Random Matrix Analysis of Cross-correlation in Energy Market of Shanxi, China

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Abstract: Energy is essential for social consumption and economic development. Different industries and sectors have different demand on the types of energy. This paper study the correlation evolution of energy price using Pearson’s correlation coefficient (PCC) in Shanxi province for the data from January 2014 to October 2015. Several fossil fuels (thermal coal, anthracite coal, blowing coal, coking coal, diesel oil, gasoline oil) are selected typically. Then, combined the empirical cross-correlation matrix with the random matrix theory (RMT), we mainly examine the statistical properties of cross-correlation coefficient, the evolution of average correlation coefficient, the distribution of eigenvalues and corresponding eigenvectors. The investigation is a way to understand the correlation structure of Shanxi energy market. The result indicated that the selected six kinds of energy are correlated. What’s more, the correlation are influenced by domestic market policy and international energy market.

Keywords: correlation coefficient; energy price; random matrix; eigenvalue; eigenvector

1 Introduction

China’s high-speed economic development has been implemented by a dramatic increase in energy use [1]. In spite of China’s rapid proportion increase in renewable in recent years, coal consumption still dominates the main position. In 2014, China’s total energy consumption was 4.26 billion tons of standard coal equivalents and coal consumption accounted for 66% [2]. By the end of 2014, China’s installed capacity of thermal power was 920 million kW, accounting for 67% of the total power generating installed capacity, what’s more, 61% of which was from coal-fired power generation [3]. The energy structure dominated by coal and the rapid development of industry have resulted in great pressure on environment, such as GHGs (greenhouse gas emissions), air pollution. Specially, the CO2 emissions in China reached 9.15 billion tons in 2015, accounting for 27.3% of the world total (BP, 2016). In November 2009, the Chinese government pledged to reduce carbon emissions by 40%-45% of 2005 levels by 2020 [4]. Under dual pressure from domestic environmental pollution and international climate negotiations, Chinese Government has made a determined effort to regulate energy market and improve environmental quality. As China’s a major energy base with rich coal and iron deposit, Shanxi is one of the most polluted cities in China [5]. Thus, it is very necessary to study the status of Shanxi energy market and explore its characteristics.

Shanxi province lies in northern China and covers 1/60 of territory of China with a population of about 34 million. There are a variety of coal production in Shanxi province, including thermal coal, coking coal, anthracite coal, blowing coal et al, which are refined from raw coal. These coal production represents about 1/4 of China’s total production [6]. Specially, Shanxi is the largest coke production region in the world. In 2007, coke production accounted for over 60% of the total global production [6]. Coal is mined for two main purposes: energy production and steel making. The consumption of coking coal and blowing coal is in higher quality and price, and mainly used in the metallurgical industry within the province [7]. Thermal coal is cheap and dependable. The extraction and transportation procedures are well-established and economically viable [8]. By the end of 2014, China’s installed thermal power capacity was 920 million
kW, 61% of which was from coal-fired power generation [9]. In recent years, Thermal coal has declined slightly due to emissions regulations and a push for alternative of natural gas generators, but the consumption proportion of thermal coal still occupy unshakable position. IEA predicts thermal coal production growth to range between 0.08% per year to 1.75% per year over the 2012-2035 period. Anthracite coal has the highest carbon content [10]. Thus, consumer in household industry most combusts anthracite coal to satisfy living requirement. In general, each type of coal serves different purposes according to its own characteristics. If there exists the correlation among the different types of coal (thermal coal, coking coal, anthracite coal, blowing coal) in different industries is a key issue that worth to study deeply in Shanxi coal market.

Meanwhile, energy prices, as the reflection of the invisible hand of the energy market [11], shows dynamic fluctuations which contains a various market information such as demand, supply, and economic situation (i.e. the financial crisis and petroleum price fluctuation). In this paper, we explore the correlation coefficient of energy price and confirm the influencing factor if they related to policy, international oil price or the distribution of coal resources. Our objection is to analyze Shanxi energy market from the angle of energy price and provide a reference for further research in Shanxi energy market. We hope this study would be benefit to policy-making, as well as improve environmental quality in Shanxi province.

Specifically, Six kinds of energy, Thermal coal, coking coal, anthracite coal, blowing coal diesel oil and gasoline oil, are selected as the study object in Shanxi energy market based on the random matrix theory (RMT). Coal diesel oil and gasoline oil, which are usually used in transportation sector are also considered in our research. Random matrix theory (RMT), which is first proposed by Wigner in 1951 [12], is a popular technical tool for investigating the cross-correlation in financial market [13, 14]. The random matrix theory mainly studies some statistical properties of the eigenvectors of random matrix. It is a hot research topic in the fields of Applied Mathematics, probability statistics and modern physics. Laloux et al [15] and Plerou et al [16] examined the cross-correlation of US stock market using RMT. Hao et al investigated the correlation matrix of price growth rate in US housing markets based on RMT [17]. Sun et al applied RMT to US electricity market and studied the correlation evolution of electricity price among 50 states [18]. Similarly, Ding et al explored the oil market feature in the perspective of oil price based on RMT [19].

The continuing parts of this paper is as follows: In next section, we present empirical data sources and describe the methodology. Section 3 analyze the evolution of average correlation coefficients, eigenvalues, eigenvector. Finally, we draw some conclusions in Section 4.

2 Data and methodology

2.1 Dataset

The dataset is chosen as daily spot price of six kinds of energy (power coal, anthracite coal, coking coal, blowing coal, gasoline and diesel oil) in Shanxi energy market covering the period from January 2014 to October 2015. Coal (power coal, anthracite coal, coking coal, blowing coal ) price data is collected from the China Coal Resource Network and the oil (gasoline and diesel oil) price data is collected from Gold Cast Net. As for the incomplete data in partial days, the study selects the average of ten days of energy price as a data point, giving a total of 66 values. All the variables are converted into the form of standard coal to ensure the rigor and accuracy in our study. When we define \( t = 1 \) as the first data, the range of time vary from 1 to 66.

2.2 Random matrix theory(RMT) and PCC coefficient

The logarithmic return of the energy \( i \) at time \( t \), \( r_i (t) \), is expressed as follows:

\[
r_i (t) = \ln p_i (t) - \ln p_i (t - 1)
\]

where \( p_i (t) \) is the spot price of the energy \( i \) at time \( t \). Thus, for each energy \( i \), the return \( r_i (t) \) has 66 observations.

We adopt the Pearson’s correlation coefficient (PCC) and compute cross-coefficient matrix \( C \) with elements

\[
C_{ij} = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sigma_i \sigma_j}
\]

where \( \langle \cdot \rangle \) denote the mean value, \( \sigma_i \) represents the standard deviation of \( r_i \) i.e. \( \sigma_i = \langle r_i^2 \rangle - \langle r_i \rangle^2 \). By construction, the elements \( C_{ij} \) are restricted in the domain \(-1 \leq C_{ij} \leq 1\), where \( C_{ij} = 1 \) corresponds to perfect correlation, \( C_{ij} = -1 \) corresponds to perfect anti - correlation, and \( C_{ij} = 0 \) corresponds to uncorrelated pairs of energy price. The difficulties
in analyzing the significance and meaning of the empirical cross-correlation coefficients $C_{ij}$ are due to several reasons, which include the following:

(i) Market conditions change with time and the cross-correlations that exist between any pair of price may not be stationary.

(ii) The finite length of time series available to estimate cross-correlations introduces "measurement noise".

Therefore, we consider a random matrix $C$

$$C = \frac{1}{L} M M^T$$

(3)

where $M$ is $N \to \infty, L \to \infty$ rectangular matrix with mean 0 and variance $\sigma^2 = 1$. $T$ denotes matrix transposition. For a random matrix $C$, we obtain

$$C u_k = \lambda u_k$$

(4)

Particularly, in the limit $N \to \infty, L \to \infty$, such that $Q = L/N (> 1)$ is fixed, the probability density function $P_{rm}(\lambda)$ of eigenvalues $\lambda$ of the random correlation matrix $C$ is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi} \sqrt{\frac{\lambda_{max} - \lambda}{\lambda - \lambda_{min}}}$$

(5)

where $\lambda \in [\lambda_{min}, \lambda_{max}]$, and $\lambda_{max}, \lambda_{min}$ given by

$$\lambda_{min,max} = 1 + \frac{1}{Q} \pm 2 \sqrt{\frac{1}{Q}}$$

(6)

If an eigenvalue is greater than $\lambda_{max}$ and thus deviates from the prediction of the RMT - its eigenvector frequently contains valuable information about market dynamics. An eigenvector $u_k$ of a random correlation matrix $C$ should obey the standard normal distribution with zero mean and unit variance.

$$P_R(u_k) = -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right)$$

(7)

### 3 Empirical analysis

In this section, we start by select moving windows [20–22], and then the distribution of correlation coefficient in the correlation coefficient matrix and random matrix are present in Fig. 1. We observe that there are deviation from the interval of eigenvalues $[\lambda_{min}, \lambda_{max}]$ predicted by random matrix theory. Evolution of the average correlation coefficient is shown in Fig. 2. In order to research the risk of energy market in Shanxi province, we investigate the evolution of three largest eigenvalues $\lambda$ and corresponding eigenvectors $u_k$ of $C(t)$ (Figs. 3 and 4).

#### 3.1 Probability density of correlation coefficients

The present study is based on daily spot price of six kinds of energy from January 2014 to October 2015. To the end, we generate $N = 6$ mutually random uncorrelated time series with length $T=66$. For each moving window $[t - s + 1, t]$ at time $t$ of the size $s$, we denote $s = 24$ and get 43 correlation coefficient matrixes $C_1(t), C_2(t) ... C_{41}(t)$ for investigation, whose elements $C_{ij}$ are the Pearson correlation coefficients between the return time series of the energy $i$ and $j$.

$$C_{ij} = \frac{1}{\sigma_i \sigma_j} \sum_{k=t-s+1}^{t} [r_i(k) - \mu_i] [r_j(k) - \mu_j]$$

(8)

where $\mu_i$ and $\mu_j$ are the sample means and $\sigma_i$ and $\sigma_j$ are the standard deviations of the two states $i$ and $j$ respectively.

In order to estimate the real correlation in the correlation coefficient, we examine if there exists difference between the correlation coefficient in random matrix and the correlation matrix of price return. If the distribution is consistent, it is called "noise". On the contrary, the different section is referred to as the "real information". It is very necessary explore the distributional difference between correlation coefficient in the correlation coefficient matrix and correlation coefficient in random matrix, which are presented in Fig. 1.
In Fig.1, compared with the mean value of random matrix and empirical correlation coefficient matrix, there exists obvious difference between them, which illustrates the selected several spot price in energy market are correlative during different time periods.

### 3.2 Evolution of average correlation coefficients

We use the average correlation coefficients of the matrix to explore the real correlation in spot price of energy market with the time evolution in Shanxi energy market. Fig. 2 gives the average correlation coefficient calculated by Eq.(8) for each moving window. The horizontal red line shows the critical value at significance level 5% of the correlation coefficient at each time $t$ and the error bar is the standard deviation of the value at each time $t$. With the moving of windows, the numerical value of matrix is gradually increasing. What’s more, the correlation of energy price fluctuates frequently and sometimes violently in 2015. It means the risk of energy market increases accordingly.

### 3.3 Evolution of three largest eigenvalues and eigenvectors

The dynamic energy price is affected by the market fluctuation, and the several different energy we chosen are mutual influences. Then, we calculate the Eigenvalues and Eigenvectors of correlation coefficient matrix. The first three maximum Eigenvalues of each matrix are shown in Fig. 3.

The horizontal dot-dashed red line means the maximum eigenvalue $\lambda_{\text{max}}$ predicted by RMT. Fig. 3 shows that $\lambda_1$ is larger than $\lambda_{\text{max}}$ for all $C(t)$ matrices. We also find $\lambda_2$ is slightly less than $\lambda_{\text{max}}$ except for several points. In
contrast, $\lambda_3$ falls well less than $\lambda_{\text{max}}$ (Fig.3). The eigenvalue $\lambda_1$ should contain information about spatio-temporal of the Shanxi energy market dynamics. Similarly, we also should put $\lambda_2$ into study. Corresponding to three largest eigenvalues, eigenvectors $\mu_1, \mu_2, \mu_3$ are described in Fig.4(a-c) respectively.

![Figure 4: Evolution of the eigenvectors of the three largest eigenvalues.](image)

Fig. 4 describe the eigenvectors of eigenvalues $\lambda_1, \lambda_2, \lambda_3$. Most of the components of the eigenvectors were positive and a few negative for Shanxi energy market in Fig.4(a). Compared to Fig.4(b) and Fig.4(c), the small fluctuation in the components of Fig.3(a) exhibit a great influence on energy market, which indicate that the eigenvectors of the largest eigenvalues contain much richer information.

In Fig.4(a), the components of the eigenvector of each eigenvalue are negative, and the value has little obvious change. It is difficult to extract clear information from the eigenvector. In Fig.4(b), the value of eigenvector turns from negative to positive obviously. The fluctuation may be associated to the higher price in international oil market recently.

To facilitate discussion, the investigated interval are divide into three periods: [2014/1,2014/9] ($T_1$), [2014/9,2015/3] ($T_2$), [2015/3,2015/12] ($T_3$). In $T_3$ period, the components of the eigenvector are both positive and stability. It means that energy market is relatively stable in this period. In contrast, in the periods of $T_1$ and $T_2$, there is an obvious fluctuation in the components of the eigenvector, especially in the period of $T_2$. In other words, the energy market in Shanxi is unbalance during this period. This situation may relate to the domestic and international energy market. (a) During this period, the international crude oil is mainly in-situ consolidation in first half of the year, and it is slightly stable in the third quarter. And the oil and gas price fell in the fourth quarter, and the maximum decline accumulated is nearly 50%. (b) The rapid development of shale gas industry has opened a new chapter in the new industry, but the OPEC countries insist not to cut production, which lead to the crude oil market from a relatively “short supply” of the times to a “oversupply” of the times in 2014. (c) The sustained weakening of dollar may be another reason which caused this phenomenon.

4 Conclusions

In this paper, we use PCC coefficient to construct empirical cross-correlation matrix and combine them with the RMT to analyze the data information in Shanxi energy market. We chosen the daily spot price of six kinds of energy from January 2014 to December 2015. With the moving of windows, 41 correlation coefficient matrices are generated. In empirical process, we investigate the statistical properties of cross-correlation coefficient, the average correlation coefficient, eigenvalues and corresponding eigenvectors. The findings are follows: (i) The spot price of six selected energy (power coal, anthracite coal, coking coal, blowing coal, gasoline and diesel oil) in Shanxi province are correlated. (ii) Market reform, the volatility of international oil market, development and utilization of shale gas etc have impact on the correlation of energy market in Shanxi province.

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References


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