. Figure 2 shows the construction process from \( T_0 \) to \( T_2 \) for the case of \( m = 1 \). According to the construction algorithm, at each step the number of edges in the system increases by a factor of \( 2m + 3 \). Thus, we can easily obtain that at generation \( g \) the total number of edges in \( T_g \) is \( E_g = (2m + 3)^g + 1 \). Here \( N_g \) is often named network order. The treelike fractal can also be constructed by another method. This generating algorithm can be described as follows: Given generation \( g \), \( T_g \) may be obtained by joining \( 2m + 3 \) copies of \( T_{g-1} \), denoted as \( T_{g-1}^{(1)} \), \( T_{g-1}^{(2)} \), ..., \( T_{g-1}^{(2m+3)} \), and merge together the discrete outmost nodes of \( T_{g-1}^{(1)} \) and \( T_{g-1}^{(3)} \) ... \( T_{g-1}^{(2m+1)} \), and merge together the discrete outmost nodes of \( T_{g-1}^{(m+3)} \), ...,
Figure 1: Iterative process of the treelike fractals. The next generation is obtained by performing the operation shown to the right of the arrow.

Figure 2: Illustration for the construction process for the case of $m = 1$. A square denotes hub Node 1 and Node 2, while a circle expresses other non-hub nodes $3, \ldots, N_g$.

Thus, $T^{(1)}_{g-1}, T^{(3)}_{g-1}, \ldots, T^{(m+2)}_{g-1}$ have a common point, denoted by $X_g$, and $T^{(m+3)}_{g-1}, \ldots, T^{(2m+3)}_{g-1}$ also have a common point, denoted by $Y_g$. Finally, we paste $X_g$ to one of the outmost nodes in $T^{(2)}_{g-1}$, and paste $Y_g$ to one of the other outmost nodes that is the farthest to node $X_g$. By the way, we denote one of the outmost nodes closer to node $X_g$ as $K_g$ (see Figure 3).

3 Trap fixed random walks

We focus on a particular case of random walks-trapping problem on $T_g$. We fix the trap on one of the hub nodes of the treelike fractals. Firstly, we’ll introduce something about the random walks. At each step, the walk jumps to any of its neighbor nodes with identical probability from its current location. For $T_g$, we label the two hub nodes $X_g$ and $Y_g$ by 1, 2, while all other non-hub nodes are labeled as 3, 4, ..., $N_g$. Since Node 1 and Node 2 is symmetrical to each other, we fix the trap on Node 1. Then we denote $F_{ij}(g)$ as the MFPT for a random walker starting from node $i$ to first reach node $j$, while $j = 1$ we denote $F_{i1}(g)$ as $F_i(g)$. We concentrated on the PMFPT presented as $\langle F \rangle_g$, which is defined as the average of $F_{i1}(g)$ over all starting nodes distributed uniformly over nodes in the network other than the trap. We often name PMFPT mean trapping time or mean time to absorption. By the definition, $\langle F \rangle_g$ is given by

$$\langle F \rangle_g = \frac{1}{N_g - 1} \sum_{i=2}^{N_g} F_i(g).$$

In this section, our main purpose is to determine explicitly $\langle F \rangle_g$ and explain how $\langle F \rangle_g$ scales with network order $N_g$.

We denote $e = (u, v)$ as an edge connecting two nodes $u$ and $v$ for $T_g$ and $C_{u<v}$ as the number of nodes in the subtree containing node $u$. Then, according to the previously obtained results (see [7]), we have

$$F_{uv} = 2C_{u<v} - 1. \quad (1)$$
Figure 3: A schematic illustration of the second construction method for $T_g$, which is obtained by merging five copies of $T_{g-1}$.

As Figure 4 described, nodes $u$ and $v$ is no longer adjacent at generation $g + 1$. According to the construction algorithm of the treelike fractals, the edge in $T_g$ connecting nodes $u$ and $v$ will generate $2m + 2$ new nodes at generation $g + 1$. Two of the new generated nodes are internal nodes, whose degree is $m + 2$, denoted by $x$ and $y$. Thus we have

$$F_{uv}(g + 1) = F_{ux}(g + 1) + F_{xy}(g + 1) + F_{yv}(g + 1).$$

The $F_{uv}(g + 1)$ means the expected time for the walker to get to node $v$ starting from node $u$ in $T_{g+1}$, which equals to three parts of time. That is to say the walker must first arrive node $x$ and node $y$ and then could be get to his target node $v$.

For $T_{g+1}$, $u$, $x$ are adjacent nodes, the same are $x$, $y$ and $y$, $v$. By Eq.(1), we have

$$F_{uv}(g + 1) = 2C_{u<x}(g + 1) + 2C_{x<y}(g + 1) + 2C_{y<v}(g + 1) - 3$$

$$= 2C_{u<x}(g + 1) + 2(C_{u<x}(g + 1) + m + 1)$$

$$+ 2(C_{u<x}(g + 1) + 2m + 2) - 3$$

$$= 6C_{u<x}(g + 1) + 6m + 3.$$ (2)
Notice the iteration algorithm, we know
\[
C_{u<v}(g+1) = C_{u<v}(g) + (2m+2)(C_{u<v}(g) - 1) \\
= (2m+3)C_{u<v}(g) - 2m - 2.
\]  
(3)

When the generation rise from \( g \) to \( g+1 \), the subtree containing node \( u \) has a big change. There are \( 2m+2 \) new nodes generated between every two adjacent nodes. The number of groups of adjacent nodes is \( C_{u<v}(g) - 1 \).

Inserting Eq.(3) into Eq.(2), we have
\[
F_{uv}(g+1) = 6(2m+3)C_{u<v}(g) - 6m - 9 \\
= 3(2m+3)(2C_{u<v}(g) - 1) \\
= 3(2m+3)F_{u<v}(g).
\]  
(4)

Eq.(4) tells us that for any two adjacent nodes \( u \) and \( v \) in the tree at a given generation, the MFPT between them will increase by a factor of \( 3(2m+3) \). According to the treelike structure the MFPT between any pair of two nodes \( i \) and \( j \), adjacent or not, obeys the following relation
\[
F_{ij}(g+1) = 3(2m+3)F_{ij}(g).
\]  
(5)

After obtained some relations of the MFPT between two nodes, we now determine the mean time to absorption averaged over all nontrap nodes in \( T_g \).

As we known, \( T_g \) could be merged by \( 2m+3 \) copies \( T_{g-1}^{(i)} \) (\( 1 \leq i \leq 2m+3 \)) of \( T_{g-1} \). Clearly, the trap Node 1, i.e., node \( X_g \) is always belong to the \( m+2 \) copies \( T_{g-1}^{(i)} \) (\( 1 \leq i \leq m+2 \)). We could classify all the nodes into two classes, according to the symmetry of \( T_g \). One class is the nodes in \( T_{g-1}^{(i)} \) (\( 1 \leq i \leq m+2 \)), including nodes \( X_g \) and \( Y_g \), and the rest of the nodes are belong to the other. Thus, we have
\[
\sum_{i=2}^{N_g} F_i(g) = \sum_{i \in T_{g-1}^{(1)}} F_i(g) + \sum_{i \in T_{g-1}^{(m+3)}} F_i(g).
\]

Since trap is fixed on node \( X_g \), the first item of this equation is equal to \( (m+2) \sum_{i \in T_{g-1}^{(1)}} F_i(g) \).

Thus,
\[
\sum_{i=2}^{N_g} F_i(g) = (m+2) \sum_{i \in T_{g-1}^{(1)}} F_i(g) + \sum_{i \in T_{g-1}^{(m+3)}} F_i(g) \sum_{i \in T_{g-1}^{(m+3)}, i \neq 2} F_i(g) \\
= (m+2) \sum_{i \in T_{g-1}^{(1)}} F_i(g) + (m+1) \sum_{i \in T_{g-1}^{(m+3)}, i \neq 2} (F_{i2}(g) + F_2(g)) \\
= (m+2) \sum_{i \in T_{g-1}^{(1)}} F_i(g) + (m+1) \sum_{i \in T_{g-1}^{(m+3)}} F_{i2}(g) \\
+ (m+1) \sum_{i \in T_{g-1}^{(m+3)}, i \neq 2} F_2(g).
\]

We denote the outmost nodes of \( T_{g-1}^{(1)} \) as \( K_g \) (see Figure 3). For the convenience of description, we denote \( K_{g-1}^{(i)} \) (\( 1 \leq i \leq 2m+3 \)) as the outmost nodes of copy \( T_{g-1}^{(i)} \) (\( 1 \leq i \leq 2m+3 \)).

\[
\sum_{i=2}^{N_g} F_i(g) = (2m+3) \sum_{i \in T_{g-1}^{(1)}} F_{iK_{g-1}}(g) + (m+1)(N_{g-1} - 1)F_2(g) \\
= (2m+3) \sum_{i \in T_{g-1}} F_{iK_{g-1}}(g) + (m+1)(N_{g-1} - 1)F_2(g).
\]  
(6)
Here, we denote $S_g = \sum_{i \in T_g} F_{iK_y}(g)$.

Hence,

$$S_g = \sum_{i \in T_g} F_{iK_y}(g)$$

$$= \sum_{i \in T_{g-1}} F_{iK_y}(g) + \sum_{i \in T_{g-1}^{(2)}, i \neq 1} F_{iK_y}(g) + \sum_{i \in T_{g-1}^{(m+3)}, i \neq 2} F_{iK_y}(g)$$

$$= \sum_{i \in T_{g-1}} F_{iK_y-1}(g) - (g - 1) + (m + 1) \times \sum_{i \in T_{g-1}^{(2)}, i \neq 1} (F_{iK_y-1}(g) + F_{K_y-1}(g))$$

$$+ (m + 1) \sum_{i \in T_{g-1}^{(m+3)}, i \neq 2} (F_{iK_y(m+3)}(g) + F_{K_y(m+3)}(g)).$$

Here, the node $Y_i$ e. g., Node 2, isn’t belong to $T_{g-1}^{(i)}$ $(m + 3 \leq i \leq 2m + 3)$.

Thus,

$$S_g = S_{g-1} + (m + 1) \sum_{i \in T_{g-1}^{(2)}, i \neq 1} F_{iK_y-1}(g) + (m + 1) \sum_{i \in T_{g-1}^{(m+3)}, i \neq 2} F_{2K_y}(g)$$

$$= S_{g-1} + (m + 1)S_{g-1} + (m + 1)(N_{g-1} - 1)F_{1K_y}(g)$$

$$+ (m + 1)S_{g-2} + (m + 1)(N_{g-1} - 1)(N_{g-2} - 1)(F_{1K_y}(g) + F_{2K_y}(g)).$$

Using Eq.(5) and the initial condition $F_{1K_y}(1) = 4m + 5$, $F_{2K_y}(1) = 6m + 8$, we can obtain

$$F_{1K_y}(g) = (4m + 5) \times (3(2m + 3))^{g-1},$$

and

$$F_{2K_y}(g) = (6m + 8) \times (3(2m + 3))^{g-1}.$$
Substituting the result given in Eq.(7) into Eq.(6), then
\[
\sum_{i \in T_g} F_i(g) = (2m + 3) \cdot S_{g-1} + (m + 1)(2m + 3)^{g-1} \times (2m + 3)(3(2m + 3))^{g-1}
\]
\[
= (2m + 3)^{g-1} \left( \frac{10m^2 + 23m + 13}{6m + 8} \right) (3(2m + 3))^{g-1}
+ \frac{38m^2 + 95m + 59}{6m + 8} + (m + 1) \times 3^{g-1} \times (2m + 3)^{2g-1}
\]
\[
= (2m + 3)^{g-1} \left( \frac{(m + 1)(12m^2 + 44m + 37)}{6m + 8} \right) (3(2m + 3))^{g-1}
+ \frac{38m^2 + 95m + 59}{6m + 8}.
\] (7)

Recalling \(N_g = (2m + 3)^g + 1\), we have \(g = \log_{2m+3}(N_g - 1)\). Hence, Eq.(7) can be recast as
\[
\sum_{i \in T_g} F_i(g) = (N_g - 1) \left( \frac{(m + 1)(12m^2 + 44m + 37)}{6m + 8} \right) \times (N_g - 1)^{1 + \log_{2m+3} 3}
+ \frac{38m^2 + 95m + 59}{6m + 8}.
\] (8)

Then
\[
\langle F \rangle_g = \frac{N_g - 1}{N_g} \left( \frac{(m + 1)(12m^2 + 44m + 37)}{6m + 8} \right) \times (N_g - 1)^{1 + \log_{2m+3} 3}
+ \frac{38m^2 + 95m + 59}{6m + 8}.
\] (9)

It is obvious that after each iteration the diameter of the fractals \(T_g\) triples. Thus, we have that the fractal dimension of the treelike fractals is \(d_f = \ln(2m + 3) / \ln 3\). Eq.(5) shows for any two nodes \(i\) and \(j\) increases by a factor of \(3(2m + 3)\) at next generation. Hence, the random-walk dimension of the weighted Koch networks is \(d_w = \ln(3(2m + 3)) / \ln 3\), and their spectral dimension is \(d_s = 2d_f / d_w = 2 \ln(2m + 3) / \ln (3(2m + 3))\) (see [7][8]).

For network with large order, i.e., \(N_g \rightarrow \infty\),
\[
\langle F \rangle_g \sim (N_g)^{1 + \log_{2m+3} 3} = N_g^{2/d_s},
\]
with the exponent greater than 1 and smaller than 2.

This confirms in the large \(g\) limit the PMFPT grows as a power-law function of the network order with the exponent, represented by \(\theta(m) = 1 + \log_{2m+3} 3\), being a decreasing function of \(m\). When \(m\) grows from 1 to infinite, the exponent drop from 1 + \(\log_5 3\) and approaches to 1, indicating that PMFPT grows superlinearly with network order. This also means that the efficiency of the trapping process depends on parameter \(m\): the larger the value of \(m\), the more efficient the trapping process.

### 4 Global mean first-passage time

In this section, a new quantity we concerned in is called global mean first-passage time(GMFPT), which is the average of MFPTs over all pairs of nodes in the network. The GMFPT is defined as
\[
\langle H \rangle_g = \frac{1}{N_g(N_g - 1)} \sum_{i=1}^{N_g} \sum_{j=1, j \neq i}^{N_g} F_{ij}(g).
\] (10)

Note that the definition of GMFPT involves a double average: The first one is over all the walkers to a given target \(j\), the second one is the target nodes uniformly distribute among all nodes in \(T_g\).

We determining \(\langle H \rangle_g\) by using the connection between random walks and electrical networks(see[9][10]). According to the relation between them, we have
\[
F_{ij}(g) + F_{ji}(g) = 2E_g R_{ij}(g) = E_g(R_{ij}(g) + R_{ji}(g)).
\]
where \( R_{ij} \) is the effective resistance between node \( i \) and \( j \). Thus, Eq.(10) could be recast as

\[
\langle H \rangle_g = \frac{1}{N_g} \sum_{i=1}^{N_g} \sum_{j=1, j \neq i}^{N_g} R_{ij}(g). \tag{11}
\]

Since this is a treelike network, the effective resistance \( R_{ij}(g) \) between a pair of nodes \( i \) and \( j \) is the shortest distance \( d_{ij} \) between them. That is

\[
R_{ij}(g) = d_{ij}(g).
\]

We define a new quantity here

\[
s_g = \sum_{i,j \in T_g,i \neq j} d_{ij}(g). \tag{12}
\]

Note that in Eq.(12) for the two nodes \( i \) and \( j (i \neq j) \) we only count \( d_{ij}(g) \) or \( d_{ji}(g) \). Then Eq.(11) could be expressed as follows:

\[
\langle H \rangle_g = \frac{2s_g}{N_g}.
\]

According to the second construction method, we can obtain this equation:

\[
s_g = (2m + 3)s_{g-1} + \Omega_g,
\]

where \( \Omega_g \) is the sum over all the shortest-path distances whose end points are not in the same branch \( T_{g-1}^{(1)} \). \( s_g \) could be obtain by

\[
s_g = (2m + 3)^2 s_0 + \sum_{n=1}^{g} ((2m + 3)^n \Omega_n). \tag{13}
\]

Then we have only to compute \( \Omega_n \) to obtain \( s_g \).

To find \( \Omega_g \), we denote \( \Omega_{g}^{i,j} \) as the sum of all the shortest paths with end points in \( T_{g-1}^{(1)} \) and \( T_{g-1}^{(2)} \), \( i \neq j \). From the construction of the networks, we can see that all the \( \Omega_{g}^{i,j} \) can be classified into two classes. One is those branches having a common vertex. Disconnected branches belong to the other one. Since the structure, we can know there are \( (m + 1)^2 \) disconnected groups in all, while \( (m + 1)(m + 2) \) groups left. Then total sum \( \Omega_g \) is given by

\[
\Omega_g = (m + 1)(m + 2)\Omega_g^{1,2} + (m + 1)^2 \Omega_g^{1,m+3}. \tag{14}
\]

In order to get the path length, we have to calculate \( \Omega_g^{1,2} \) and \( \Omega_g^{1,m+3} \), respectively.

Firstly, as the definition of \( \Omega_g^{\nu,\beta} \) we know \( \Omega_g^{1,2} \) is the sum of the shortest paths when the two endpoints are in \( T_{g-1}^{(1)} \) and \( T_{g-1}^{(2)} \):

\[
\begin{align*}
\Omega_g^{1,2} &= \sum_{i \in T_{g-1}^{(1)}, i \neq j \in T_{g-1}^{(2)}, j \neq 1} d_{ij}(g) \\
&= \sum_{i \in T_{g-1}^{(1)}, i \neq j \in T_{g-1}^{(2)}, j \neq 1} (d_i(g) + d_{ij}(g)) \\
&= (N_{g-1} - 1) \sum_{i \in T_{g-1}^{(1)}, i \neq X} d_i(g) + (N_{g-1} - 1) \sum_{j \in T_{g-1}^{(2)}, j \neq 1} d_{ij}(g) \\
&= 2(N_{g-1} - 1) \sum_{i \in T_{g-1}^{(1)}} d_i(g) \\
&= 2(N_{g-1} - 1)L_{g-1}. \tag{15}
\end{align*}
\]
Here we denote $L_g = \sum_{i \in T_g} d_iK_s(g)$ and $D_g$ the diameter of $T_g$. We can obtain that $D_g = 3^g$ and

$$L_g = \sum_{i \in T_g} d_iK_s(g) = \sum_{i \in T_g^{(1)}} d_iK_s(g) + (m + 1) \sum_{i \in T_g^{(2)} \setminus i \neq 1} d_iK_s(g) + (m + 1) \sum_{i \in T_g^{(m+3)} \setminus i \neq 2} d_iK_s(g) = L_{g-1} + (m + 1) \sum_{i \in T_g^{(2)} \setminus i \neq 1} (d_i(g) + d_1K_s(g)) + (m + 1) \sum_{i \in T_g^{(m+3)} \setminus i \neq 2} (d_iY(g) + d_2K_s(g)) = L_{g-1} + (m + 1) \sum_{i \in T_g^{(2)} \setminus i \neq 1} d_i(g) + (m + 1) \sum_{i \in T_g^{(m+3)} \setminus i \neq 2} d_2(g) + (m + 1) \sum_{i \in T_g^{(m+3)} \setminus i \neq 2} d_2K_s(g)
+ (m + 1) \sum_{i \in T_g^{(m+3)} \setminus i \neq 2} d_2K_s(g)
= L_{g-1} + (m + 1)L_{g-1} + (m + 1)(N_{g-1} - 1)D_{g-1} + (m + 1)L_{g-1} + 2(m + 1)(N_{g-1} - 1)D_{g-1}.

Then we get

$$L_g = (2m + 3)L_{g-1} + 3(m + 1)(N_{g-1} - 1)D_{g-1}. \quad (16)$$

Considering the initial condition $L_0 = 1$, Eq.(16) is solved to obtain

$$L_g = (2m + 3)^{g-1} \left( \frac{m + 1}{2} 3^{g+1} + \frac{m + 3}{2} \right). \quad (17)$$

Inserting Eq.(17) into Eq.(15), we have

$$\Omega_g^{1,2} = 2 \times (2m + 3)^{2g-3} \left( \frac{m + 1}{2} 3^g + \frac{m + 3}{2} \right).$$

Similarly, we compute $\Omega_g^{1,5}$ and finally get

$$\Omega_g^{1,m+3} = 2(N_{g-1} - 1)L_{g-1} + (N_{g-1} - 1)^2D_{g-1} = (2m + 3)^{2g-3}((m + 1)3^g + m + 3) + 3^{g-1}(2m + 3)^{2g-2}.$$ From Eq.(14), we have

$$\Omega_g = (m + 1)(2m + 3)^{2g-2}(4m + 7)3^{g-1} + m + 3. \quad (18)$$

Substituting Eq.(18) into (13) and using the initial condition $s_0 = 1$, we can obtain the expression for the total distance

$$s_g = \frac{(2m + 3)^{g-1}}{3m + 4} ((m + 1)(4m + 7)(3(2m + 3))^g + (m + 3)(3m + 4)(2m + 3)^g - 5(m + 1)^2).$$

Till here we arrive at the explicit solution to $\langle H \rangle_g$:

$$\langle H \rangle_g = \frac{2s_g}{N_g} = \frac{(2m + 3)^{g-1}}{(3m + 4)((2m + 3)^g + 1)} ((m + 1)(4m + 7)(3(2m + 3))^g + (m + 3)(3m + 4)(2m + 3)^g - 5(m + 1)^2).$$

We can also express Eq.(19) in terms of network order $N_g$ as

$$\langle H \rangle_g = \frac{N_{g-1} - 1}{(3m + 4)N_g} ((m + 1)(4m + 7)(N_g - 1)^{1+\log_{2m+3}3} + (m + 3)(3m + 4)(N_g - 1) - 5(m + 1)^2).$$

**JNS email for contribution:** editor@nonlinearscience.org.uk
For network with large order, i.e., $N_g \to \infty$,

$$\langle H \rangle_g \sim (N_g)^{1+\log_2 m + 3} = N_g^{2/d_s}, \quad \text{(19)}$$

showing that the GMFPT $\langle H \rangle_g$ obeying the same scaling as $\langle F \rangle_g$.

From Eq. (9) and Eq. (19) indicate that $\langle F \rangle_g$ and $\langle H \rangle_g$ in the treelike fractal networks show a similar behavior, both of which grow approximately as a power-law function of network order $N_g$ with the exponent being a decreasing function of $m$.

5 Conclusions

In this paper, we constructed a new class of treelike fractal networks and presented the processes to determine the explicit solutions to both PMFPT and GMFPT on them. We find that in the limit of the large network order $N_g$, both of the PMFPT and the GMFT are growing superlinearly with the number of network nodes, which reveals that the trap’s location has no qualitative impact on the trapping efficiency. As the conclusions of trapping problems on another fractal networks [6], we know the result on these two networks are similar, which make us sure that the trapping problem can be done by classifying numbers of central nodes or hub nodes.

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