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Topology Evolving Model with Scale-free Property for Wireless Sensor Networks

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Abstract: In the view of the survivability of wireless sensor networks(WSNs), a topology evolution model for WSNs based on scale-free network is proposed from the angle of complex networks. The topology of this model evolves according to a certain strategy of connection and deletion with probability, also takes energy of nodes into consideration. The corresponding topology evolution algorithms are given according to the proposed model, and the dynamic characteristics of the model are analyzed. The simulation results show that the network topology formed by the new model is more fit for the practical applications of WSNs and has a good survivability by taking these factors into consideration, which can maintain the robustness of scale-free networks against random removal or failures of nodes.

Keywords:wireless sensor network; complex network; scale-free; robustness; topology evolution

1 Introduction

In recent years, complex networks have received increasing attention for exhibiting the topological structure, function and dynamical properties of many real-world networks such as the World Wide Web, the Internet[1,2], social networks[3], biological networks[4],and ad hoc networks[5]. Wireless sensor networks (WSN) can be described as a special case of complex networks and are composed of plenty of sensor nodes. The sensor nodes are usually deployed in some inaccessible and dangerous places to gather information. WSN has been applied in monitoring environmental changes (such as weather, gas in coal mine)[6], health and animal tracking[7], military surveillance (battle field, biological and chemical attack detection)[8]. Authors in Ref.[9] studied a new evolving mechanism for deducing the fault-tolerant communication topology among the cluster heads with complex network theory. Authors in Ref.[10] adopted two evolving algorithms for WSNs, which are based on complex network theory.

In this paper, in order to provide and maintain a reconfigurable, robust and self-healing topology for WSNs, We propose a topology evolution model, which should take into account the following factors: node addition and deletion with probability, edge addition and deletion with probability, and energy of nodes.

The remainder of this paper is organized as follows. In section 2, we propose an Evolution model of WSNs. In section 3, we give the analysis by the mean field theory. In the section 4, we give the numerical experiments to present the features of the networks generated by the proposed model. Finally, section 5 gives the conclusion of this paper.

2 Evolution model of WSNs

The model evolves from an initial configuration consisting of m_0 nodes and e_0 edges at first. At each time step t evolution of WSNs incorporates the three mechanisms:

(1) Growth: starting with a small number of nodes (m_0) , at every time step t , we add a new node with m edges which will be connected to the nodes already present in the network with probability α_1.

(2) Preferential attachment: When a new node comes into the network, it will choose some nodes in its local-area to connect. We assume that the probability $\prod_1(k_i)$ of a new node will be connected to node i depending on the connectivity

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k_i and its remaining energy E_i of that node. In this paper, the more energy a node has, the more ability it will have of being connected to the new coming nodes. Therefore, the form of $\prod_1(k_i)$ is

$$\prod_1(k_i) = \frac{E_i k_i}{\sum_j^N E_j k_j} \tag{1}$$

(3) With deletion probability α_2 , a uniformly chosen sensor node and its entire links are deleted.

$$\prod_2(k_i) = \frac{1}{N(t)} \tag{2}$$

where $N(t)$ is the total number of nodes in the network, so we can get

$$N(t) = m_0 + \alpha_1 t - \alpha_2 t = m_0 + (\alpha_1 - \alpha_2)t \tag{3}$$

3 The mean field theory of model

In this section, the theoretic analysis of the degree distribution $p(k)$ of the nodes is given in the model by using mean field theory. So the increasing rate of $k_i(t)$ satisfies the following dynamical equation:

$$\frac{\partial k_i}{\partial t} = \alpha_1 m \prod_1 - \alpha_2 \frac{k_i}{N(t)} = \alpha_1 m \frac{E_i k_i}{\sum_j^N E_j k_j} - \alpha_2 \frac{k_i}{N(t)}$$

$$= \alpha_1 m \frac{E_i k_i}{\overline{E} < k(t) > N(t)} - \alpha_2 \frac{k_i}{N(t)} \tag{4}$$

where $\overline{E}$ is the expected energy value of the network, and $\langle k(t) \rangle$ is the average degree of the network.
Case 1: $0 \leq \alpha_1 \leq 1$, $\alpha_2 = 0$

In this case, there are only link and node additions without link and node deletions in the evolving process. So k_i satisfies:

$$\frac{\partial k_i}{\partial t} = \alpha_1 m \prod_1 = \alpha_1 m \frac{E_i k_i}{\overline{E} < k(t) > N(t)} \tag{5}$$

where $N(t) = m_0 + \alpha_1 t, < k(t) >= \frac{2m\alpha_1 t + e_0}{m_0 + \alpha_1 t}$,then we can get

$$\frac{\partial k_i}{\partial t} \approx \frac{E_i k_i}{2\overline{E} t} \tag{6}$$

By the initial condition $k_i(t_i) = m$, then we can get

$$k(i,t) = m \left(\frac{t}{i} \right)^{\frac{E_i}{2\overline{E}}} \tag{7}$$

The probability that a node has a connectivity which satisfy $k_i(t) < k$ is

$$P(k_i(t) < k) = P(t_i > (\frac{m}{k})^{\frac{2\overline{E}}{E_i}} t) \tag{8}$$

Assuming that we add the node to the network at equal time intervals in evolving process for WSNs, the probability density at the time step t_i is $P(t_i) = \frac{1}{m_0 + t}$. Therefore, we get

$$P(k_i(t) < k) = 1 - (\frac{m}{k})^{\frac{2\overline{E}}{E_i}} \frac{t}{m_0 + t} \tag{9}$$

The probability density function of the degree of a node with energy E is

$$P(k_E) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{2m^{\frac{2\overline{E}}{E_i}} t}{m_0 + t} k^{-(1 + \frac{2\overline{E}}{E_i})} \tag{10}$$

The overall probability density function is

$$P(k) = \int_{E_{mn}}^{E_{\max}} \rho(E)P(k_E)dE = \int_{E_{\min}}^{E_{\max}} \rho(E)\frac{2tm^{\frac{2\overline{E}}{E_i}}}{m_0+t}k^{-(1+\frac{2\overline{E}}{E_i})}dE$$

$$\propto k^{-(1+\frac{2\overline{E}}{E_i})} \quad t\to\infty \tag{11}$$

where $\rho(E)$ is the probability density distribution of node energy E in the whole network, $E_{\min}$ and $E_{\max}$ are the bounds of node energy values. $1+\frac{2\overline{E}}{E_i}$ is the exponent of power-law. The degree distribution follows the same power law as the BA scale-free model when $\overline{E}/E_i$is a constant.

Case 2: $0\le\alpha_1\le 1, 0\le\alpha_2<1$

In this case, there are not only node addition and deletion but also edge addition and deletion in the evolving process. So k_i satisfies

$$\frac{\partial k_i}{\partial t} = \alpha_1 m\frac{E_i k_i}{\overline{E}<k(t)>N(t)} - \alpha_2\frac{k_i}{N(t)} \tag{12}$$

Let $S(t)$ be the sum of the degree of all nodes appeared in the network and $e(t)$ be the total number links in the network. thus

$$e(t) = \frac{S(t)}{2} \tag{13}$$

$$\langle k(t)\rangle = \frac{S(t)}{N(t)} \tag{14}$$

we can get the rate equation for$e(t)$

$$\frac{de(t)}{dt} = \alpha_1 m - \alpha_2\langle k(t)\rangle \tag{15}$$

Substituting Eq.(13)and Eq.(14) into Eq.(15), we get

$$S(t) = \frac{2\alpha_1(\alpha_1-\alpha_2)mt}{\alpha_1+\alpha_2} \tag{16}$$

Substituting Eq.(14)and Eq.(16) into Eq.(12), we get

$$\frac{\partial k_i}{\partial t} = \alpha_1 m\frac{E_i k_i}{\overline{E}<k(t)>N(t)} - \alpha_2\frac{k_i}{N(t)} = \alpha_1 m\frac{aE_i k_i}{\overline{E}\frac{2\alpha_1(\alpha_1-\alpha_2)mt}{\alpha_1+\alpha_2}} - \alpha_2\frac{k_i}{m_0+(\alpha_1-\alpha_2)t}$$

$$\approx \frac{(\alpha_1+\alpha_2)E_i - 2\overline{E}\alpha_2}{2\overline{E}(\alpha_1-\alpha_2)}\frac{k_i}{t} \quad t\to\infty \tag{17}$$

By the initial condition $k_i(t_i)=m$, then we can get

$$k(i,t) = m\left(\frac{t}{i}\right)^{\beta} \tag{18}$$

Where $\beta = \frac{(\alpha_1+\alpha_2)(E_i/\overline{E})-2\alpha_2}{2(\alpha_1-\alpha_2)}$, we define $E_i/\overline{E}$ as energy coefficient.

The probability that a node has a connectivity which satisfy $k_i(t)<k$ is

$$P(k_i(t)<k) = P(t_i > (\frac{m}{k})^{\frac{1}{\beta}}t) \tag{19}$$

Assuming that we add the node to the network at equal time intervals in evolving process for WSNs, the probability density at the time step t_i is $P(t_i)=\frac{1}{m_0+t}$. Therefore, we get

$$P(k_i(t)<k) = 1-(\frac{m}{k})^{\frac{1}{\beta}}\frac{t}{m_0+t} \tag{20}$$

The probability density function of the degree of a node with energy E is

$$P(k_E) = \frac{\partial P(k_i(t)<k)}{\partial k} = \frac{t}{m_0+t}m^{\frac{1}{\beta}}k^{-(\frac{1}{\beta}+1)} \tag{21}$$

The overall probability density function is

$$P(k) = \int_{E_{mn}}^{E_{\max}} \rho(E) P(k_E) dE = \int_{E_{\min}}^{E_{\max}} \rho(E) \frac{t}{m_0 + t} m^{\frac{1}{\beta}} k^{-(1+\frac{1}{\beta})} dE$$
$$\propto k^{-(1+\frac{1}{\beta})} \qquad t \to \infty \tag{22}$$

where $\gamma = 1 + \frac{1}{\beta}$ is the power-law exponent of network.

4 Simulation and analysis

To clearly understand the influence of α_1,α_2 and $E_i/\overline{E}$ on the network, we use MATLAB to simulate the case 2, which is more fit for common status.

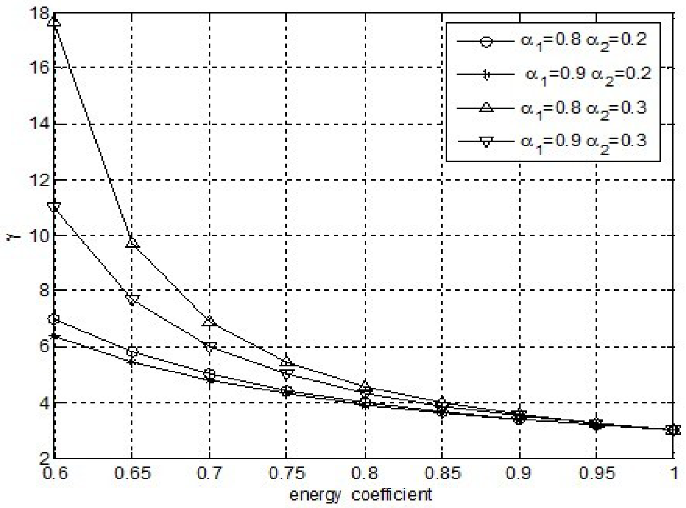

Figure 1: relationship between the power-law exponent γ and energy coefficient $E_i/\overline{E}$

Since the power-law exponent of scale-free in real network generally ranges from 2 to 4. As shown in Fig.1, in order to achieve the best degree distribution of the network, the best number of energy coefficient $E_i/\overline{E}$ should range from 0.85 to1, which provides a hint that an appropriate $E_i/\overline{E}$ can make WSNs maintain a good network topology.

In this simulation we set $E_i/\overline{E}$ to be 0.9. As shown in Fig.2, when α_2 range from0 to0.35, the network can achieve an ideal degree distribution, which have the characteristics of scale-free network. They are robust against random removal or failures of nodes. in harsh environment.

5 Conclusion

In this paper, we introduced complex network theory into the topology evolution of WSNs and proposed a topology evolution models according to the characteristics of WSNs. We also use mean-field theory to make an analysis for the dynamic characteristics of the models. Our numerical simulations above show that our new model reaches the requirements and improves on the network robustness.

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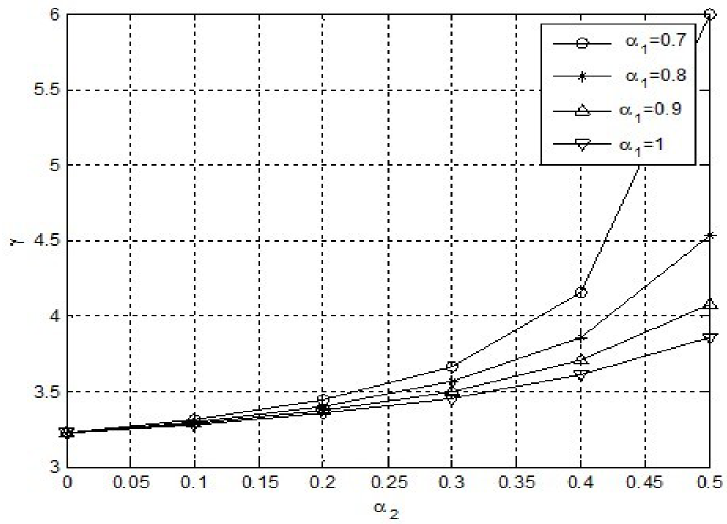

Figure 2: relationship between the power-law exponent$\gamma$and deletion probabilityα_2.

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