ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.24(2017) No.1, pp.44-52

A Generalized Anti-synchronization of Discrete Chaotic Maps via Linear Transformations

Debjani Maity[1], Mohammad Ali Khan[2] *, Banshidhar Sahoo[3], Anindita Tarai[4]
[1] Department of Mathematics, Kasthakhali N.K. High School, Purba Medinipur, West Bengal, India
[2]Department of PG Mathematics, Ramananda College, Bankura, West Bengal, India
[3]Department of Mathematics, Hiralal Bhakat College, Birbhum, West Bengal, India
[4]Department of Mathematics, Aligunj R.R.B. High School, Paschim Medinipur, West Bengal, India
(Received 13 September 2013, accepted 15 June 2017)

Abstract: In this paper, we propose the theory of generalized anti-synchronization (GAS) of discrete chaotic maps via linear transformations. We find the necessary and sufficient condition for generalized anti-synchronization of chaotic maps via linear transformations. This synchronization method based on the stability criteria of linear system. This method does not require calculation of the Lyapunov exponents. Our proposed method is able to find the functional relationship between the driving system and the driven system after synchronization. We have taken 3D chaotic Hennon map as an example to discuss our proposed theory. Finally, some numerical simulation results are presented to show the efficiency of our theory.

Keywords: Chaos; Chaotic system; Generalized anti-synchronization(GAS); Hennon map.

1 Introduction

Experimental observations have pointed out that chaotic systems are common in nature. It is found that in Chemistry (Belouov-Zhabotinski reaction), in Nonlinear Optics (lasers), in Electronics (Chua-Matsumoto circuits), in Fluid Dynamics (Rayleigh-Benard convection), etc. chaotic systems exist. Chaos is found in meteorology, solar system, heart and brain of living organisms and so on. Synchronization and control of interacting chaotic oscillators is one of the fundamental phenomena of nonlinear dynamics and chaos. Experimental realization of chaos synchronization and control have been achieved with a magnetoelastic ribbon, a heart, a thermal convection loop, a diode oscillator, an optimal multimode chaotic solid-state laser, a Belousov-Zhabotinski reaction diffusion chemical system, and many other experiments.

One of the most striking discoveries in the study of chaos is that chaotic systems can be made to synchronize with each other. Synchronization of chaos is a phenomenon that may occur when two or more chaotic dynamical systems are coupled. This was discovered by Pecora and Carroll in 1990 [1]. Since Pecora and Carroll's [1] work many effective methods namely, OGY method [2], adaptive control [3], differential geometric method [4], inverse optimal control [5], active control [6], lag synchronization [7, 8], projective synchronization [9–11], spatiotemporal synchronization [12] etc. for chaos control and synchronization have been proposed. Usually two dynamical systems are called synchronized if the distance between their corresponding states converges to zero as time goes to infinity. This type of synchronization is known as identical synchronization [1]. A generalization of the concept for unidirectionally coupled dynamical systems was proposed by Rulkov et. al.[13], where two systems are called synchronized if a static functional relationship exists between the states of the systems. They called this kind of synchronization a generalized synchronization (GS). Kocarev and Parlitz [14] formulated a condition for the occurrence of GS between two coupled continuous dynamical systems. Yang and Chua [15] proposed GS of continuous dynamical systems via linear transformations. Tarai et.al.[16] introduces synchronization between two generalized bidirectionally coupled chaotic system. The anti-synchronization is a phenomenon that the state variables of synchronized systems have the same absolute values but opposite signs. We say that anti-synchronization of two systems S_1 and S_2 are achieved if the following holds:

* **Corresponding author**. *E-mail address*: mdmaths@gmail.com

IJNS.2017.08.15/969

$$\lim_{t\to\infty} |x_1(t) + x_2(t)| = 0$$

where $x_1(t)$, $x_2(t)$ are the state vectors of the systems S_1 and S_2 respectively. It was well known that the first observation of synchronization of oscillators by Huygens in the seventeenth century was, in fact anti-synchronization(AS) between the pendulum clocks. Kim et.al.[17] have observed anti-synchronization phenomena in coupled identical chaotic oscillators. Zhang and Sun [18] have studied anti synchronization based on suitable separation technique. In 2005 Hu et.al.[19] have observed adaptive control for anti synchronization of Chua's chaotic system. Anti synchronization of colpitts oscillators using active control was studied by Hui [20]. An observer based anti synchronization was investigated by Li et.al. [21]. In 2008, Li et.al. [22] have investigated anti-synchronization of two different chaotic systems. Recently Sawalha et.al. [23] have studied anti synchronization of two hyperchaotic systems via nonlinear control.

In this paper, we propose the theory of GAS of discrete chaotic maps via linear transformations. We also find the necessary and sufficient conditions for GAS of chaotic maps via linear transformation. We have taken 3D chaotic Hennon map as an example to discuss our proposed theory. Finally some numerical simulation results are presented to show the efficiency of our proposed method.

2 Generalized anti-synchronization via linear transformation

Any discrete dynamical system can be decomposed into two parts

$$X(k+1) = AX(k) + \Psi(X(k)) \tag{1}$$

where $X = (x_1, x_2, x_3, ...x_n)^T$, A is an $n \times n$ constant matrix and $\Psi : R^n \to R^n$ function. We assume that driving system transmit the signal $\Psi(x)$ to the driven system through the following unidirectional coupling scheme:

$$\begin{aligned} X(k+1) &= AX(k) + \Psi(X(k)) \\ Y(k+1) &= AY(k) - \Lambda\Psi(X(k)) \end{aligned} \tag{2}$$

where Λ is $n \times n$ matrix. Notice that the matrix Λ may be a time dependent matrix, not necessarily a constant matrix.
Theorem *If the matrix Λ commutes with A then two dynamical systems are in a state of generalized anti-synchronization (GAS) via linear transformation*

$$\mathbf{Y}(\infty) = \mathbf{H}(\mathbf{x}) = -\mathbf{\Lambda X}$$

if and only if A has spectral radius less than 1. i.e, if all eigen values of the matrix A has modulus less than 1.

Proof. Let $Z(k) = Y(k) + \Lambda X(k)$, be the generalized anti-synchronization error. Then we have the following dynamical system for the error

$$\begin{aligned} Z(k+1) &= Y(k+1) + \Lambda X(k+1) \\ &= AY(k) - \Lambda\Psi(X(k)) + \Lambda AX(k) + \Lambda\Psi(X(k)) \\ &= A(Y(k) + \Lambda X(k)) \\ &= AZ(k). \end{aligned} \tag{3}$$

Therefore $\lim_{k\to\infty} Z(k) = 0$ if and only if all eigen values (real or complex) of the matrix A having modulus less than 1. Therefore the matrix A can be taken as any real matrix with all eigen values having modulus less than 1. So there are infinite ways to choose the matrix A.
The matrices X which commute with $n \times n$ matrices which satisfies the following equation:

$$AX = XA. \tag{4}$$

Clearly the above equation has infinite number of solutions; therefore we can construct several methods of linear generalized anti-synchronization between two chaotic systems. ■

3 Generalized anti-synchronization of generalized Hennon map

In this section, we discuss the GAS of two discrete time chaotic generalized Hennon map via linear transformation. We consider the following 3D Hennon map

$$\begin{aligned} x_1(k+1) &= -bx_3(k) \\ x_2(k+1) &= bx_3(k) + x_1(k) \\ x_3(k+1) &= 1 + x_2(k) - ax_3^2(k) \end{aligned} \tag{5}$$

where a and b are parameters. For $a = 1.07$ and $b = 0.3$, the Hennon map displays chaotic behavior. The Hennon map can be decomposed into two parts as

$$X(k+1) = AX(k) + \Psi(X(k)) \tag{6}$$

in many ways. Here we consider the two types of decomposition. Firstly we consider

$$A = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix} \tag{7}$$

where $X(k) = (x_1, x_2, x_3)^T$ and $\Psi(X) = [-\alpha_1 x_1(k) - bx_3(k), x_1(k) - \alpha_2 x_2(k) + bx_3(k), 1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)]^T$. Clearly the matrix A has eigenvalues which are all less than 1 in modulus if and only if $\alpha_1 < 1$, $\alpha_2 < 1$ and $\alpha_3 < 1$. Lastly we take

$$A = \begin{pmatrix} \alpha_1 & 0 & -b \\ 0 & \alpha_2 & b \\ 0 & 0 & \alpha_3 \end{pmatrix} \tag{8}$$

where $X(k) = (x_1, x_2, x_3)^T$ and $\Psi(X) = [-\alpha_1 x_1(k), x_1(k) - \alpha_2 x_2(k), 1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)]^T$. Clearly the matrix A has eigenvalues which are all less than 1 in Modulus also, if and only if $\alpha_1 < 1$, $\alpha_2 < 1$ and $\alpha_3 < 1$. Now the driven Hennon map can be taken as

$$Y(k+1) = AY(k) - \Lambda\Psi(X(k)) \tag{9}$$

where the matrix Λ commutes with A. Therefore the driving Hennon map and the driven Hennon map will anti-synchronize in the generalized sense.

4 Results and Discussions

We did the numerical simulation of the generalized Hennon map taking random initial conditions for both driving and driven system. First two simulations have done for decomposition of the Hennon map as in equation (7) and rest three simulations are done using decomposition of the Hennon map as in equation (8).

Simulation 1

In this simulation, we take

$$\Lambda = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \tag{10}$$

where λ is real. Obviously the sufficient condition for GAS, $\Lambda A = A\Lambda$ is satisfied. Here the driving Hennon map is given by (5) and the driven Hennon map is given by

$$\begin{aligned} y_1(k+1) &= \alpha_1 y_1(k) + \lambda(\alpha_1 x_1(k) + bx_3(k)) \\ y_2(k+1) &= \alpha_2 y_2(k) - \lambda(bx_3(k) + x_1(k) - \alpha_2 x_2(k)) \\ y_3(k+1) &= \alpha_3 y_3(k) - \lambda(1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)) \end{aligned} \tag{11}$$

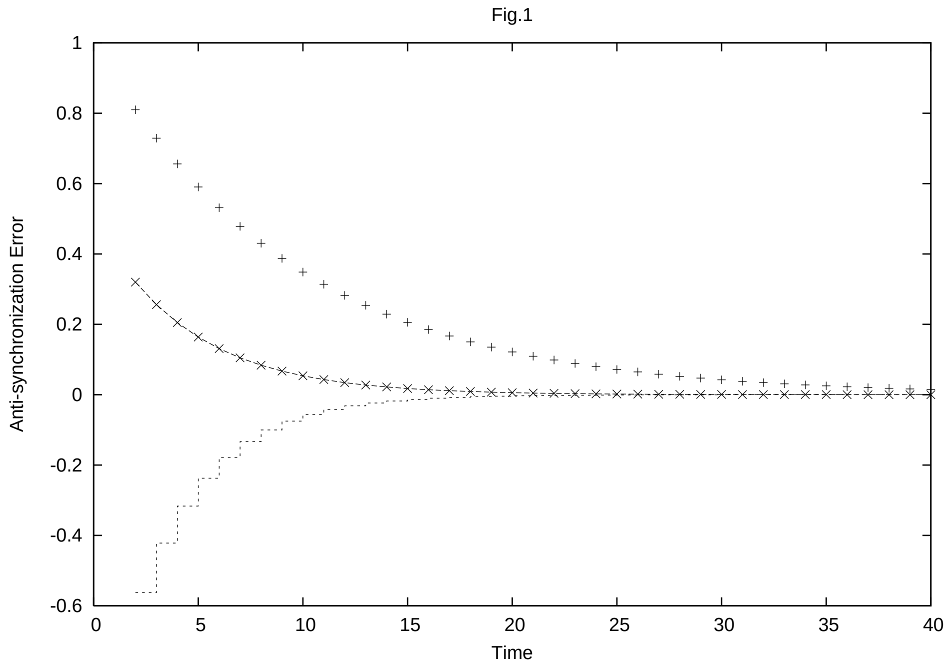

Figure 1: Time evaluation of generalized anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown for $\alpha_1 = 0.9, \alpha_2 = 0.8$ and $\alpha_3 = 0.75$ for simulation 1.

In this case, we define GAS error as

$$\begin{aligned} e_1(k) &= y_1(k) + \frac{1}{2}x_1(k) \\ e_2(k) &= y_2(k) + \frac{1}{2}x_2(k) \\ e_3(k) &= y_3(k) + \frac{1}{2}x_3(k) \end{aligned} \quad (12)$$

Taking $\alpha_1 = 0.9, \alpha_2 = 0.8$ and $\alpha_3 = 0.75$ and for $\lambda = 0.5$ the time evaluation of generalized anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ is shown in Fig.1. It is obvious from the Fig.1. that anti-synchronization between the systems (5) and (11) is happening. Here the state variable of the driving system and the driven system are connected by the linear transformation

$$\begin{aligned} y_1(k) + \frac{1}{2}x_1(k) &= 0 \\ y_2(k) + \frac{1}{2}x_2(k) &= 0 \\ y_3(k) + \frac{1}{2}x_3(k) &= 0 \end{aligned} \quad (13)$$

Simulation 2

In this simulation, we take

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \quad (14)$$

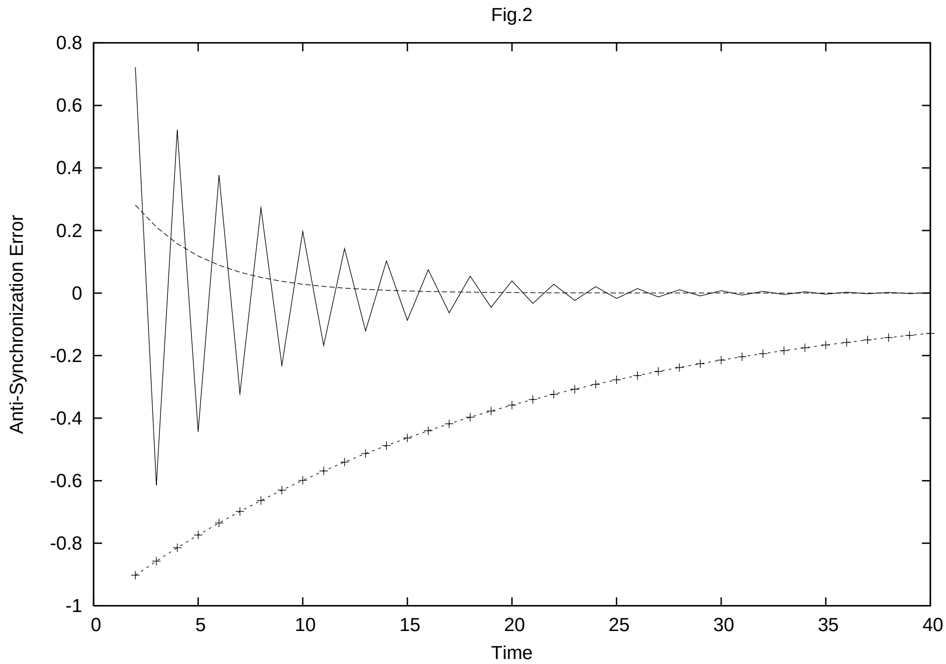

Figure 2: Time evaluation of generalized anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown for $\alpha_1 = -0.85, \alpha_2 = 0.75$ and $\alpha_3 = 0.95$ for simulation 2.

where $\lambda_1, \lambda_2, \lambda_3$ are all real. In this case the driven system are given by

$$\begin{aligned} y_1(k+1) &= \alpha_1 y_1(k) + \lambda_1(\alpha_1 x_1(k) + bx_3(k)) \\ y_2(k+1) &= \alpha_2 y_2(k) - \lambda_2(bx_3(k) + x_1(k) - \alpha_2 x_2(k)) \\ y_3(k+1) &= \alpha_3 y_3(k) - \lambda_3(1 + x_2(k) - \alpha_3 x_3(k) - a x_3^2(k)) \end{aligned} \tag{15}$$

In this case, we define GAS error as

$$\begin{aligned} e_1(k) &= y_1(k) + x_1(k) \\ e_2(k) &= y_2(k) + 2x_2(k) \\ e_3(k) &= y_3(k) + 3x_3(k) \end{aligned} \tag{16}$$

Taking $\alpha_1 = -0.85, \alpha_2 = 0.75$ and $\alpha_3 = 0.95$ and for $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$, the effectiveness of the proposed scheme is shown in Fig.2, by drawing the time evaluation of GAS errors $e_1(t)$, $e_2(t)$ and $e_3(t)$.

Here the state variable of the driving system and the driven system are connected by the linear transformation

$$\begin{aligned} y_1(k) + x_1(k) &= 0 \\ y_2(k) + 2x_2(k) &= 0 \\ y_3(k) + 3x_3(k) &= 0 \end{aligned} \tag{17}$$

Simulation 3

In this simulation, we take

$$\Lambda = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \tag{18}$$

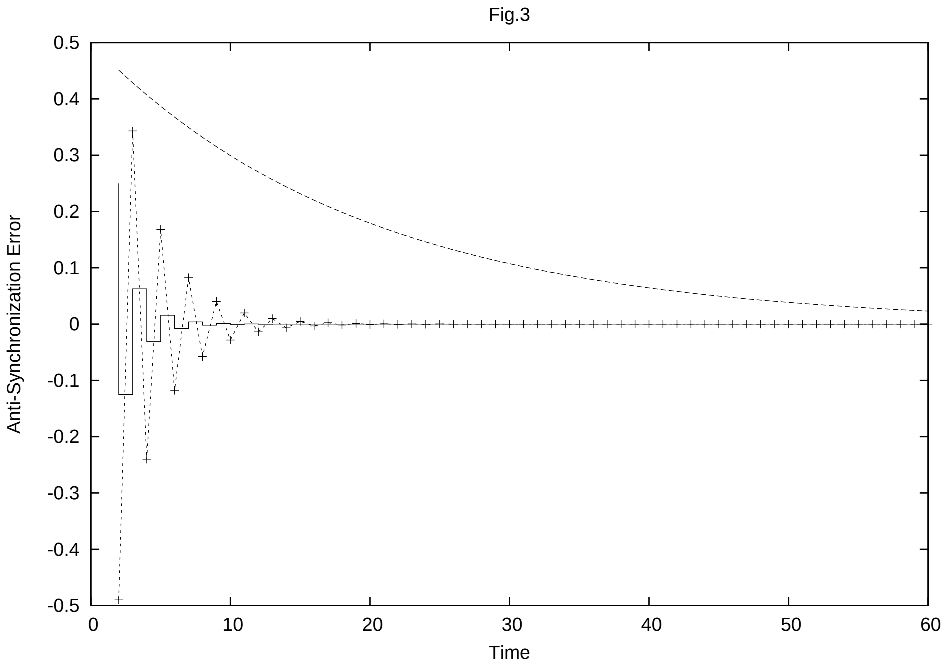

Figure 3: Time evaluation of generalized anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown for $\alpha_1 = -0.5, \alpha_2 = 0.95$ and $\alpha_3 = -0.70$ for simulation 3.

where λ is real. In this case the driven system are given by

$$\begin{aligned} y_1(k+1) &= \alpha_1 y_1(k) - by_3(k) - \lambda\alpha_1 x_1(k) \\ y_2(k+1) &= \alpha_2 y_2(k) + by_3(k) - \lambda(x_1(k) - \alpha_2 x_2(k)) \\ y_3(k+1) &= \alpha_3 y_3(k) - \lambda(1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)) \end{aligned} \tag{19}$$

In this case, we define GAS error as

$$\begin{aligned} e_1(k) &= y_1(k) + 2x_1(k) \\ e_2(k) &= y_2(k) + 2x_2(k) \\ e_3(k) &= y_3(k) + 2x_3(k) \end{aligned} \tag{20}$$

Taking $\alpha_1 = -0.5, \alpha_2 = 0.95$ and $\alpha_3 = -0.70$ and for $\lambda = 2$, the successfulness of our method is shown in Fig.3. For the state variable of the driving system and the driven system are connected by the linear transformation

$$\begin{aligned} y_1(k) + 2x_1(k) &= 0 \\ y_2(k) + 2x_2(k) &= 0 \\ y_3(k) + 2x_3(k) &= 0 \end{aligned} \tag{21}$$

Simulation 4

In this simulation, we take

$$\Lambda = A = \begin{pmatrix} \alpha_1 & 0 & -b \\ 0 & \alpha_2 & b \\ 0 & 0 & \alpha_3 \end{pmatrix} \tag{22}$$

In this case the driven Hennon system are given by

$$\begin{aligned} y_1(k+1) &= \alpha_1 y_1(k) - by_3(k) - \alpha_1^2 x_1(k) + b(1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)) \\ y_2(k+1) &= \alpha_2 y_2(k) + by_3(k) - \alpha_2(x_1(k) - \alpha_2 x_2(k)) - b(1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)) \\ y_3(k+1) &= \alpha_3 y_3(k) - \alpha_3(1 + x_2(k) - \alpha_3 x_3(k) - ax_3^2(k)) \end{aligned} \tag{23}$$

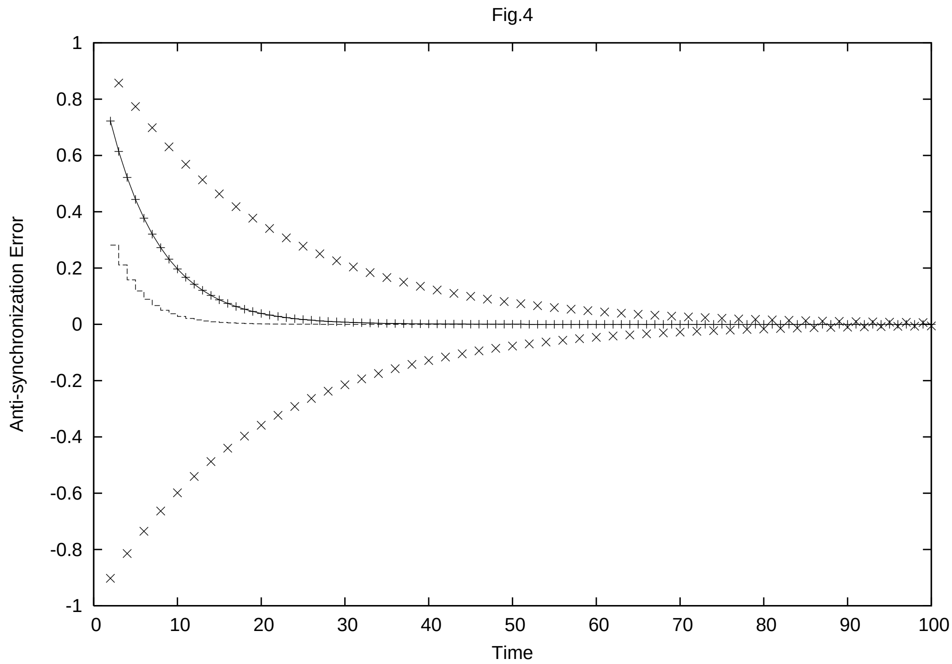

Figure 4: Time evaluation of generalized anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown for $\alpha_1 = 0.85, \alpha_2 = 0.75$ and $\alpha_3 = -0.95$ for simulation 4.

The GAS error can be expressed as

$$\begin{aligned} e_1(k) &= y_1(k) + \alpha_1 x_1(k) - b x_3(k) \\ e_2(k) &= y_2(k) + \alpha_2 x_2(k) + b x_3(k) \\ e_3(k) &= y_3(k) + \alpha_3 x_3(k) \end{aligned} \tag{24}$$

Taking $\alpha_1 = 0.85, \alpha_2 = 0.75$ and $\alpha_3 = -0.95$ and for $\lambda = 2$, the time evaluation of GAS errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown in Fig.4. In this case the state variables of the driving system and driven system are connected by the linear transformation

$$\begin{aligned} y_1(k) + \alpha_1 x_1(k) - b x_3(k) &= 0 \\ y_2(k) + \alpha_2 x_2(k) + b x_3(k) &= 0 \\ y_3(k) + \alpha_3 x_3(k) &= 0 \end{aligned} \tag{25}$$

Simulation 5

In this simulation, we take

$$\Lambda = A^{-1} = \begin{pmatrix} \frac{1}{\alpha_1} & 0 & \frac{b}{\alpha_1 \alpha_3} \\ 0 & \frac{1}{\alpha_2} & -\frac{b}{\alpha_2 \alpha_3} \\ 0 & 0 & \frac{1}{\alpha_3} \end{pmatrix} \tag{26}$$

For the above choice of matrix, the driven driven Hennon system are given by

$$\begin{aligned} y_1(k+1) &= \alpha_1 y_1(k) - b y_3(k) + x_1(k) - \frac{b}{\alpha_1 \alpha_3}(1 + x_2(k) - \alpha_3 x_3(k) - a x_3^2(k)) \\ y_2(k+1) &= \alpha_2 y_2(k) + b y_3(k) - \frac{1}{\alpha_2}(x_1(k) - \alpha_2 x_2(k)) + \frac{b}{\alpha_2 \alpha_3}(1 + x_2(k) - \alpha_3 x_3(k) - a x_3^2(k)) \\ y_3(k+1) &= \alpha_3 y_3(k) - \frac{1}{\alpha_3}(1 + x_2(k) - \alpha_3 x_3(k) - a x_3^2(k)) \end{aligned} \tag{27}$$

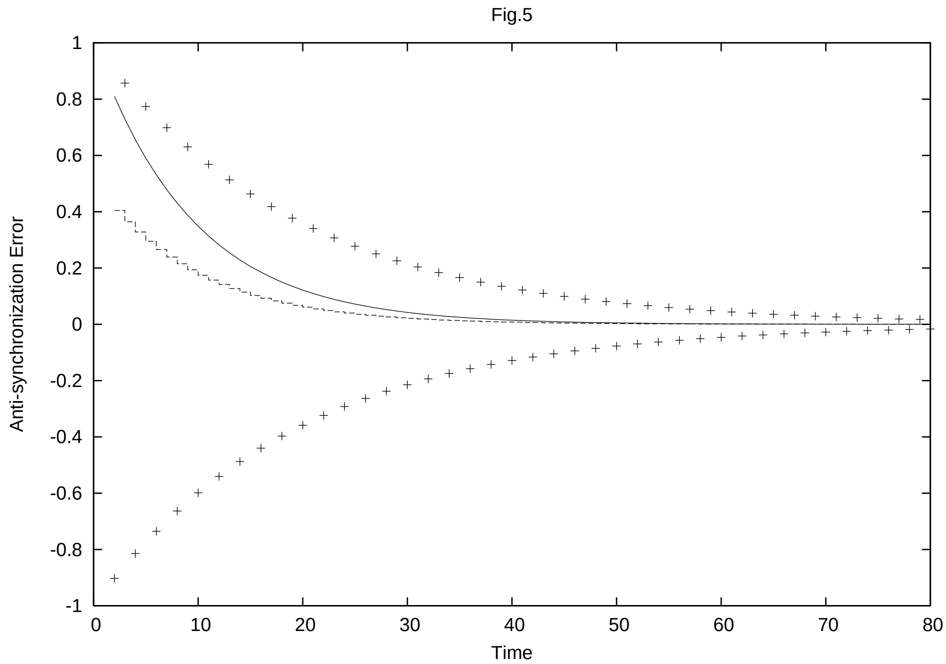

Figure 5: Time evaluation of generalized anti-synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown for $\alpha_1 = 0.90, \alpha_2 = 0.90$ and $\alpha_3 = -0.95$ for simulation 5.

For Simulation 5, we define GAS error as

$$\begin{aligned} e_1(k) &= y_1(k) + \frac{1}{\alpha_1}x_1(k) + \frac{b}{\alpha_1\alpha_3}x_3(k) \\ e_2(k) &= y_2(k) + \frac{1}{\alpha_2}x_2(k) - \frac{b}{\alpha_2\alpha_3}bx_3(k) \\ e_3(k) &= y_3(k) + \frac{1}{\alpha_3}x_3(k) \end{aligned} \tag{28}$$

The time evaluation of GAS errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ for system of Eq. (28) are shown in Fig.5. The state variables of the driving system and the driven system are connected by the linear transformation

$$\begin{aligned} y_1(k) + \frac{1}{\alpha_1}x_1(k) + \frac{b}{\alpha_1\alpha_3}x_3(k) &= 0 \\ y_2(k) + \frac{1}{\alpha_2}x_2(k) - \frac{b}{\alpha_2\alpha_3}x_3(k) &= 0 \\ y_3(k) + \frac{1}{\alpha_3}x_3(k) &= 0 \end{aligned} \tag{29}$$

5 Conclusions

A generalized approach for constructing chaotically anti-synchronized discrete dynamical systems via linear transformation is proposed in this paper. The proposed method is simpler than the conventional methods because here we need not require calculation of the Lyapunov exponents. In our method the functional relationship between the states of the driving system and the driven system can be determined. Therefore knowing the driving system the behavior of the driven system can be predicted in advance here. If the matrix associated with the linear transformation is invertible then from the behavior of the driven system it is possible to predict the behavior of the driving system. This synchronization scheme may be used for sending secret message for the purpose of communications. For the communication purpose our goal is to send secretes information of the driving system to the driven system. Our method proposes a rule by which one can construct a response system which will anti-synchronized with the driving system. For communication purpose GAS is more effectively secret because for the outsiders who want to extract information transmitted by the driving system, apart from knowing the mechanism of synchronization, become very difficult. A hacker has to know another code the functional relationship between the variables of the drive and response system. In GAS there exist infinite ways to choose the secret key. Therefore, the techniques based on GAS is seemed to be more practical in secrete communication application.

References

[1] Pecora and Carroll, *Synchronization in chaotic systems.*, Physical Review Letters. 1990, 64:821-824.
[2] E.Ott, C.Grebogi and J.A.York, *Controlling chaos.*, Physical Review Letters. 1990, 64:1196-1199.
[3] S.Chen and J.Lu, *Synchronization of an uncertain unified chaotic system via adaptive control.*, Chaos, Soliton and Fractals.2002, 14:643.
[4] C.C.Fuh and P.C.Tung, *Controlling chaos using differtial geometric method.*, Phys.Rev.Lett. 1995, 75:2952.
[5] E.N.Sanchez, J.P.Perez, M.Martinez and G.Chen, *Chaos stabilization: an inverse optimal control approach.*, Latin Am Appl Res:Int J.2002, 32:111.
[6] M.A.Khan, *Synchronization of different 3D chaotic systems by generalized active control.*, Journal of Information and Computing Science. 2012 7:272-283.
[7] D.Pazo, M.A.Zaks and J.Kurths, *Role of unstable periodic orbit in phase and lag synchronization between coupled chaotic oscillators.*, Chaos. 2003, 13:309-318.
[8] S.Pal, B.Sahoo and S.Poria, *Generalized lag synchronization of delay coupled chaotic systems via linear transformations.*, Physica Scripta. 2013, 87:045011.
[9] Ronnie Mainieri and Jan Rehacek, *Projective synchronization in three dimensional chaotic systems*, Phys.Rev.Lett. 1999, 82:3042-3045.
[10] A.Tarai and S.Poria, *Projective synchronization of bidirectionally coupled chaotic systems via linear transformations*, Inter.J.of Appl. Math. Research. 2012, 1:531-540.
[11] M.A.Khan and S.Poria, *Projective synchronization of chaotic systems via backstepping design*, Inter.J.of Appl. Math. Research. 2012, 1:541-548.
[12] S.Poria, M.A.Khan and M.Nag, *Spatiotemporal synchronization of coupled Ricker maps over a complex network.*, Physica Scripta. 2013, 88:015004.
[13] N.F.Rulkov,M.M.Sushchik,L.S.Tsimring and H.D.I Abarbanel, *Generalized synchronization of chaos in directionally coupled chaotic systems.*, Phys. Rev.E. 1995, 51:980-994.
[14] L.Kocarev and U.Parlitz, *Generalized synchronization, predictability, an equivalence of unidirectionally coupled dynamical systems.*, Phys.Rev.Lett. 1996, 76:1816-1819.
[15] T.Yang and L.O.Chua, *Generalized synchronization of chaos via linear transformations*,International Journal of Bifurcation and Chaos. 1999, 9:215-219.
[16] A. Tarai, S.Poria and P.Chatterjee *Synchronization of generalised linearly bidirectionally coupled unified chaotic system*, Chaos, Soliton & Fractals. 2009, 40:885-892.
[17] Chil-Min Kim, S. Rim, WH. Kye and W. Key, *Anti synchronization of chaotic oscillators*, Physics Letters A. 2003, 320:39-46.
[18] Y.Zhang and J.Sun, *Chaotic synchronization and anti-synchronization based on suitable separation*, Physics Letters A. 2004, 330:442-447.
[19] J.Hu, S.Chen and L.Chen, *Adaptive control for anti-synchronization of Chua's chaotic system*, Physics Letters A. 2005, 339:455-460.
[20] LG Hui,*Synchronization and anti-synchronization of Colpitts oscillators using active control*, Chaos, Soliton & Fractals. 2005, 26:87-93.
[21] GH Li and SP Zhou, *An observed based anti-synchronization*, Chaos, Solitons and Fractals. 2006, 29:495-498.
[22] W.Li,X.chen and S.Zhiping, *Anti-synchronization of two different chaotic systems*, Physica A. 2008, 387:3747-3750
[23] M.Mossa Al-Sawalha and M.S.M.Noorani, *Anti-synchronization of two hyperchaotic systems via nonlinear control*, Commun Nonlinear Sci Numer Simulat. 2009, 14:3402-3411.