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Common Fixed Point for Two Self-maps Satisfying a Generalized ${}^{\psi}\int_{\varphi}$ Weakly Contractive Condition of Integral Type

Vishal Gupta *, Naveen Mani
Department of Mathematics, Maharishi Markandeshwar University, Mullana, Ambala—133001 (Haryana), India
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Abstract: In this paper we establish coincidence fixed point and common fixed point theorem for two self mappings which satisfies a ${}^{\psi}\int_{\varphi}$ weakly contractive condition of integral type in complete metric space. Our result extended and generalizes the results of many authors existing in the literature.

Keywords: Coincidence point, complete metric space, compatible maps, ${}^{\psi}\int_{\varphi}$- weakly contractive condition of integral type

1 Introduction

The Banach contraction principle [14] is a very popular tool for solving existence problems in many branches of mathematics and used in many applications. This famous theorem can be stated as

Theorem 1 *[14] - Let (X, d) be a complete metric space,$\alpha \in (0, 1)$ and F be a self-maps of X such that for all $x, y \in X$*

$$d(Fx, Fy) \leq \alpha(d\,(x, y))$$

Then a point $x \in F$ is a fixed point of F, that is $Fx = x$

After this, in 2002, Branciari [1] obtained a fixed point theorem for a mapping satisfying an analogue of Banach Contraction Principle for an integral type inequality stated as follows.

Theorem 2 *[1] Let (X, d) be a complete metric space,$\beta \in (0, 1)$, and C be a self-maps of X such that,for each $x, y \in X$*

$$\int_0^{d(Cx,Cy)} \varphi(t)dt \leq \beta \int_0^{d(x,y)} \varphi(t)dt$$

where $\varphi : [0, +\infty) \to [0, +\infty)$ is a Lebesgue-integrable function which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$,

$$\int_0^{\epsilon} \varphi(t)dt > 0$$

then C has a unique fixed point $z \in X$ such that for each $x \in X$, $C^n x \to z$ as $n \to +\infty$.

Rhoades [10] also extend the result of Branciari for more contractive conditions. Many authors extended and generalized the results of Banach and Branciari, and prove some important result in the field of fixed point theory. Some are mentioned in[2,3,4,6,7,8,9,11,12,16,17,18] .
The concept of altering distances between the points with the use of certain control function is an interesting aspect. In 1984, Khan et al [10] establish a new category of fixed point for single map using such type of control function which they called an altering distance function.

*Corresponding author. *E-mail address*: vishal.gmn@gmail.com

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Definition 1 *[10] The function $\psi : [0,+\infty) \to [0,+\infty)$ is called an altering distance function if the following properties are satisfied:*

- *ψ is contionous and non- decreasing*
- *$\psi(t) = 0$ implies $t = 0$*

and then prove the following theorem:

Theorem 3 *[10] Let (X, d) be a complete metric space and let C be a self-maps of X such that for each $x, y \in X$*

$$\psi(d(Cx, Cy)) \le k\psi(d(x, y))$$

where ψ is an altering distance function and $0 < k < 1$. Then C has a unique fixed point.

Continuing in this process in 2008, P.N.Dutta and B.S. Choudhury [13] proves a generalization of theorem -3 as follows.

Theorem 4 *[13] Let (X, d) be a complete metric space and let C be a self-maps of X such that for each $x, y \in X$*

$$\psi(d(Cx, Cy)) \le \psi(d(x, y)) - \phi(d(x, y))$$

where ψ and ϕ are altering distance function. Then C has a unique fixed point.

In 2010, Sumitra et al [15] generalizes the above result for compatible maps by using above defined control functions. In 2010, N.V.Luong and N.X.Thuan [17] define a $^{\psi}\int_{\varphi}$ -weakly contractive mapping using control function and prove a fixed point theorem for a $^{\psi}\int_{\varphi}$ -weakly contractive mapping in metric space stated as:

Theorem 5 *[17] Let (X, d) be a complete metric space and let $F : X \to X$ is $^{\psi}\int_{\varphi}$ -weakly contractive mapping where $\varphi : [0,+\infty) \to [0,+\infty)$ is a Lebesgue-integrable function which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Then F has a unique fixed point.*

Recently in 2011, Vishal Gupta and Naveen Mani [17] prove a common fixed point theorem for two weakly compatible mappings satisfying a new contractive condition of integral type stated as follows:

Theorem 6 *[17] Let S and T be self compatible maps of a complete metric space (X, d) satisfying the following condition:*

- *$S(x) \subset T(X)$*
- *$\psi\left(\int_0^{d(Sx,Sy)} \varphi(t)dt\right) \le \psi\left(\int_0^{d(Tx,Yy)} \varphi(t)dt\right) - \phi\left(\int_0^{d(Tx,Yy)} \varphi(t)dt\right)$*

for each $x, y \in X$ where $\psi : [0,+\infty) \to [0,+\infty)$ is a continuous and non-decreasing function and $\phi : [0,+\infty) \to [0,+\infty)$ is a lower semi continuous and non-decreasing function such that $\psi(t) = \phi(t) = 0$ if and only if $t = 0$. Also $\varphi : [0,+\infty) \to [0,+\infty)$ is a "Lebesgue-integrable function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Then S and T have a unique common fixed point.

The main result of this paper is the following theorem which extended and generalizes the result of many authors.

2 Main result

Theorem 7 *Let (X, d) be a complete metric space and S and T be two self mappings satisfying the condition*

$$\psi\left(\int_0^{d(Tx,Ty)} \varphi(t)dt\right) \le \psi\left(\int_0^{\lambda(x,y)} \varphi(t)dt\right) - \phi\left(\int_0^{\lambda(x,y)} \varphi(t)dt\right) \tag{1}$$

for each $x, y \in X$ where $\psi : [0,+\infty) \to [0,+\infty)$ is a continuous and non-decreasing function and $\phi : [0,+\infty) \to [0,+\infty)$ is a lower semi continuous and non-decreasing function such that $\psi(t) = \phi(t) = 0$ if and only if $t = 0$.

Also $\varphi : [0, +\infty) \to [0, +\infty)$ is a "Lebesgue-integrable function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$,

$$\int_0^{\epsilon} \varphi(t)dt > 0 \tag{2}$$

and

$$\lambda(x, y) = max\left\{d(Sx, Sy), d(Sx, Tx), d(Sy, Ty), \frac{d(Sx, Ty) + d(Sy, Tx)}{2}\right\} \tag{3}$$

If $T(x) \subset S(X)$ then S and T have a coincidence point in X. Further if S and T are weakly compatible then they have a unique common fixed point in X.

Proof. Since $T(x) \subset S(X)$ so let us choose a point $x_1 \in X$ such that $Sx_1 = Tx_0$. Continuing this process, in general, choose $x_{n+1} \in X$ such that $y_{n+1} = Sx_{n+1} = Tx_n$ and $y_n = Sx_n = Tx_{n-1}\ \forall \quad n = 0, 1, 2....$
Now for each $n \geq 1$, from (1)

$$\psi\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right) = \psi\left(\int_0^{d(Tx_{n-1}, Tx_n)} \varphi(t)dt\right) \leq \psi\left(\int_0^{\lambda(x_{n-1}, x_n)} \varphi(t)dt\right) - \phi\left(\int_0^{\lambda(x_{n-1}, x_n)} \varphi(t)dt\right) \tag{4}$$

Now from (3)

$$\lambda(x_{n-1}, x_n) = max\left\{d(Sx_{n-1}, Sx_n), d(Sx_{n-1}, Tx_{n-1}), d(Sx_n, Tx_n), \tfrac{d(Sx_{n-1}, Tx_n) + d(Sx_n, Tx_{n-1})}{2}\right\}$$

$$\lambda(x_{n-1}, x_n) = max\left\{d(y_{n-1}, yx_n), d(y_{n-1}, y_n), d(y_n, y_{n+1}), \tfrac{d(y_n, y_n) + d(y_{n-1}, y_{n+1})}{2}\right\}$$

$$\lambda(x_{n-1}, x_n) = max\left\{d(y_{n-1}, y_n), d(y_n, y_{n+1}), \tfrac{d(y_n, y_n) + d(y_{n-1}, y_{n+1})}{2}\right\}$$

$$\lambda(x_{n-1}, x_n) = max\left\{d(y_{n-1}, y_n), d(y_n, y_{n+1})\right\}$$

Thus from (4)

$$\psi\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right) \leq \psi\left(\int_0^{max\{d(y_{n-1}, y_n), d(y_n, y_{n+1})\}} \varphi(t)dt\right) - \phi\left(\int_0^{max\{d(y_{n-1}, y_n), d(y_n, y_{n+1})\}} \varphi(t)dt\right)$$

$$\Rightarrow \quad \psi\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right) \leq \psi\left(\int_0^{max\{d(y_{n-1}, y_n), d(y_n, y_{n+1})\}} \varphi(t)dt\right) \tag{5}$$

Now if $d(y_n, y_{n+1}) \geq d(y_{n-1}, y_n)$ for some n, then from (5)

$$\psi\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right) < \psi\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right)$$

This is a contradiction. Thus $d(y_n, y_{n+1}) < d(y_{n-1}, y_n)$ this implies that

$$\psi\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right) \leq \psi\left(\int_0^{d(y_{n-1}, y_n)} \varphi(t)dt\right)$$

Thus $\int_0^{d(y_n, y_{n+1})} \varphi(t)dt$ is monotone decreasing and lower bounded sequence. Therefore there exist $r \geq 0$ such that

$$\lim_{n\to\infty}\left(\int_0^{d(y_n, y_{n+1})} \varphi(t)dt\right) = r \tag{6}$$

Suppose that $r > 0$. Using equation (5), (6) and taking limit as $n \to \infty$ on both side of (3) and also using that ϕ is lower semi continuous, we get

$$\psi(r) \le \psi(r) - \phi(r) < \psi(r)$$

this is a contradiction. Therefore $r = 0$. This implies that

$$\lim_{n\to\infty}\left(\int_0^{d(y_n,y_{n+1})}\varphi(t)dt\right) = 0 \tag{7}$$

$$\lim_{n\to\infty}(d(y_n, y_{n+1}) = 0 \tag{8}$$

Now we prove that $\{y_n\}$ is a Cauchy sequence. Suppose it is not.Therefore there exist an $\epsilon > 0$ and subsequence $\{y_{m(p)}\}$ and $\{y_{n(p)}\}$ of $\{y_n\}$ with $m(p) < n(p) < m(p+1)$ such that

$$d(y_{m(p)}, y_{n(p)}) \ge \epsilon, d(y_{m(p)}, y_{n(p-1)}) < \epsilon \tag{9}$$

Consider

$$\psi\left(\int_0^{\epsilon}\varphi(t)dt\right) \le \psi\left(\int_0^{d(y_{m(p)},y_{n(p)})}\varphi(t)dt\right) = \psi\left(\int_0^{d(Tx_{m(p)-1},Tx_{n(p)-1})}\varphi(t)dt\right)$$

Using (1)

$$\psi\left(\int_0^{\epsilon}\varphi(t)dt\right) \le \psi\left(\int_0^{\lambda(x_{m(p)-1},x_{n(p)-1})}\varphi(t)dt\right) - \phi\left(\int_0^{\lambda(x_{m(p)-1},x_{n(p)-1})}\varphi(t)dt\right) \tag{10}$$

Now from (3),

$\lambda(x_{m(p)-1}, x_{n(p)-1}) = max$

$\left\{d(Sx_{m(p)-1}, Sx_{n(p)-1}), d(Sx_{m(p)-1}, Tx_{m(p)-1}), d(Sx_{n(p)-1}, Tx_{n(p)-1}), \frac{d(Sx_{m(p)-1},Tx_{n(p)-1})+d(Sx_{n(p)-1},Tx_{m(p)-1})}{2}\right\}$

$=max\left\{d(y_{m(p)-1}, y_{n(p)-1}), d(y_{m(p)-1}, y_{m(p)}), d(y_{n(p)-1}, y_{n(p)}), \frac{d(y_{m(p)-1},y_{n(p)})+d(y_{n(p)-1},y_{m(p)})}{2}\right\}$

Let

$$k(m,n) = \frac{d(y_{m(p)-1}, y_{n(p)}) + d(y_{n(p)-1}, y_{m(p)})}{2} \tag{11}$$

Therefore

$$\lambda(x_{m(p)-1}, x_{n(p)-1}) = max\left\{d(y_{m(p)-1}, y_{n(p)-1}), d(y_{m(p)-1}, y_{m(p)}), d(y_{n(p)-1}, y_{n(p)}), k(m,n)\right\} \tag{12}$$

Consider

$\int_0^{\lambda(x_{m(p)-1},x_{n(p)-1})}\varphi(t)dt = \int_0^{max\left\{d(y_{m(p)-1},y_{n(p)-1}),d(y_{m(p)-1},y_{m(p)}),d(y_{n(p)-1},y_{n(p)}),k(m,n)\right\}}\varphi(t)dt$

$\int_0^{\lambda(x_{m(p)-1},x_{n(p)-1})}\varphi(t)dt$

$$= max\left\{\int_0^{d(y_{m(p)-1},y_{n(p)-1})}\varphi(t)dt, \int_0^{d(y_{m(p)-1},y_{m(p)})}\varphi(t)dt, \int_0^{d(y_{m(p)-1},y_{m(p)})}\varphi(t)dt, \int_0^{d(y_{n(p)-1},y_{n(p)})}\varphi(t)dt,\right\} \tag{13}$$

$$\int_0^{k(m,n)}\varphi(t)dt \tag{14}$$

By using (9) and triangle inequality, we get

$$d(y_{m(p)-1}, y_{n(p)-1}) \leq d(y_{m(p)-1}, y_{m(p)}) + d(y_{m(p)}, y_{n(p)-1}) \leq d(y_{m(p)-1}, y_{m(p)}) + \epsilon$$

$$\lim_{p\to\infty} \int_0^{d(y_{m(p)-1}, y_{n(p)-1})} \varphi(t)dt \leq \int_0^{\epsilon} \varphi(t)dt \tag{15}$$

Now

$k(m,n) = \frac{d(y_{m(p)-1}, y_{n(p)}) + d(y_{n(p)-1}, y_{m(p)})}{2}$

$k(m,n) \leq \frac{d(y_{m(p)-1}, y_{m(p)}) + d(y_{m(p)}, y_{n(p-1)}) + d(y_{n(p)-1}, y_{n(p)}) + d(y_{n(p)-1}, y_{m(p)})}{2}$

$k(m,n) \leq \frac{d(y_{m(p)-1}, y_{m(p)}) + d(y_{n(p)-1}, y_{n(p)})}{2} + \epsilon$

Taking $\lim_{p\to\infty}$ and using (8), we get

$$\lim_{p\to\infty} \int_0^{k(m,n)} \varphi(t)dt \leq \int_0^{\epsilon} \varphi(t)dt \tag{16}$$

Taking $\lim_{p\to\infty}$ in equality (10) and using (11),(13),(15),(16), we get

$$\psi\left(\int_0^{\epsilon} \varphi(t)dt\right) \leq \psi\left(\int_0^{\epsilon} \varphi(t)dt\right) - \phi\left(\int_0^{\epsilon} \varphi(t)dt\right)$$

Which is a contradiction. Therefore $\{y_n\}$ is a Cauchy sequence. Call the limit z such that

$$\lim_{n\to\infty} y_n = z$$

$$i.e \qquad \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \tag{17}$$

Since $T(X) \subset S(X)$ therefore there exist some $u \in X$ such that $Su = z$. Hence $\lim_{n\to\infty} Sx_n = Su$. Now consider

$$\lim_{n\to\infty} \psi\left(\int_0^{d(Tx_n, Tu)} \varphi(t)dt\right) \leq \lim_{n\to\infty} \left\{\psi\left(\int_0^{\lambda(x_n,u)} \varphi(t)dt\right) - \phi\left(\int_0^{\lambda(x_n,u)} \varphi(t)dt\right)\right\} \tag{18}$$

where

$$\lim_{n\to\infty} \lambda(x_n, u) = \lim_{n\to\infty} \left[max\left\{d(Sx_n, Su), d(Sx_n, Tx_n), d(Su, Tu), \frac{d(Sx_n, Tu) + d(Su, Tx_n)}{2}\right\}\right]$$

implies

$$\lim_{n\to\infty} \lambda(x_n, u) = d(z, Tu)$$

Hence from (18), we get

$$\psi\left(\int_0^{d(z,Tu)} \varphi(t)dt\right) \leq \psi\left(\int_0^{d(z,Tu)} \varphi(t)dt\right) - \phi\left(\int_0^{d(z,Tu)} \varphi(t)dt\right) \tag{19}$$

This implies that $\phi\left(\int_0^{d(z,Tu)} \varphi(t)dt\right) \leq 0$ which is possible only if $d(z, Tu) = 0$.
Thus $Su = z = Tu$ and so u is the coincidence point of S and T. Since S and T are weakly compatible so they commute at their coincidence point. That is $STu = TSu$ this implies that $Tz = Sz$. Now we claim that z is a fixed point of T. Consider

$$\psi\left(\int_0^{d(z,Tz)} \varphi(t)dt\right) = \psi\left(\int_0^{d(Tu,Tz)} \varphi(t)dt\right) \leq \psi\left(\int_0^{\lambda(u,z)} \varphi(t)dt\right) - \phi\left(\int_0^{\lambda(u,z)} \varphi(t)dt\right) \tag{20}$$

where

$$\lambda(u,z)=max\left\{d(Su,Sz),d(Su,Tu),d(Sz,Tz),\frac{d(Su,Tz)+d(Sz,Tu)}{2}\right\}=d(z,Tz)$$

Hence from (20)

$$\psi\left(\int_0^{d(z,Tz)}\varphi(t)dt\right)\leq\psi\left(\int_0^{d(z,Tz)}\varphi(t)dt\right)-\phi\left(\int_0^{d(z,Tz)}\varphi(t)dt\right)$$

which is a contradiction. Thus $d(z,Tz)=0$. Therefore z is the fixed point of map T and so the fixed point of S. Thus z is the common fixed point of S and T.
For Uniqueness assume that there exist another point w such that $Sw=w=Tw$. Now from (1)we have

$$\psi\left(\int_0^{d(Tz,Tw)}\varphi(t)dt\right)\leq\psi\left(\int_0^{\lambda(z,w)}\varphi(t)dt\right)-\phi\left(\int_0^{\lambda(z,w)}\varphi(t)dt\right) \tag{21}$$

where

$$\lambda(z,w)=max\left\{d(Sz,Sw),d(Sz,Tz),d(Sw,Tw),\frac{d(Sz,Tw)+d(Sw,Tz)}{2}\right\}=d(z,w)$$

hence from (21), we arrive at contradiction. This establish the uniqueness of the the theorem and so the result. ■

3 Remarks

1 - If in theorem 2.1, we assume that $\varphi(t)=1\quad\forall\quad t>0$ then we obtain the result of Sumitra et al [15].
2 - If in theorem 2.1, we assume that $\lambda(x,y)=d(x,y)$ then we obtain the result of N.V.Luong and N.X.Thuan [17].
3 - If we assume that $\varphi(t)=1\quad\forall\quad t>0$ and $\lambda(x,y)=d(x,y)$ then we obtain the result of P.N.Dutta and B.S. Choudhury [13].
This conclude that Theorem (7) generalizes recent existing results in the literature.

4 Example

Here we give an example to verify and support our result.
Let $X=[0,1]$ and d is usual metric on X. Define two self maps S and T where $T(X)\subset S(X)$ such that

$$Tx=\frac{x}{2}\quad and\quad Sx=x\quad\forall\quad x\in X.$$

Let us define $\psi,\phi,\varphi:[0,+\infty)\to[0,+\infty)$ as

$$\psi(t)=t,\quad\phi(t)=\frac{t^2}{2},\quad\varphi(t)=2t\quad\forall\quad t\in[0,+\infty)$$

then for each $\epsilon>0$,

$$\int_0^{\epsilon}\varphi(t)dt=\epsilon^2$$

Verification: Since d is usual metric for all $x,y\in X$

$L.H.S=\psi\left(\int_0^{d(Tx,Ty)}\varphi(t)dt\right)=\psi\left(\int_0^{\frac{|x-y|}{2}}\varphi(t)dt\right)=\psi\left(\frac{|x-y|}{2}\right)^2=\frac{|x-y|}{4}^2$

Now

$R.H.S.=\psi\left(\int_0^{\lambda(x,y)}\varphi(t)dt\right)-\phi\left(\int_0^{\lambda(x,y)}\varphi(t)dt\right)$

where

$\lambda(x,y) = max\left\{|x-y|, \frac{|x|}{2}, \frac{|y|}{2}, \left|x-\frac{y}{2}\right|, \left|y-\frac{x}{2}\right|\right\} = |x-y|$

Hence

$R.H.S. = \psi\left(\int_0^{|x-y|}\varphi(t)dt\right) - \phi\left(\int_0^{|x-y|}\varphi(t)dt\right) \geq \frac{|x-y|}{4}^2$

Thus L.H.S $\leq$ R.H.S and hence inequality of theorem (7) verified.
Also $TX = [0, \frac{1}{2}] \subset SX = [0,1]$. Moreover maps S and T are weakly compatible in X. Hence all the condition of theorem (7)are satisfied.
Clearly '0' is the unique fixed point of S and T.

Corollary 8 *Let (X,d) be a complete metric space and S and T be two self mappings satisfying the condition*

$$\int_0^{d(Tx,Ty)}\varphi(t)dt \leq \int_0^{\lambda(x,y)}\varphi(t)dt - \phi\left(\int_0^{\lambda(x,y)}\varphi(t)dt\right)$$

for each $x,y \in X$ where $\phi : [0,+\infty) \to [0,+\infty)$ is a lower semi continuous and non-decreasing function such that $\phi(t) = 0$ if and only if $t = 0$. Also $\varphi : [0,+\infty) \to [0,+\infty)$ is a "Lebesgue-integrable function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$,

$$\int_0^{\epsilon}\varphi(t)dt > 0$$

and

$$\lambda(x,y) = max\left\{d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), \frac{d(Sx,Ty)+d(Sy,Tx)}{2}\right\}$$

If $T(x) \subset S(X)$ then S and T have a coincidence point in X. Further if S and T are weakly compatible then they have a unique common fixed point in X.

Proof. By using Theorem(7) where $\psi(t) = t$ for all $t \in [0,+\infty)$. ■

Corollary 9 *Let (X,d) be a complete metric space and T be a self map satisfying the condition*

$$\int_0^{d(Tx,Ty)}\varphi(t)dt \leq \int_0^{\lambda(x,y)}\varphi(t)dt - \phi\left(\int_0^{\lambda(x,y)}\varphi(t)dt\right)$$

for each $x,y \in X$ where $\phi : [0,+\infty) \to [0,+\infty)$ is a lower semi continuous and non-decreasing function such that $\phi(t) = 0$ if and only if $t = 0$. Also $\varphi : [0,+\infty) \to [0,+\infty)$ is a "Lebesgue-integrable function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$,

$$\int_0^{\epsilon}\varphi(t)dt > 0$$

and

$$\lambda(x,y) = max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$$

then T has a unique fixed point in X.

Proof. By using Theorem(7) where $\psi(t) = t$ and by assuming $S = I$ for all $t \in [0,+\infty)$. ■

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