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Economic Growth Under the Constraints of Environment, Capital and Carbon-Based Energy

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Abstract: This paper constructs a model of economic growth under the triple constraints of environment, capital and carbon-based energy, and introduces factors such as energy intensity, labor force and technological level into production function. Dynamic optimization theory, static analysis and the analysis technique of optimal equilibrium are used to discuss the existence of dynamic equilibrium stability of economic system and analyze the influence of related parameters on economic growth. The results show that increasing the carbon-based energy tax rate will inhibit consumers' appetite for future consumption and the enhancement of economic growth rate on the balanced growth path. The increase of environmental self-purification ability is not conducive to increase the economic growth rate, but it can inhibit the increase of the growth rate of pollution intensity. Under certain conditions, although the rise of the pollution degree index can enhance the economic growth rate, it cannot restrain the increase of the growth rate of pollution intensity. The enhancement of the growth rate of technology and labor force can both inhibit the increase of the growth of the carbon-based energy intensity and can enhance the economic growth rate, and under certain conditions, the enhancement of them can both inhibit the rise of the growth rate of pollution intensity and is conducive to improve the growth rate of environmental quality. Under certain conditions, improving the efficiency of environmental protection investment can be beneficial to decrease the growth rate of the pollution intensity.

Keywords: Environment; Pollution intensity; Carbon tax; Carbon-based energy intensity; Economic growth rate

1 Introduction

At present, with the continuous improvement of people's living standard and the rapid development of economy, science and technology in our country, environmental pollution and energy consumption have increasingly become one of the core and focus of human concern. Although energy is an indispensable resource for human survival and development, it also brings about some problems such as environment and pollution emission. In particular, carbon-based energy sources, which emit large quantities of greenhouse gases such as sulfur dioxide and carbon dioxide emitted during combustion, and it is the culprit responsible for the greenhouse effect, acid rain, haze and atmospheric brown clouds. Environmental pollution and emission not only affect the sustainable economic development of our country, but also do harm to the living environment, physical and mental health of human beings. Therefore, the mode of China's economic growth in the future should be shifted from the "three high model" of high pollution, high consumption and high emission to the "three low" model of low pollution, low consumption and low emission. Energy should be developing toward "low carbon, green, high efficiency". Improving energy exploration, development and production technology would reduce carbon emissions and improve energy efficiency, which can alleviate the high energy consumption, environmental pollution and discharge pressure to a certain extent, to ensure long-term stability and development of China's economic system.

As for economic growth, energy consumption, environmental and pollution emissions, carbon tax and other issues, scholars at home and abroad have been related to many studies. In Ref. [1], Yu et al. based on the growth model within the R&D, constructed a sustainable growth model with considering environmental threshold limit, cost of environmental

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governance and energy resource depletion, and used the method of optimal control to discuss the balanced growth solution of having constructed a model. Yang et al. [2] used the method of dynamic system to study the competition evolution under Chinese energy alternative path and the impact of energy policy on economic growth, considering the relationship between energy and economic growth, then building a dynamic evolution model. The method of artificial neural network was used to identify the related parameters in the system, and the effect of energy policy on economic growth was proved and analyzed. Saidi and Mbarek [3] studied the causal relationship between renewable energy, nuclear energy consumption, carbon dioxide emissions and per capita real GDP by using panel data from nine developed countries during the period 1990 to 2013, while capital and labor are included as additional variables. Zhou et al. [4]used a dynamic energy-environment-economy general equilibrium model to explore the impact of carbon tax policy on China's economic growth and carbon dioxide emission reduction effect. Yang [5] based on the 28 provincial panel data from 1995 to 2006, the economic growth of China was decomposed on the basis of non-parametric environmental production frontier model to judge the way and motive force of economic growth, and discussed the conduction mechanism of pollution emission and energy consumption constraints on economic growth. Azlina et al. [6] studied the causal relationship between pollutant discharge, energy consumption and economic growth in Malaysia from 1970 to 2010, and used the cointegration and vector error correction model to show that there was a one-way causal relationship from economic growth to energy consumption, from pollutant discharge to economic growth, from the pollutant discharge to energy consumption. Yang et al. [7] established the dynamic change equation of energy utilization, and introduced the energy factor into the economic growth model, then discussed the economic growth model of Lucas and neoclassical under the condition of energy equation constraint, as well as analyzed the sustainable economic growth. Alam et al. [8] studied the effects of population growth, energy consumption and income on carbon dioxide emissions by using annual time series data from China, India, Brazil and Indonesia in the period of 1970 to 2012 and auto-regressive distribution lagging boundary test methods. Zhang [9] established a new classical economic growth model of containing energy constraints, explained the obstructive effect of energy consumption on the sustained and balanced economic growth, and analyzed the problems in the process of energy development and utilization in Inner Mongolia. Tang et al. [10] established the neoclassical Solow growth model, and analyzed the relationship between energy consumption and economic growth in Vietnam(1971-2011) by using co-integration test and Granger causality test. Vita [11]first established the model of non-renewable energy replaced by renewable energy on the assumption of whether the technical replacement rate is 1, and then compared the advantages and disadvantages of different policies in the correction of market failure.

In conclusion, the domestic and foreign scholars have studied the environment and energy intensity from different angles. But the energy intensity, the labor force and the technical level, and the environmental factors are simultaneously introduced into the production function, and combined with carbon taxes, energy and environmental factors such as the relatively few studies. Based on the existing research, this paper has constructed an endogenous growth model under the triple constraints of carbon-based energy, capital, and the environment, and introduces simultaneously the factors of energy intensity, labor force, the level of production technology and pollution intensity into the production function. And then we discuss and analyze their impacts on China's sustainable economic growth and development.

2 Model

2.1 Physical capital

According to the model idea of Gupta et al. [12], it is assumed that the output of physical capital is used as follows: (a) household consumption C; (b) environmental investment T (used for the improvement of environment); (c) carbon-based energy tax, where $\tau(0<\tau<1)$ denotes the energy tax rate levied by the government on carbon-based energy production of per unit of enterprise energy producer, the change equation of capital accumulation at the moment of t is

$$\dot{K}_t=(1-\tau)Y_t-C_t-T_t. \tag{1}$$

Where Y_t represents the economic output in the economy at the moment of t. It is further assumed that there is a relationship between capital stock K_t and environmental investment T_t as follows

$$T_t=\omega K_t. \tag{2}$$

Where K_t represents the capital stock at the moment of t ; $\omega(0<\omega<1)$ is the proportion of environmental investment in the capital stock. Substituting equation (2) into equation (1) can be obtained

$$\dot{K}_t=(1-\tau)\,Y_t-C_t-\omega K_t. \tag{3}$$

2.2 Energy

Assuming that there is a certain amount of carbon-based energy in the natural world (ie, carbon-containing energy represented by coal, natural gas and oil), and it will continue to decrease with the large amount of human exploitation and use. It is further assumed that the initial stock of carbon-based energy is S_0, then the dynamic equation of carbon-based energy stock at the moment of t is as follows

$$S_t = S_0 - \int_0^t R_\mu d\mu. \tag{4}$$

Formula (4) can be derived from both sides for t at the same time, as the above can be obtained

$$\dot{S}_t = -R_t. \tag{5}$$

Where $\dot{S}_t$ is the change rate of carbon-based energy stock at the moment of t and R_t is the consumption of carbon-based energy at the moment of t . For the convenience of the writing process, time subscripts are omitted below.

Draw lessons from the model idea of Xu [13], further assumption is as follows

$$R = \pi Y. \tag{6}$$

Where π represents the intensity of the carbon-based energy, the formula (6) is substituted into equation (5) can be obtained

$$\dot{S} = -\pi Y. \tag{7}$$

2.3 Environmental quality

Assuming that the environmental quality is influenced by both the environmental self-purification capacity $\eta(\eta > 0)$ and the pollutant discharge $D\left(Y, z\right)$ [14, 15], the dynamic equation of environmental quality in the economy at the moment of can be expressed as

$$\dot{E} = D\left(Y, z\right) - \eta E. \tag{8}$$

It is further assumed that the pollution emissions $D\left(Y, z\right)$ are affected by economic output, pollution intensity and environmental investment T [16], so $D\left(Y, z\right)$ can be expressed as

$$D\left(Y, z\right) = -\left(T\right)^{-h} z^{\upsilon} Y. \tag{9}$$

Where $z\left(0 \leqslant z \leq 1\right)$ and $\upsilon\left(\upsilon > 0\right)$ respectively indicate the pollution intensity of the moment of t and pollution degree index in the economy, and the government will properly control the pollution degree index to a certain extent, which will be helpful to reduce the amount of pollution emissions; $h\left(0 < h < 1\right)$ Represents the efficiency of environmental investment (ie, the regular environmental protection investment may contribute to the enhancement of environmental quality and the improvement of environment). Substituting equation (9) into equation (8), we have

$$\dot{E} = -\left[\omega K\right]^{-h} z^{\upsilon} Y - \eta E. \tag{10}$$

Where $\left[\omega K\right]^{-h}$ represents the positive effect of environmental investment on environment (that is, contribution to the environment).

2.4 Utility function

It is assumed that there is a rational social representative consumer, its program goal is to maximize the utility of their own. Assuming further that the maximum utility obtained is determined by the material consumption C and θ ($\theta > 0$, which is the marginal utility elasticity), the utility function obtained by the social representative consumer is expressed as follows

$$u\left(C\right) = \frac{C^{1-\theta} - 1}{1 - \theta}. \tag{11}$$

Ie,$maxu\left(C\right) = \int_0^\infty \frac{C^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$, where ρ represents the utility discount rate of the consumer.

2.5 Production function

This section draws on the model thoughts and the neoclassical economic growth theory (economic growth is affected by labor and capital) to establish the CD-type production function as follows

$$Y = K^{\alpha} R^{\beta} [AL]^{\gamma} z. \tag{12}$$

Where A and L respectively represent the technical level and labor force in the process of carbon-based energy extraction and production at the moment of t; α, β and γ respectively represent the output elasticity of physical capital, carbon-based energy consumption, technology and labor at the moment of t $(\alpha, \beta, \gamma \in (0,1))$, and $\alpha + \beta + \gamma = 1$.

By substituting equation (6) into equation (12), the production function can be further expressed as follows

$$Y = K^{\frac{\alpha}{1-\beta}} \pi^{\frac{\beta}{1-\beta}} [AL]^{\frac{\gamma}{1-\beta}} z^{\frac{1}{1-\beta}}. \tag{13}$$

Based on the model idea of Solow [17] and the basic assumptions of his model, the change rate of the technical level and the labor force is easily obtained as follows

$$\dot{A} = gA, \dot{L} = nL. \tag{14}$$

Where g, n respectively represent the growth rate of technology and labor force at the moment of t, and $g, n \in (0,1)$.

3 Model solving

According to the above hypothesis, the dynamic optimization problem established is as follows

$$maxu(C) = \int_0^{\infty} \frac{C^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt.$$

$$Y = K^{\frac{\alpha}{1-\beta}} \pi^{\frac{\beta}{1-\beta}} [AL]^{\frac{\gamma}{1-\beta}} z^{\frac{1}{1-\beta}}.$$

$$\dot{K} = (1-\tau) Y - C - \omega K.$$

$$\dot{S} = -\pi Y.$$

$$\dot{E} = -[\omega K]^{-h} z^{\upsilon} Y - \eta E.$$

3.1 Optimal conditions

For the above model, the following Hamilton functions are established

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \lambda_1 [(1-\tau) Y - C - \omega K] - \lambda_2 \pi Y - \lambda_3 \left[(\omega K)^{-h} z^{\upsilon} Y + \eta E\right]. \tag{15}$$

In the formula (15), λ_1, λ_2 , λ_3 respectively represent the shadow price of the state variable K, S, E. C, π, z for the control variables. From the equations (13), (15) and the optimization theory, the first order condition and the Euler equation can be respectively obtained as follows

$$\lambda_1 = C^{-\theta}, \lambda_2 \pi = \left[\lambda_1 (1-\tau) - \lambda_3 (\omega K)^{-h} z^{\upsilon}\right] \beta, \lambda_2 \pi = \lambda_1 (1-\tau) - \lambda_3 (\omega K)^{-h} z^{\upsilon} [(1-\beta) \upsilon + 1]. \tag{16}$$

$$\dot{\lambda}_1 = \lambda_1 (\rho + \omega) - \left[\lambda_1 (1-\tau) - \lambda_2 \pi - \lambda_3 (\omega K)^{-h} z^{\upsilon} \frac{\alpha - h(1-\beta)}{\alpha}\right] \frac{\alpha}{1-\beta} \frac{Y}{K}, \dot{\lambda}_2 = \rho \lambda_2, \dot{\lambda}_3 = (\rho + \eta) \lambda_3. \tag{17}$$

By Eq.(16) can be further obtained

$$\lambda_1 (1-\tau) = \lambda_3 (\omega K)^{-h} z^{\upsilon} (1+\upsilon). \tag{18}$$

From the formulas (16), (17) and (18), it can be easily seen that

$$g_{\lambda_1} = \rho + \omega - \frac{(1-\tau)(\alpha \upsilon + h)}{1+\upsilon} \frac{Y}{K}. \tag{19}$$

3.2 The impact of carbon tax on economic growth rate

On the path of balanced growth, it is easily known from the relationship between output, investment and consumption as follows

$$g_Y = g_C = g_K. \tag{20}$$

From the formulas (16), (19) and (20) can be seen that

$$g_Y = g_C = \frac{1}{\theta}\left[\frac{(1-\tau)(\alpha\upsilon+h)}{1+\upsilon}\frac{Y}{K} - \rho - \omega\right]. \tag{21}$$

From the formula (21), it can be seen that both g_C and g_Y are the decreasing functions of the carbon-based energy tax rate τ and the proportion of environmental investment ω, so the growth rates of consumption and economy will decrease with the increase of τ and ω when other factors remain unchanged. However, the growth rate of economy and consumption will increase as the efficiency of environmental investment increases. Based on this, we can see that if the government and its local governments properly reduce their carbon-based energy tax levy and environmental investment and improve the efficiency of environmental investment more effectively, they will be able to promote economic growth to some extent.

3.3 stability of dynamic equilibrium

It can be seen as follows that the growth rate of variables is solved by defining the major variables. Defined as follows

$$b = \frac{Y}{K}, d = \frac{C}{K}, m = \frac{\pi Y}{S}, p = \frac{z}{E}. \tag{22}$$

From the formulas (3), (7), (10), (13), (16), (17) to (19) and (22) can be further obtained

$$g_{\lambda_1} = \rho + \omega - \frac{(1-\tau)(\alpha\upsilon+h)}{1+\upsilon}b. \tag{23}$$

$$g_{\lambda_2} = \rho. \tag{24}$$

$$g_{\lambda_3} = \rho + \eta. \tag{25}$$

$$g_K = (1-\tau)b - d - \omega. \tag{26}$$

$$g_S = -m. \tag{27}$$

$$g_E = g_Y + \upsilon g_z - h g_K. \tag{28}$$

$$g_C = \frac{1}{\theta}\left[\frac{(1-\tau)(\alpha\upsilon+h)}{1+\upsilon}b - \rho - \omega\right]. \tag{29}$$

$$g_{\lambda_1} = g_{\lambda_3} + \upsilon g_z - h g_K. \tag{30}$$

$$g_\pi = g_{\lambda_3} + \upsilon g_z - g_{\lambda_2} - h g_K. \tag{31}$$

$$g_Y = \frac{\alpha}{1-\beta}g_K + \frac{\beta}{1-\beta}g_\pi + \frac{\gamma}{1-\beta}[g_A + g_L] + \frac{1}{1-\beta}g_z. \tag{32}$$

From the Eqs.(23) to (26), (30) and (31) are available that

$$g_\pi = g_{\lambda_1} - g_{\lambda_2} = \omega - \frac{(1-\tau)(\alpha\upsilon+h)}{1+\upsilon}b. \tag{33}$$

$$g_z = \frac{(1-\tau)(h-\alpha)}{1+\upsilon}b - \frac{h}{\upsilon}d + \frac{(1-h)\omega-\eta}{\upsilon}. \tag{34}$$

By substituting the formulas (14), (26), (33) and (34) into equation (32) can be seen that

$$g_Y = \frac{(1-\tau)(\alpha\upsilon+h)}{1+\upsilon}b - \frac{(\alpha\upsilon+h)}{(1-\beta)\upsilon}d + \frac{(\beta-\alpha)\omega+\gamma(g+n)}{1-\beta} + \frac{(1-h)\omega-\eta}{(1-\beta)\upsilon}. \tag{35}$$

Combining expressions (10), (26)-(29) with (33)-(35) can be obtained

$$g_b=\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1\right](1-\tau)\,b+\frac{\gamma\upsilon-h}{(1-\beta)\,\upsilon}d+\frac{(1-\alpha)\,\omega+\gamma\,(g+n)}{1-\beta}+\frac{(1-h)\,\omega-\eta}{(1-\beta)\,\upsilon}. \tag{36}$$

$$g_d=\left[\frac{\alpha\upsilon+h}{(1+\upsilon)\,\theta}-1\right](1-\tau)\,b+d-\frac{\rho+\omega}{\theta}+\omega. \tag{37}$$

$$g_m=-\frac{\alpha\upsilon+h}{(1-\beta)\,\upsilon}d+m+\frac{(1-\alpha)\,\omega+\gamma\,(g+n)}{1-\beta}+\frac{(1-h)\,\omega-\eta}{(1-\beta)\,\upsilon}. \tag{38}$$

$$g_p=\frac{(1-\tau)\,(h-\alpha)}{1+\upsilon}b-\frac{h}{\upsilon}d+(\omega K)^{-h}\,z^{\upsilon-1}Yp+\frac{(1-h)\,\omega+(\upsilon-1)\,\eta}{\upsilon}. \tag{39}$$

The Eqs(36)-(39) is a four-dimensional system with respect to $M=(b,d,m,p)$, and there is a relation on the equilibrium growth path: $\dot b=\dot d=\dot m=\dot p=0$. In which the conversion values of the variables on the equilibrium growth path are denoted by superscript $*$. The Jacobian matrix (where $i,j=1,2,3,4$) at the $M^*=(b^*,d^*,m^*,p^*)$ on the right side of the formulas (36) -(39) is denoted by $Q=(q_{ij})$, and the equations (36)-(39) can be linearized at M^* as follows (ie, the dynamic equations of the economic system)

$$\dot M=Q\,(M-M^*)\,. \tag{40}$$

The formulas (36)-(39) at the Jacobian matrix Q of $M^*=(b^*,d^*,m^*,p^*)$ is

$$Q=\begin{bmatrix}\frac{\partial\dot b}{\partial b}&\frac{\partial\dot b}{\partial d}&\frac{\partial\dot b}{\partial m}&\frac{\partial\dot b}{\partial p}\\ \frac{\partial\dot d}{\partial b}&\frac{\partial\dot d}{\partial d}&\frac{\partial\dot d}{\partial m}&\frac{\partial\dot d}{\partial p}\\ \frac{\partial\dot m}{\partial b}&\frac{\partial\dot m}{\partial d}&\frac{\partial\dot m}{\partial m}&\frac{\partial\dot m}{\partial p}\\ \frac{\partial\dot p}{\partial b}&\frac{\partial\dot p}{\partial d}&\frac{\partial\dot p}{\partial m}&\frac{\partial\dot p}{\partial p}\end{bmatrix}=\begin{bmatrix}\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1\right](1-\tau)\,b^*&\frac{\gamma\upsilon-h}{(1-\beta)\upsilon}b^*&0&0\\ \left[\frac{\alpha\upsilon+h}{(1+\upsilon)\theta}-1\right](1-\tau)\,d^*&d^*&0&0\\ 0&-\frac{\alpha\upsilon+h}{(1-\beta)\upsilon}m^*&m^*&0\\ \frac{(1-\tau)(h-\alpha)}{1+\upsilon}p^*&-\frac{h}{\upsilon}p^*&0&(\omega K)^{-h}\,z^{\upsilon-1}Yp^*\end{bmatrix} \tag{41}$$

All the eigenvalues of the formula (41) can be obtained by letting $\left|\mu_iI-Q\right|=0$, where I is the unit matrix and $\mu_i\,(1,2,3,4)$ is the eigenvalue of the Jacobian matrix in equation (41), because

$$\begin{aligned}\left|\mu_iI-Q\right|&=\begin{vmatrix}\mu_i-\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1\right](1-\tau)\,b^*&-\frac{\gamma\upsilon-h}{(1-\beta)\upsilon}b^*&0&0\\ \left[1-\frac{\alpha\upsilon+h}{(1+\upsilon)\theta}\right](1-\tau)\,d^*&\mu_i-d^*&0&0\\ 0&\frac{\alpha\upsilon+h}{(1-\beta)\upsilon}m^*&\mu_i-m^*&0\\ -\frac{(1-\tau)(h-\alpha)}{1+\upsilon}p^*&\frac{h}{\upsilon}p^*&0&\mu_i-(\omega K)^{-h}\,z^{\upsilon-1}Yp^*\end{vmatrix}\\ &=\left(\mu_i-(\omega K)^{-h}\,z^{\upsilon-1}Yp^*\right)(\mu_i-m^*)\begin{vmatrix}\mu_i-\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1\right](1-\tau)\,b^*&-\frac{\gamma\upsilon-h}{(1-\beta)\upsilon}b^*\\ \left[1-\frac{\alpha\upsilon+h}{(1+\upsilon)\theta}\right](1-\tau)\,d^*&\mu_i-d^*\end{vmatrix},\end{aligned} \tag{42}$$

so by the equation (42) and $\left|\mu_iI-Q\right|=0$ can be easily obtained

$$\mu_1=(\omega K)^{-h}\,z^{\upsilon-1}Yp^*>0,\mu_2=m^*>0. \tag{43}$$

In the following, we will solve the characteristic roots of the second order determinant on the right side of the equation (42)(where we only need to prove that the sign of the product of the two characteristic roots of the second order determinant is negative). According to the equation (43), it can be assumed that the two characteristic roots of the second order determinant in equation (42) are: μ_3, μ_4. Because

$$\begin{aligned}\begin{vmatrix}\mu_i-\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1\right](1-\tau)\,b^*&-\frac{\gamma\upsilon-h}{(1-\beta)\upsilon}b^*\\ \left[1-\frac{\alpha\upsilon+h}{(1+\upsilon)\theta}\right](1-\tau)\,d^*&\mu_i-d^*\end{vmatrix}&=\mu_i^2-\left[d^*+\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1\right](1-\tau)\,b^*\right]\mu_i\\ &+\left[\frac{\alpha\upsilon+h}{1+\upsilon}-1+\left[1-\frac{\alpha\upsilon+h}{(1+\upsilon)\,\theta}\right]\frac{\gamma\upsilon-h}{(1-\beta)\,\upsilon}\right](1-\tau)\,b^*d^*=0.\end{aligned} \tag{44}$$

Therefore, according to the above hypothesis we can see that μ_3 and μ_4 are the two roots of the unity quadratic equation of μ_i in equation (44), ie

$$\mu_3 \cdot \mu_4 = \left[\frac{\alpha\upsilon + h}{1+\upsilon} - 1 + \left[1 - \frac{\alpha\upsilon + h}{(1+\upsilon)\theta}\right]\frac{\gamma\upsilon - h}{(1-\beta)\upsilon}\right](1-\tau)\,b^* d^* = -\left[\beta\upsilon\theta + \gamma\upsilon + \theta - h\right]\frac{(\alpha\upsilon + h)(1-\tau)}{(1+\upsilon)(1-\beta)\upsilon\theta}b^* d^*. \tag{45}$$

From the formula (45), when $\beta\upsilon\theta + \gamma\upsilon + \theta > h$, then $\mu_3 \cdot \mu_4 < 0$. We can see that the signs of characteristic roots μ_3 and μ_4 must be: a positive one and a negative one. By combining the Eq.(43), we can see that there are one negative and three positive eigenvalues of the Jacobi matrix Q in the equation (41). So the dynamic system of the economy in the equation (40) has a certain dynamic equilibrium stability when $\beta\upsilon\theta + \gamma\upsilon + \theta > h$ [18].

3.4 The solution of the economic growth rate

According to equations (20), (28) to (30), (33) to (35) (where b^*, d^*, m^* and p^* replace respectively the values of b, d, m and p), it can be respectively obtained

$$g_Y^* = g_K^* = g_C^* = \frac{1}{\theta}\left[\frac{(1-\tau)(\alpha\upsilon + h)}{1+\upsilon}b^* - \rho - \omega\right]. \tag{46}$$

$$g_\pi^* = \omega - \frac{(1-\tau)(\alpha\upsilon + h)}{1+\upsilon}b^*. \tag{47}$$

$$g_z^* = \frac{(1-\tau)(h-\alpha)}{1+\upsilon}b^* - \frac{h}{\upsilon}d^* + \frac{(1-h)\,\omega - \eta}{\upsilon}. \tag{48}$$

$$g_E^* = -\frac{\alpha\upsilon + h}{(1-\beta)\upsilon}d^* + \frac{(1-\alpha)\,\omega + \gamma(g+n)}{1-\beta} + \frac{(1-h)\,\omega - \eta}{(1-\beta)\upsilon} - \eta. \tag{49}$$

Where b^*, d^*, m^* and p^* are respectively expressed as

$$b^* = \frac{1+\upsilon}{(1-\tau)\left[1+\upsilon - (\alpha\upsilon + h)\right]}\left[\frac{\gamma\upsilon - h}{(1-\beta)\upsilon}d^* + \frac{(1-\alpha)\,\omega + \gamma(g+n)}{1-\beta} + \frac{(1-h)\,\omega - \eta}{(1-\beta)\upsilon}\right]. \tag{50}$$

$$d^* = \frac{(1-\tau)[(1+\upsilon)\theta - (\alpha\upsilon + h)]}{(1+\upsilon)\,\theta}b^* + \frac{\rho + \omega}{\theta} - \omega. \tag{51}$$

$$m^* = \frac{\alpha\upsilon + h}{(1-\beta)\upsilon}d^* - \frac{(1-\alpha)\,\omega + \gamma(g+n)}{1-\beta} - \frac{(1-h)\,\omega - \eta}{(1-\beta)\upsilon}. \tag{52}$$

$$p^* = (\omega K)^h z^{1-\upsilon} Y^{-1}\left[\frac{(1-\tau)(\alpha - h)}{1+\upsilon}b^* + \frac{h}{\upsilon}d^* - \frac{(1-h)\,\omega - \eta}{\upsilon} - \eta\right]. \tag{53}$$

By combining the Eq. (50) with (51) can be seen that

$$b^* = \frac{(1+\upsilon)\theta\upsilon}{(1-\tau)(\alpha\upsilon + h)[\gamma\upsilon + (1+\beta\upsilon)\theta - h]}[\frac{(1-h)\omega - \eta}{\upsilon} + (1-\alpha)\omega + \gamma(g+n) + \frac{\gamma\upsilon - h}{\upsilon}(\frac{\rho + \omega}{\theta} - \omega)]. \tag{54}$$

From the formulas (46), (48), (49), (51) and (54) can be obtained

$$g_Y^* = \frac{1}{\gamma\upsilon + \theta(1+\beta\upsilon) - h}\left[\gamma\upsilon(g+n) - \eta - \rho(1+\beta\upsilon)\right]. \tag{55}$$

$$g_z^* = \frac{-1}{\gamma\upsilon + \theta(1+\beta\upsilon) - h}\left[\rho(\gamma + h\beta) + \gamma(\theta - h)(n+g) + \eta(\gamma + \beta\theta)\right]. \tag{56}$$

$$g_E^* = \frac{\gamma\upsilon(1-\theta)(g+n) - \rho\left[1 - h + (1-\alpha)\upsilon\right] - \eta(1 + \beta\upsilon\theta + \gamma\upsilon - h)}{\gamma\upsilon + \theta(1+\beta\upsilon) - h}. \tag{57}$$

3.5 Carbon-based energy intensity

The growth rate of carbon-based energy intensity can be obtained by combining equation (47) with equation (54)

$$g_\pi^* = \frac{1}{\gamma\upsilon + \theta(1+\beta\upsilon) - h}\left[\theta\eta - \rho(\gamma\upsilon - h) - \gamma\upsilon\theta(g+n)\right]. \tag{58}$$

3.6 Comparative static analysis

From the formulas (55)-(58) we can see that the g_Y^*, g_π^*, g_E^* and g_z^* all contain υ, g, n, η and ρ after the adjustment of the dynamic system of the economy. Under the premise of satisfying formula (45) $\mu_3 \cdot \mu_4 < 0$, i.e. $\beta\upsilon\theta + \gamma\upsilon + \theta > h$, this paper focuses on the impact of pollution intensity υ, environmental self-purification ability η, labor growth rate n, technology growth rate g and the utility discount rate of consumer ρ on g_Y^*, g_π^*, g_E^* and g_z^*. Through g_Y^*, g_π^*, g_E^* and g_z^* respectively on the parameter υ, g, n, η and ρ for partial guidance, and obtained the following results (static analysis of Table 1)

Table 1: An static analysis

	$x=\upsilon$	$x=\eta$	$x=n$	$x=g$	$x=\rho$
$\frac{\partial g_Y^*}{\partial x}$	>0, if $h<\theta$	<0	>0	>0	<0
$\frac{\partial g_\pi^*}{\partial x}$	<0, if $h<\theta$	>0	<0	<0	>0,if $h>\gamma\upsilon$; <0,if $h<\gamma\upsilon$
$\frac{\partial g_E^*}{\partial x}$	/	/	>0,if $\theta<1$; <0,if $\theta>1$	>0 ,if $\theta<1$; <0 ,if $\theta>1$	<0
$\frac{\partial g_z^*}{\partial x}$	>0, if $h<\theta$	<0	<0 , if $h<\theta$; >0 , if $h>\theta$	<0 , if $h<\theta$; >0 , if $h>\theta$	<0

From Table 1 we can see that $\frac{\partial g_Y^*}{\partial \upsilon} > 0$ and $\frac{\partial g_z^*}{\partial \upsilon} > 0$ when $h < \theta$, ie, g_Y^* and g_z^* are both about the increasing function of parameter υ, it indicates that the rise of the degree index of pollution can enhance the economic growth rate, but not conducive to reduce the growth rate of pollution intensity. However, $\frac{\partial g_\pi^*}{\partial \upsilon} < 0$ when $h < \theta$, which shows that g_π^* is the the decreasing function of υ. Ie, the growth rate of carbon-based energy intensity g_π^* is decreased with the increase of υ, that is, the amount of energy consumption of the per unit of output value will decrease and the energy efficiency is improved. The influence of υ on the growth rate of environmental quality g_E^* is restricted by various factors such as marginal utility elasticity θ and discount rate ρ.

It can be seen from $\frac{\partial g_\pi^*}{\partial \eta} > 0$ that g_π^* increases with the increase of environmental self-purification ability η, that is, the increase of η can not inhibit the increase of the growth rate of energy intensity. While $\frac{\partial g_Y^*}{\partial \eta} < 0$ and $\frac{\partial g_z^*}{\partial \eta} < 0$, it can be shown that g_Y^* and g_z^* are both the reduction function of η, that is, enhancing the environmental self-purification capacity η is not conducive to improve the economic growth rate, but it can inhibit the increase of pollution intensity growth rate. In addition, the impact of η on the growth rate of environmental quality is constrained by such factors as marginal utility elasticity and pollution intensity.

From $\frac{\partial g_\pi^*}{\partial n} < 0$, $\frac{\partial g_\pi^*}{\partial g} < 0$, $\frac{\partial g_Y^*}{\partial n} > 0$ and $\frac{\partial g_Y^*}{\partial g} > 0$, we can see that the increase of the growth rate of technology and labor force not only can both restrain the growth rate of carbon-based energy intensity, but also can enhance the economic growth rate. It can be seen that $\frac{\partial g_z^*}{\partial n} < 0$ and $\frac{\partial g_z^*}{\partial g} < 0$ from Table 1 when $h < \theta$, which means that the improvement of g and n can both inhibit the increase of the growth of pollution intensity. From the table 1 we can see that the influence of g and n on g_E^* are both limited by the condition of θ. It can be seen from the table 1 that the improvement of the growth rate of labor and technology can improve the growth rate of environmental quality when $\theta < 1$. It can be concluded from Table 1 that increasing the discount rate ρ can not promote the increase of g_Y^* and g_E^*. The increase of ρ can restrain the increase of g_π^* when $h < \gamma\upsilon$.

By equation (55), we have $\frac{\partial g_Y^*}{\partial h} = \frac{\gamma\upsilon(g+n)-\eta-\rho(1+\beta\upsilon)}{[\gamma\upsilon+\theta(1+\beta\upsilon)-h]^2}$, then $\frac{\partial g_Y^*}{\partial h} < 0$ when $\gamma\upsilon(g+n) < \eta + \rho(1+\beta\upsilon)$,which is an decreasing function of environmental investment efficiency h, that is, enhancing the efficiency of environmental protection investment can be not conductive to the growth of economy.

By the formula (56), we have $\frac{\partial g_z^*}{\partial h} = \frac{(\gamma+\beta\theta)[\gamma\upsilon(g+n)-\eta-\rho(1+\beta\upsilon)]}{[\gamma\upsilon+\theta(1+\beta\upsilon)-h]^2}$, then $\frac{\partial g_z^*}{\partial h} < 0$ when$\gamma\upsilon(g+n) < \eta + \rho(1+\beta\upsilon)$, that is, the increase of environmental protection investment efficiency h is beneficial to decrease the growth rate of pollution intensity.

4 Conclusion

Based on the existing research, this paper establishes a model of economic growth under the triple constraints of capital, carbon-based energy and environment, and introduces simultaneously factors such as energy intensity, technical level, labor force and pollution intensity into the production function. The dynamic optimization theory, static analysis and the analysis technique of optimal equilibrium are used to discuss the dynamic equilibrium stability of the economic system and analyze the influence of the related parameters on the economic growth in China. It is revealed that in the path of balanced growth if the government increases the carbon-based energy tax rate and environmental investment, the consumer's future consumption tendency and economic growth rate will decrease. The increase of pollution degree index can increase the economic growth rate and decrease the growth rate of carbon-based energy intensity, but not conducive to decline the pollution intensity when $h < \theta$. The enhancement of environmental self-purification ability is not conducive to increase the economic growth rate, but it can be beneficial to reduce the pollution intensity. The enhancement of the growth rate of the technology and labor force can both restrain the increase of the growth rate of carbon-based energy intensity and promote the economic growth, and the increase of them could both inhibit the increase of pollution intensity when $h < \theta$. Enhancing the growth rate of labor and technology can improve the environmental quality when $\theta < 1$. The increase of the discount rate can be not conductive to enhance the growth rate of economy. The enhancement of the efficiency of environmental protection investment can be not beneficial to increase the economic growth rate, but it is conducive to decrease the growth rate of the pollution intensity when $\gamma\upsilon(g + n) < \eta + \rho(1 + \beta\upsilon)$.

According to the analysis results in this paper, we can put forward relevant policy recommendations: the government can make appropriate coping strategies according to the specific place in different regions, such as appropriately lowering the carbon-based energy tax rate and other policies, which will enhance China's economic growth to a certain extent. Strengthening environmental supervision and pollution control in all key industries, strictly controlling the rise of the pollution index can improve the environment. Increasing the efficiency of environmental protection investment will be conducive to decrease the pollution intensity. Further increasing the investments in technological innovation and labor force will be beneficial to reduce the energy intensity and enhance the energy efficiency as well as China's economic growth and sustainable development. Under certain conditions, it can also improve the environment.

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