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Projective Synchronization Via Active Control of Identical Chaotic Oscillators with Parametric and External Excitation

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Abstract: Using the active control technique with Lyapunov stability theory and Routh-Hurwirtz criteria, the control functions are designed to achieve projective synchronization between two identical ϕ^6 Van der Pol Oscillator (ϕ^6 VDPOs) and two identical ϕ^6 Duffing Oscillator (ϕ^6 DOs), comprising ϕ^6 VDPO and ϕ^6 DO for the triple well configuration of the ϕ^6 potential. The coefficient matrix of the error dynamics between each pair of projective synchronized systems is chosen such that the number of active control functions reduces from two to one, thereby, significantly reducing controller complexity in the design. The designed controllers enable the state variables of the response system to achieve projective synchronization with those of the drive system. Also, the coupling parameter that leads to the fastest synchronization time was determined. The results are validated using numerical simulations.

1 Introduction

The Duffing and Van der Pol oscillators are simple models of dynamical systems with complex structural behavior that can be parametrically and externally excited.

The Duffing oscillator in the normal form is given by Eq. (1)

$$\ddot{x} + \alpha\dot{x} + w_0^2 x + \beta x^3 = F(t) \tag{1}$$

where α is the damping constant, w_0 is the natural frequency and β is the stiffness constant which plays the role of nonlinear parameter and $F(t)$ is an external time dependent periodic driving force.

Many research have been conducted on this system. The Duffing oscillator has been the study of many investigations theoretically and experimentally. Nijmeijer and Berghuis [1] proposed and developed a Lyapunov-based controller to control the chaotic Duffing equation. Ravichandran et al [2] considered the effects of various periodic forcing F(t) including sine wave, square wave, rectified sine wave, symmetric saw-tooth wave, asymmetric saw-tooth wave, and rectangular wave on bifurcation on the Duffing oscillator. The system was applied to weak signal detection by Jalilvand and Fotoohabadi [3] based on frequency spectrum analysis and filtering. The routes to chaos have been reported in three, five and six coupled Duffing oscillators by Musielak et al [4]. The response of a Duffing oscillator excited harmonically was studied experimentally by Patil and Mallik [5]. Two identical and non-identical Duffing oscillators were synchronized using Active control method by Njah and Vincent [6].

The van der Pol oscillator is given by Eq. 2. This model was proposed by Balthasar van der Pol (1889-1959) in 1920 when he was an engineer working for Philips Company (in the Netherlands). Van der Pol oscillator has enjoyed similar popularity among researchers as the Duffing oscillator.

$$\ddot{x} + \mu(1 - x^2)\dot{x} + x = F(t) \tag{2}$$

The synchronization of extended Bonhoffer-van der Pol oscillator was done by Zribi and Alshamali [7] using both Lyapunov based controller and sliding mode controller while an active control scheme was used by Njah [8] for the extended

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Bonhoffer-van der Pol oscillators. A van der Pol oscillator was synchronized with a Chen chaotic circuit using a nonlinear controller by Elabbasy and El-Dessoky [9]. The dynamics of two coupled van der Pol oscillator was investigated by Pastor-Diaz and Lopez-Fraguas [10] using return maps, Poincare section and probability distribution functions. A new form for coupling in the van der Pol oscillator was introduced by Camacho et al [11] using a bath. Comparison of projective synchronization of the system using backstepping and a modified active control was done by Ojo et al [12]. Applications of van der Pol oscillators is well documented in literature [13–16].

The Van der Pol oscillator model describes periodically self-excited oscillators in Physics, Engineering, Electronics, Biology, Neurology and many other disciplines [17] while the Duffing oscillator model describes various physical, electrical, mechanical and engineering devices [18] for different potentials $V(x)$. Synchronization work on Duffing-Van der Pol oscillator systems include sliding mode controller by Kakmeni et al [19].

Chaos has become an established field of study with applications in many disciplines including medicine, engineering, space studies etc. One of the most important interesting aspect of chaos is the ability to map the waveform of a chaotic system to that of another, through a process called synchronization. It was first proposed by Pecora and Carrol [20] in 1990.

Many methods have been proposed to achieve chaos synchronization, such as the passive control method [21], backstepping design method [22], impulsive control method [23], adaptive control, active control [8], sliding mode control. Bai and Lonngren [24] proposed the method of identical chaos synchronization using active control. The technique was latter generalized to non-identical systems by Ho and Hung [25].

The ubiquitous application of active control techniques has encouraged researchers to introduced active control based on different stability criteria. For instance, Lei et al [26] introduces active control based on Lyapunov stability theory and Routh Hurwitz criteria which has the advantage of possible implementation and has been used to synchronize a few chaotic systems [8]. However, active control technique always gives rise to as many as is the dimension of the systems been synchronized, thereby, making controllers unsuitable for practical implementation.

Lyapunov stability theory and Routh–Hurwirth criteria incorporated in the original active control technique enables us to successful reduced the number of controllers to only one. Hence, this improved the original active control technique. Only few papers on synchronization of chaotic systems have been reported using this active control with Rourth–Hurwitz criteria. Idowu et al [27] considered synchronization of chaos in non-identical parametrically excited systems via active control technique with Lyapunov stability theory and Routh–Hurwitz criteria using two controllers while, Njah [8] considered the synchronization considered synchronization via active control with lyapunov stability and Routh–Hurwitz of identical and non-identical ϕ^6 chaotic oscillators with external excitations using only one controller. To the best of our knowledge all the papers so far reported on active control with Lyapunov stability theory and Routh–Hurwitz criteria were based on complete synchronization of chaotic systems. To this end we are presenting projective synchronization via active control with Lyapunov stability theory and Routh–Hurwitz criteria of identical ϕ^6 chaotic oscillators with both parametric and external excitations which is of great importance in secure communications.

2 Models Description

When parametric and external excitation are added to the general form of Eq. (1) and (2), we have

$$\ddot{x} - \mu(1-x^2)\dot{x} + \alpha\eta\cos(2\omega t)x + \frac{dV(x)}{dx} = f\cos(\omega t) \tag{3}$$

$$\ddot{x} + \delta\dot{x} - \alpha\eta\cos(2\omega_1 t)x_1 + \frac{dV(x)}{dx} = f_1\cos(\omega_1 t) \tag{4}$$

Substituting the value of $V(x) = \frac{1}{2}\alpha x^2 + \frac{1}{4}\lambda x^4 + \frac{1}{6}\beta x^6$ in (4) and (4) yields ϕ^6 Duffing van der Pol oscillator and ϕ^6 Duffing oscillator respectively. The potential $V(x)$ is shown in figure (1).

3 Projective Synchronization

3.1 Projective Synchronization of Identical ϕ^6 Van der Pol Oscillators

The Van der Pol oscillator can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \mu(1-x_1^2)x_2 - \alpha\eta\cos(2\omega t)x_1 - \alpha x_1 - \lambda x_1^3 - \beta x_1^5 + f\cos\omega t \end{aligned} \tag{5}$$

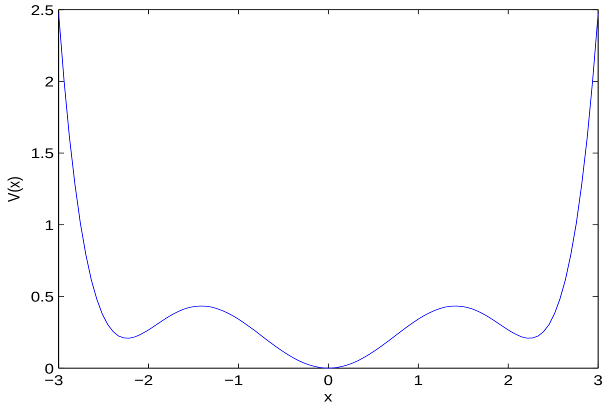

Figure 1: The ϕ^6 potential for parameter values $\alpha = 1.0,\ \lambda = -0.7,\ \beta = 0.1$

where $x = x_1$ and $\dot{x} = x_2$. Let system (5) be the drive system and the system and the system (6) be the response system:

$$\begin{aligned}\dot{y}_1 &= y_2 + u_1(t)\\ \dot{y}_2 &= \mu(1-y_1^2)y_2 - \alpha\eta\cos(2\omega t)y_1 - \alpha y_1 - \lambda y_1^3 - \beta y_1^5 + f\cos\omega t + u_2(t)\end{aligned} \tag{6}$$

where $u_1(t)$ and $u_2(t)$ are control functions to be determined. Subtracting (5) from (6) and using the notation $e_1 = y_1 - px_1$ and $e_2 = y_2 - px_2$ where p is the scaling factor, then we have

$$\begin{aligned}\dot{e}_1 &= e_2 + u_1(t)\\ \dot{e}_2 &= \mu e_2 - \mu(y_1^2y_2 - px_1^2x_2) - e_1\alpha(\eta\cos 2\omega t + 1) - \lambda(y_1^3 - px_1^3) - \beta(y_1^5 - px_1^5) + (1-p)f\cos(\omega t + u_2(t)\end{aligned} \tag{7}$$

we now redefine the control functions such as to eliminate terms in (7) which cannot be expressed as linear terms in $e_1(t)$ and $e_2(t)$ as follows:

$$\begin{aligned}u_1(t) &= v_1(t)\\ u_2(t) &= \mu(y_1^2y_2 - px_1^2x_2) + \lambda(y_1^3 - px_1^3) + \beta(y_1^5 - px_1^5) - (1-p)f\cos\omega t + v_2(t)\end{aligned} \tag{8}$$

substituting (8) into (7) we have

$$\begin{aligned}\dot{e}_1 &= e_2 + v_1(t)\\ \dot{e}_2 &= \mu e_2 - \alpha e_1(\eta\cos 2\omega t + 1) + v_2(t)\end{aligned} \tag{9}$$

Eq. (9) is the error dynamics, which can be interpreted as a control problem where the system to be controlled is a linear system with control input $v_1(t) = v_1(e_1(t), e_2(t))$ and $v_2(t) = v_2(e_1(t), e_2(t))$. as long as these feedbacks stabilize the system, $\mid e_i(t)_{i=1,2} \mid \to 0$ as $t \to \infty$. This implies that the two systems (5) and (6) evolving from different initial conditions are synchronized. As functions of $e_1(t)$ and $e_2(t)$ we choose $v_1(t)$ and $v_2(t)$ as follows:

$$\begin{pmatrix} v_1(t)\\ v_2(t)\end{pmatrix} = D\begin{pmatrix} e_1(t)\\ e_2(t)\end{pmatrix} \tag{10}$$

where $D = \begin{pmatrix} a & b\\ c & d\end{pmatrix}$ is a 2×2 constant feedback matrix to be determined. Hence, the error system (9) can be written as

$$\begin{pmatrix} \dot{e}_1(t)\\ \dot{e}_2(t)\end{pmatrix} = C\begin{pmatrix} e_1(t)\\ e_2(t)\end{pmatrix} \tag{11}$$

where $C = \begin{pmatrix} a & 1+b \\ c-\alpha(1+\eta\cos 2\omega t) & \mu+d \end{pmatrix}$ is the coefficient matrix. According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$\begin{aligned} a+\mu+d<0 \\ (1+b)(c-\alpha(1+\eta\cos 2\omega t)-a(\mu+d)<0 \end{aligned} \tag{12}$$

then the eigenvalues of the coefficient matrix error system (9) must be real negative or complex with negative real parts and, hence stable synchronized dynamics between system (5) and (6) is guaranteed. Let

$$\begin{aligned} a+\mu+d=-E \\ (1+b)(c-\alpha(1+\eta\cos 2\omega t))-a(\mu+d)=-E \end{aligned} \tag{13}$$

where $E>0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements a,b,c,d of matrix D in order to satisfy Eq. (12). We optimize the way this choice is made so that not only the synchronization time is short but also the controller complexity is reduced. This can be achieved by letting $a=b=0$ in (13) to obtain

$$D = \begin{pmatrix} 0 & 0 \\ (\alpha(1+\eta\cos 2\omega t)-E) & (-\mu-E) \end{pmatrix} \tag{14}$$

which satisfies (12) and gives $u_1(t)=0$, thereby, leading to a single active control function $u_2(t)$ as follows:

$$u_2(t)=\mu(y_1^2y_2-px_1^2x_2)+\lambda(y_1^2-px_1^2)+\beta(y_1^5-px_1^5)+e_1(\alpha(1+\eta\cos 2\omega t)-E)-e_2(E+\mu)-(1-p)f\cos\omega t \tag{15}$$

3.2 Numerical Simulation for ϕ^6 Van der Pol Oscillator

Numerical Simulations were carried out using MATLAB. The Fourth order Runge-Kunta algorithm with initial conditions $(x_1,\ x_2)=(0.1,0.2)$, $(y_1,\ y_2)=(2.2,0.05$ with time step of 0.001 and parameter values given as $\mu=0.4,\ \alpha=1.0,\ \lambda=-0.7,\ \beta=0.1,\ f=9,\ \omega=3.14$. The error dynamics when the controls are deactivated and activated are shown in figures 2 and 3 respectively. The convergence of the error dynamics and magnitude of the errors to zero in figures 3 and 4 indicate that projective synchronization has been achieved. Figure 5 shows the dependence of the synchronization time on the coupling parameter.

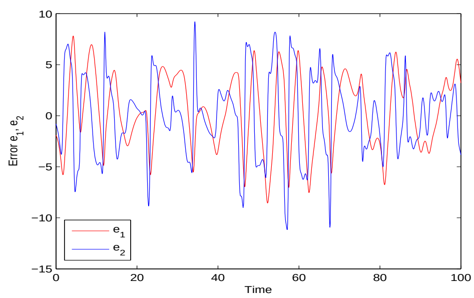

Figure 2: Error dynamics of ϕ^6 (triple well) VDPO when the controls are deactivated for $0\le t\le 100$ where $e_1=x_1-y_1, e_2=x_2-y_2$

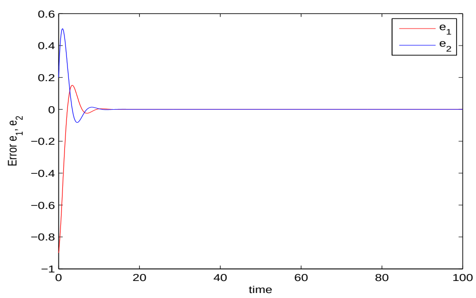

Figure 3: Error dynamics of ϕ^6 (triple well) Van der Pol oscillator when the controls are activated for $t\ge 0$ where $e_1=x_1-y_1, e_2=x_2-y_2$

4 Projective Synchronization of ϕ^6 Duffing Oscillator

The Duffing oscillator can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\delta x_2+\alpha_1\eta\cos(2\omega_1 t)x_1-\alpha_1 x_1+\lambda_1 x_1^3-\beta_1 x_1^5+f_1\cos\omega_1 t \end{aligned} \tag{16}$$

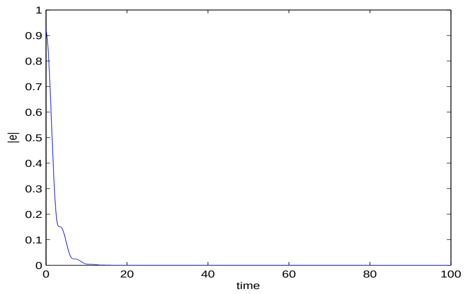

Figure 4: Error dynamics of ϕ^6 (triple well) Van der Pol oscillator when the controls are activated for $t \geq 0$ where $\mid e \mid = \sqrt{e_1^2 + e_2^2}$

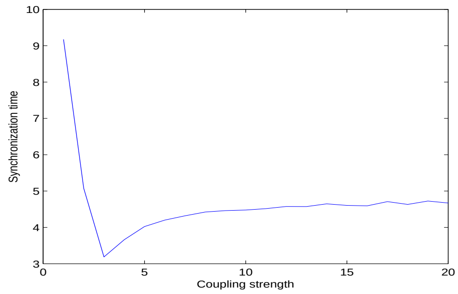

Figure 5: Dependence of the synchronization time on the coupling parameter

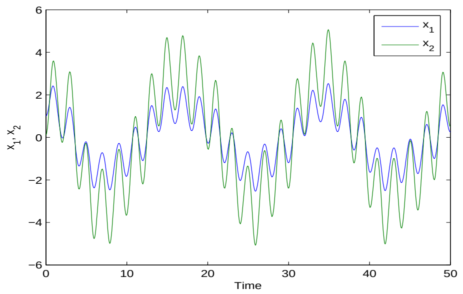

Figure 6: Time evolution of variables x_1 and y_1 when controls are activated for $t \geq 0$

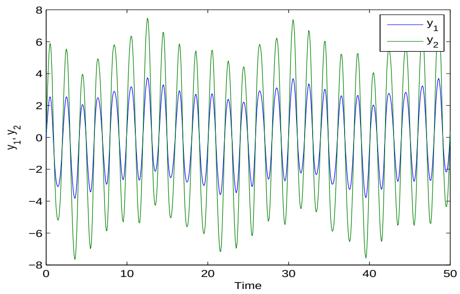

Figure 7: Time evolution for variables x_2 and y_2 when controls are activated for $t \geq 0$

where $x = x_1$ and $\dot{x} = x_2$. Let system (16) be the drive system and system (17) be the response system.

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1(t) \\ \dot{y}_2 &= -\delta y_2 + \alpha_1 \eta \cos(2\omega_1 t) y_1 - \alpha_1 y_1 + \lambda_1 y_1^3 - \beta_1 y_1^5 + f_1 \cos \omega_1 t + u_2(t) \end{aligned} \tag{17}$$

where $u_1(t)$ and $u_2(t)$ are control functions to be determined. Subtracting (16) from (17) and using the notations $e_1 = y_1 - px_1$ and $e_2 = y_2 - px_2$ we have the error dynamics

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1(t) \\ \dot{e}_2 &= -\delta e_2 - \alpha_1 e_1 (1 - \eta \cos 2\omega_1 t) + \lambda_1 (y_1^3 - px_1^3) - \beta_1 (y_1^5 - px_1^5) + (1-p) f_1 \cos \omega_1 t + u_2(t) \end{aligned} \tag{18}$$

We now re-define the control functions such as to eliminate terms in (18) which cannot be expressed as linear terms in $e_1(t)$ and $e_2(t)$ as follows:

$$\begin{aligned} u_1(t) &= v_1(t) \\ u_2(t) &= -\lambda_1 (y_1^3 - px_1^3) + \beta_1 (y_1^5 - px_1^5) - (1-p) f_1 \cos \omega_1 t + v_2(t) \end{aligned} \tag{19}$$

substituting (19) into (18) we have

$$\begin{aligned} \dot{e}_1 &= e_2 + v_1(t) \\ \dot{e}_2 &= -\delta e_2 - \alpha_1 e_1 (1 - \eta \cos 2\omega_1 t) + v_2(t) \end{aligned} \tag{20}$$

Eq. (20) corresponds to the expression for the ϕ^6 Van der Pol oscillator. Following the same procedure from (9) to (11), we obtain the coefficient matrix C as

$$C = \begin{pmatrix} a & 1+b \\ c - \alpha_1 (1 - \eta \cos 2\omega_1 t) & d - \delta \end{pmatrix} \tag{21}$$

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$\begin{aligned} a+d-\delta &< 0 \\ (c-\alpha_1(1-\eta\cos 2\omega_1 t))(1+b)-a(d-\delta) &< 0 \end{aligned} \tag{22}$$

Then the eigenvalues of the coefficient matrix of matrix of error system (20) must be real negative or complex with negative real parts and, hence, stable synchronized dynamics between systems (16) and (17) is guaranteed. Let

$$\begin{aligned} a+d-\delta &= -E \\ (c-\alpha_1(1-\eta\cos 2\omega_1 t))(1+b)-a(d-\delta) &= -E \end{aligned} \tag{23}$$

where $E > 0$ is a real number as earlier explained. From (23) we again choose $a = b = 0$ to obtain D as

$$D = \begin{pmatrix} 0 & 0 \\ (\alpha_1(1-\eta\cos 2\omega_1 t)-E) & (\delta-E) \end{pmatrix} \tag{24}$$

which satisfies (22) and gives $u_1(t) = 0$, thereby,leading to a single active control function $u_2(t)$ as follows:

$$u_2(t) = e_1(-E+\alpha_1(1-\eta\cos 2\omega_1 t)) - e_2(E-\delta) - \lambda_1(y_1^3 - px_1^3) + \beta_1(y_1^5 - px_1^5) - (1-p)f_1\cos\omega_1 t \tag{25}$$

4.1 Numerical Simulation for ϕ^6 Duffing Oscillator

Simulations were carried for ϕ^6 Duffing oscillator using Runge-Kutta algorithm with a time step of 0.001 and initial conditions $(x_1,\ x_2) = (0,\ 1.5)$, $(y_1,\ y_2) = (0.5,\ 1.0)$ to ensure chaotic dynamics of the state variables. The error dynamics of the system without activating the control functions is given in figure 8. When the control functions are activated (figure 9), the error dynamics converges to zero as shown which indicates that projective synchronization has been achieved. To justify the evidence of the projective synchronization each pair of the corresponding state variables of the master and slave system are confirm after the control functions have been activated as shown in Figure 10 and Figure 11. The result obtained show that the size of the state variable of the slave is twice that of the master system which again confirm the realization of projective synchronization.

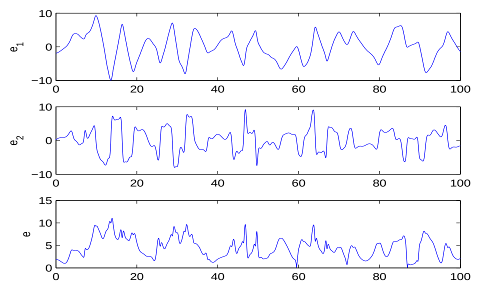

Figure 8: Error dynamics of ϕ^6 (triple well) Duffing oscillator when the controls are deactivated for $0 \le t \le 100$ where $e_1 = x_1 - y_1$, $e_2 = x_2 - y_2$

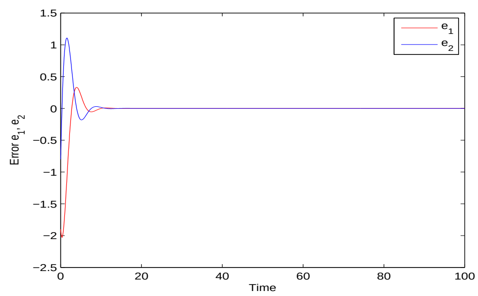

Figure 9: Error dynamics of ϕ^6 (triple well) VDPO when the controls are activated for $t \ge 0$ where $e_1 = x_1 - y_1$, $e_2 = x_2 - y_2$

5 Conclusion

The active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria has been used to design control functions for the projective synchronization of 2-D chaotic system in such a way that the number of control functions decreased from two to one. As examples of 2-D systems,ϕ^6 Van der Pol with triple-well potential and ϕ^6 Duffing oscillators with triple well potential was introduced, which exhibit highly complex dynamics and whose projective synchronization behaviour has not been well investigated. Single control functions were designed for the purpose of

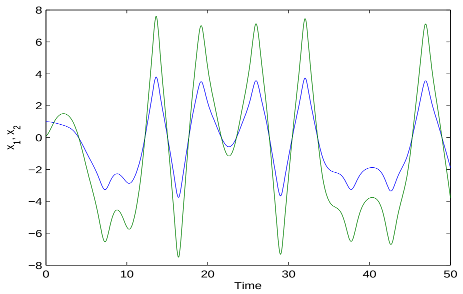

Figure 10: Time evolution for variables x_1 and y_1 when controls are activated for $t \geq 0$

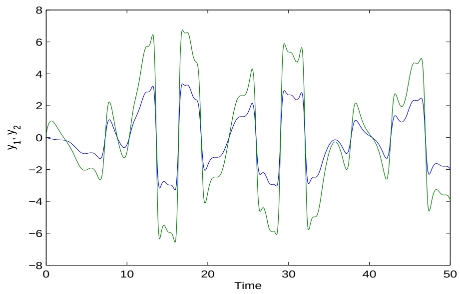

Figure 11: Time evolution for variables x_2 and y_2 when controls are activated for $t \geq 0$

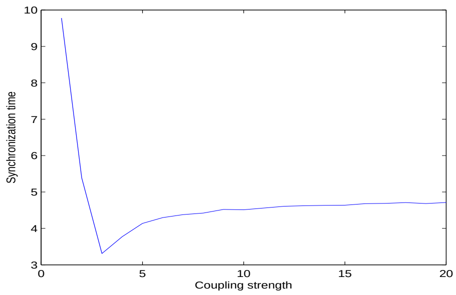

Figure 12: Dependence of the synchronization time on the coupling parameter

achieving projective synchronization between two identical ϕ^6 (Triple well) Van der Pol and ϕ^6 (Triple well) Duffing oscillators. A parameter E that can be tuned to optimize the synchronization time was also introduced in the design which enable us to determine the value of the coupling parameter that yields fastest synchronization time. The single controller design significantly reduced the complexity and hence the cost of the controller, thereby, making it suitable for practical implementation. The synchronization of complex chaotic systems using simple controllers has potential application in secure communications.

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