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A Review on Opinion Spreading Models Based on Complex Networks

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Abstract: In recent year, the study of opinion spreading models based on complex networks has become a research hot spot. This paper introduces the research development of opinion spreading models on complex networks at home and abroad presently. Combining the front theory of complex networks, the limitations of current research are discussed and the future research direction is prospected.

Keywords: complex networks; opinion spreading; opinion spreading models

1 Introduction

Stephen Hawking, the famous British physicist said that the 21st century is a century of complexity science. This means that the task of the theoretical science in 21st century is to deal with complex systems, and construct a theory system which is different from before. In recent years, complex network has become one of the hottest and most advanced topic in many disciplines [1–6]. With the development of complex network research, researchers began to focus on using complex network theory to study the emergence phenomenon of collective behavior in social network, such as the avalanche of epidemic, the formation and synchronization of opinion [7, 8]. In this review, we provide a research profile of opinion spreading models at home and abroad. Combining the front theory of complex networks, the limitation of current research are discussed and the future research direction is prospected.

2 Opinion spreading models study based on complex networks

In opinion dynamics, the couple of spins represent the interactions between individuals, magnetic moment represents the majority social opinion. Researchers have mapped the topology of social connections onto complete networks in which the nodes represent agents and the links represent the interactions between agents. Various versions of opinion models based on spin models have been proposed and studied, such as the voter model, the majority rule model, Sznajd model, Deffuant model and so on.

2.1 The voter model

The voter model is named basing on a intuitively simple phenomenon in opinion dynamics —- to convince. Since the definition is somewhat trivial, the model is applied to probability theory, population genomic and other fields. The voter model was first proposed by Clifford and Sudbury when considering special competition [9], and named the voter model in Ref. [10]. The model is designed as follows: one can imagine there is a voter at each point on a connected graph, where the connections indicate that there is same form of interaction between a pair of voters (modes). A voter's opinion at any given time can take one of two values, labeled 0 and 1. In each time step, a random individual is selected and that voter's opinion and changed according to a stochastic rule. Specifically, for one of the chosen voter's neighbors is chosen according to a given set of probabilities and that individual is opinion is transferred to the chosen voter.

On regulation lattice network, the voter model can be accurately mapped to random walk model, an individual choose a neighbor and take the neighbor's opinion. Therefore, the random walk theory can be used to study the voter model. In

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d-dimension lattice network, each node point represents an individual. Let $s = s_i$ denotes the state of system, where s_i represents the opinion of node i. A probability function $w_k(s) = w(s_k - s_k)$ [11] is introduced to denote the reversal of an individual's opinion could occur. Then we can get the probability distribution function $p(s)$. Known from the master equation, the correlation functions with arbitrary given order's convergent. Therefore, the correlation functions is solvable [12]. Many scholars have analyzed the first and the second order correlation functions and obtained some meaningful results [12, 13]. In complete graph, the Fokker-Planck equation of magnetisation has the same form as one-dimensional diffusion equation with probability distributing diffusion parameter, which is dependent on position [14].

In recent years, many scholars have studied the effect of network heterogeneity to the evolution of voter models [15–19] , and the effect of small world network topology to the evolution of voter model [20, 21]. Yang et al. [22] have proposed the nonlinear voter model and studied the relationship between the time to achieve consistent phase and adjustable parameter in regular lattice network and scale-free network.

2.2 The majority rule model

Majority Rule (MR) is a decision rule that selects alternatives which have a majority, that is, more than half the votes. In fact, as early as 1982, MR was applied in a statistical geometric model, which showed continuous phase transition. After MR was proposed in 1984 by Serge Galam, many models emerged based on this idea. These models could conform to the reality very well. Ref. [23] made a classification of these models from five different perspectives.

MR model is proposed to describe the public decision-making problem [24]. The system consists of N individuals. There are p_+ fraction of individuals with variable s_i=+1, p_-=1-p_+ fraction of individuals with variable s_i=-1. In the simplest case, each individual is interacted with all the other individuals in the system, namely, the system is a totally connected graph. In each iteration step, r individuals are selected randomly as a group. Each individual adopts the opinions of the majority in the group. The size of the group r is not fixed, but follows a certain distribution. If r is odd, there exists a majority opinion. If r is even, choose a majority opinion, such as r=+1.

Chen et al. extended MR rule to multi-state majority model [25]. MR models, with the individual moving in the space, or with changeable number of neighbors, show interesting properties [26, 27].

2.3 Sznajd model

Sznajd model is proposed by theoretical physicists a father and his daughter in Poland in 2000 [28]. " Three people make a tiger " is the essential theoretical principle of Sznajd model. The rules are as following: in each time step, a pair of spins s_i and s_{i+1} is chosen to change their nearest neighbor, i.e. the spins s_{i+1} and s_{i+2}. If s_i=s_{i+1}, then

$$s_{i-1} = s_i = s_{i+1} = s_{i+2}. \tag{1}$$

But if $s_i \neq s_{i+1}$, then

$$s_{i-1} = s_{i+1}, s_i = s_{i+2}. \tag{2}$$

If individuals i and $i + 1$ have the same opinion, they will affect the opinions of their neighbors according to the Eq (1). Instead, if i and $i + 1$ cannot agree, each of them only affects the pinion of the other neighbor.

The initial states of all the individuals in the system are entirely random. Opinions are randomly assigned to all individuals. Agents update their opinions in a random order. At the end, there are only two states both up (or down) and an equal split of up and down. According to Eq. (2), the latter state can be obtained with a probability of 1/2, and the probability of reaching a consensus is 1/4. The time, which the system reaches attractive point, follows logarithmic distribution. The number of agents, who never change their opinions decreases with time in power function, until reaches a fixed finite value [29]. Some analytical solutions have been obtained under random or other related initial condition [14, 30].

Behera et al. [31] have proved that in one dimensional model, the information flow direction of opinion spreading model is not relevant, and the Sznajd model is equivalent to the voter model. The only difference with the classical voter model is that agent is not affected by its neighbors, but by its neighbors' neighbor. Ref. [30] has found the analytic solution of Sznajd model in complete graph, where Eq. (2) has been changed with the following condition. First, agent pair i and j, and their neighbor k are randomly selected. If $s_i = s_j$, then $s_i = s_j = s_k$, otherwise each agent stick to its own opinion. Sznajd Model has been widely applied in various fields.

In politics, it has been used to describe voting behavior of voters [32]. It has also been used to study the competitive relationship of different products in the markets, and the opinion spreading across a group of traders and so on [33]. Fontoura has studied how to generate complex networks from Sznajd rules [34].

2.4 Deffuant model

The above models are special discrete dynamic opinion models. However, in some cases, the opinion agent holds is continuously changing from one to another [29]. Deffuant model is a typical continuous opinion model [35].

A network with N nodes is used to describe the a group with N agents, where the node represent agent, the link connecting two nodes presents these two nodes will impact on each other. At the initial time, each node i randomly select a number from $[0, 1]$ as its opinion s_i. In each time step, select one model and one of its neighbors randomly. Suppose at time t, a pair of neighbor nodes i and j are chosen, and assigned $s_i(t)$ and $s_j(t)$, respectively. The rules are as following: if $|s_i(t) - s_j(t)| > \varepsilon$, nodes i and j stick to their opinions; but if $|s_i(t) - s_j(t)| < \varepsilon$, their opinions shift toward each other. Therefore, the evolution of Deffuant model is on the basis of compromise principle, most results about Deffuant model have been obtained by simulation. Under special conditions, some scholars studied analytically the evolutionary dynamics of Deffuant modelfor example, Ben-Naim studied Deffuant dynamic model in complete figure using probability equation [36].

Inspired by Deffuant model, a large number of researchers studied the improved Deffuant model. Lorenz et al. considered the diversity of trust parameters of different agents [37–41]. Ben-Naim considered that agents would spontaneously change their opinions [42], Carletti and his collaborators studied the impact of the external periodic disturbance on the agents [43]. Guo et al. considered the diversity of convergence parameters of different agents [44].

2.5 Other models

At the beginning of this century, Hegselmann and Krause established Hegselmann-Krause (HK) model [45]. As HK model has simple structure and complex evolvement behaviors, it has attracted wide attention from many researchers. These studies showed that HK model extended to discrete variables was completely equivalent to the Voter model, with confidence region [45–47]. Fortunato et al. studied the damage spreading and opinion dynamics using the rule of averaging opinions of different neighbors [34]. Lorenz et al. studied the power to enforce and prevent consensus by main puluting communication rules [48]. Iterative Markov chain was applied to study the Deffuant and Hegselmann-Krause dynamics models, and so on [49, 50].

Almost all spin-like opinion models are based on short-range interactions that reach an ordered steady state, with a consensus opinion. In real life, however, different opinions coexist. The realify has motivated scientists to explore opinion spreading models that are more realistic, one in which two opinions can stably coexist. In 2009, Shao et al. have proposed a non consensus opinion (NCO) model in Phys. Rev. L. Unlike the voter model and the majority rule in which the dynamic of an agent's opinion is not influenced by the agent's own current opinion but only by its neighbors. The NCO model takes each node's own current opinion into consideration, and this is a unifical condition for reaching a non-consensus steady state [51]. Exact solutions of the NCO model in one dimension and in a Cayley tree have been developed by Ben-Avraham [52]. Recently, Borghesi et al. studied a spin-like non-consensus opinion model that is able to explain the distribution of voters in several elections [53]. Li et al. developed a spin-type inflexible contrarian opinion (ICO) model in which inflexible contrarian agents are introduced dmto the steady state of the NCO model [54].

3 Existing problems and research prospects

Above all, research on opinion spreading model based on complex network theory has achieved stunning results. However, considering the real world, there are still exist some problems and insufficiency. On one hand, the opinion strength of different individuals are also different. The opinion spreading model with weight has rarely been studied. On the other hand, most studies of opinion spreading model is restricted to undirected network, while the opinion spreading model based on directed network has been rarely studied yet. In addition, in the past most of the results are limited to a single network. However, human activities are not always isolated, but interdependent. Therefore, opinion is not spreading on a single network, but among interdependent networks.

Therefore, combining with the frontier theory of complex network, building the opinion spreading model which can describe more precisely the real-world system is of great significance. How to introduce weight to the model, build opinion spreading model on the directed network, study the laws of opinion spreading on fully and partially interdependent networks, will be the research focus of the next stage.

4 Conclusions

Opinion spreading model based on complex networks has been highly concerned by the fields of academe. But because of the complexity and variety of the real network, setting up the opinion spreading model in line which the acturality has important theoretical significance and practical value. This paper introduces several basic opinion models detailedly, the voter model, Sznajd model, Deffuant model and HK model and so on and describes the research development of the models based on complex networks at home and abroad. Then combining the forefront complex network theory, the limitations of current research are discussed and the future direction is prospected.

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