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On the Quantum Zakharov-Kuznetsov Equation

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Abstract: Here, an extended ($\frac{G'}{G}$)-expansion method and truncated expansion method with the aid of a symbolic computational method is used for constructing the traveling wave solutions. The solutions obtained are general solutions which are in the form of hyperbolic, trigonometric and rational functions and a variety of special solutions like kink shaped, anti-kink shaped, bell type solido solutions etc., can easily be derived from the general results under certain domain. The validity and reliability of the two methods are tested by their applications for nonlinear physical problem, namely, Zakharov-Kuznetsov equation that is derived to describe the ion acoustic waves in magnetized plasma. The two methods are straight-forward and concise and they can also be applied to other nonlinear evolution equations in physics. The exact solutions obtained here are pretty new in literature.

Keywords: Extended ($\frac{G'}{G}$)-expansion method; Truncated Expansion method; Zakharov-Kuznetsov equation; Traveling wave solutions

1 Introduction

Nonlinear evolution equations are frequently used to describe many problems of solid state physics, fluid mechanics,plasma physics, population dynamics, chemical kinetics, nonlinear optics, protein chemistry, theory of Bosie Einstein condensates etc [1-5]. The basic strategies one may adopt to predict, control and quantify the underlying features of a system under investigation are to model the system in terms of mathematical equations, which are generally nonlinear and then find exact analytic solutions of such model equations using some suitable methods [10]. In the past few decades, considerable efforts have been made to obtain exact analytical solutions of such nonlinear equations and a number of powerful and efficient methods have been developed for obtaining explicit traveling wave solutions [12-15].

Seeking exact solutions of nonlinear partial differential equations is of great significance as it appears that these (NLPDEs) are mathematical models of complex physics phenomena arising in physics, mechanics, biology, chemistry and engineers [31, 36]. In order to help engineers and physicists to better understand the mechanism that governs these physical models or to better provide knowledge to the physical problem and possible applications, a vast variety of the powerful and direct methods have been derived [39-44]. Various powerful methods for obtaining explicit traveling solitary wave solutions to nonlinear equations have proposed such as [47-50].

The investigation of ion-acoustic waves and structures in dense quantum plasmas has attracted much attention in recent years [7, 9, 22, 33]. It was shown that the quantum effects play a crucial role in plasma dynamics when the de-Broglie wavelength of the charge carriers becomes comparable to the spatial scale of the system. In dense quantum plasma, the quantum hydrodynamic (QHD) model is one of the most popular models. The QHD model is a generalization of classical fluid model of plasmas where QHD transport equations are expressed in terms of the conservation laws of particles momentum and energy.

This paper is organized as follows: In Section 2, the governed equations of the plasma system is given and transformed into the quantum Zakharov-Kuznetsov (QZK) equation. We simply provide the mathematical framework of the two methods in Section 3. Finally, conclusions are given.

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2 Governed equation of the problem

The importance of quantum effects in ultra-small electronic devices, in dense astrophysical plasma system and in laser plasma have produced interest on the investigation of the quantum counterpart of some of the classical plasma physics phenomena. For instance, the quantum plasma has attracted much attention waves in a cohesionless demagnetized quantum plasma amongst the plasma physics community because of its potential applications in different scientific areas either in laboratory or in astrophysics [45].

The nonlinear propagation of the electrostatic waves, in a dense Thomas–Fermi magneto-plasma whose constituents are the electrons and singly charged ions confined in an external magnetic field of strength B_0 along the x-axis, is governed by the dimensionless ion continuity and momentum equations represented by

$$\frac{\partial n_i}{\partial t}+\frac{\partial(n_i u_{ix})}{\partial x}+\frac{\partial(n_i u_{iy})}{\partial y}=0, \tag{1}$$

$$\frac{\partial u_{ix}}{\partial t}+[u_{ix}\frac{\partial}{\partial x}+u_{iy}\frac{\partial}{\partial y}]u_{ix}+\frac{3}{2}\frac{\partial \phi}{\partial x}=0, \tag{2}$$

$$\frac{\partial u_{iy}}{\partial t}+[u_{ix}\frac{\partial}{\partial x}+u_{iy}\frac{\partial}{\partial y}]u_{iy}+\frac{3}{2}\frac{\partial \phi}{\partial y}-u_{iz}=0, \tag{3}$$

$$\frac{\partial u_{iz}}{\partial t}+[u_{ix}\frac{\partial}{\partial x}+u_{iy}\frac{\partial}{\partial y}]u_{iz}+u_{iy}=0, \tag{4}$$

with Tomas-Fermi law for degenerate electrons

$$n_e=(1+\phi)^{3/2}. \tag{5}$$

Eqs.(1)-(5) are closed by the Poisson's equation.

$$\Omega[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}]\phi=\frac{2}{3}(n_e-n_i). \tag{6}$$

Through these Eqs.(1)-(6), $n_i(x,y,t)$, and $n_e(x,y,t)$ are the number densities of the ion and electron species, respectively and are normalized by the unperturbed electron/ion number density $n_{i0}=n_{e0}$ and h is Planck's constant. $u_i(x,y,t)$ is the ion fluid velocity, which is normalized by the Fermi ion-sound speed $C_{si}=\sqrt{2E_F/(3m_i)}$ where $E_F=2k_BT_F$, k_B is the Boltzmann constant, T_F is the Fermi electron temperature and m_i is the ion mass. $\phi(x,y,t)$ is the electrostatic potential normalized by E_F/e where e is the magnitude of the electron charge. The time and space variables are in units of the ion gyro-frequency $\omega_{ci}=eB_0/(m_ic)$ and ion sound gyro-radius $\rho_s=C_{si}/\omega_{ci}$, respectively, where B_0 is the strength of the magnetic field taken along the x-axis and c is the speed of light in vacuum. Furthermore, $\Omega=(3c9_{ci}/3c9_{pi})^2$ where $\omega_{pi}=\sqrt{4\pi e^2n_0/m_i}$ is the ion plasma frequency.

The reductive perturbation method [37] introduced the stretching space-time coordinates

$$X=3b5^{1/2}(x-t),\ Y=3b5^{1/2}y,\ \text{and}\ T=3b5^{3/2}t, \tag{7}$$

where $3b5$ is a smallness parameter measuring the weakness of the amplitude or dispersion. The dependent variables $n_{i(e)}$, u_{ix}, u_{iy}, and ϕ are expanded about the unperturbed values in power series of $3b5$. Inserting the above stretching and the expansions into Eqs.(1)-(6) and separating various powers of $3b5$ lead to the following Zakharov-Kuznetsov equation.

$$\frac{\partial \phi_1}{\partial T}+A\,\phi_1\frac{\partial \phi_1}{\partial X}+B\,\frac{\partial^3\phi_1}{\partial X^3}+C\,\frac{\partial^3\phi_1}{\partial X\partial Y^2}=0, \tag{8}$$

$$A=2,\ B=\frac{1}{2}\Omega,\ \text{and}\ C=\frac{1}{2}(1+\Omega). \tag{9}$$

3 Description of the two proposed methods

3.1 Extended $(\frac{G'}{G})$-expansion method

For a given a nonlinear evolution equations,say in two independent variables x and t, is given by

$$\phi(u, u_t, u_x, u_t, u_{xx}, \cdots) = 0, \tag{10}$$

where $u = u(x,t)$ is an unknown function, ϕ is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear term are involved. Combining the independent variables x and t ito one variable $\xi = x - ct$, we suppose that

$$u(x,t) = u(\xi), \xi = x - ct, \tag{11}$$

the traveling wave variable Eq.(14) permits us reducing Eq.10 yo an ODE for $u = u(\xi)$

$$\psi(u, -cu', u', c^2u'', -cu'', \cdots) = 0. \tag{12}$$

Suppose that the solution of ODE Eq.(10) can be expressed by a polynomial in $(\frac{G'(\xi)}{G(\xi)})$,

$$u(\xi) = \sum_{i=-m}^{m} a_i (\frac{G'(\xi)}{G(\xi)})^i, \tag{13}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{14}$$

where $G'(\xi) = \frac{dG(\xi)}{d\xi}$, $G''(\xi) = \frac{d^2G(\xi)}{d\xi^2}$, a_i, λ, and μ are constants to be determined later, $a_i \neq 0$, the unwritten part in Eq.(4) is also a polynomial in $(\frac{G'}{G})$, but the degree of which is generally equal to or less than $m-1$, the positive inter m can be determined by balancing the highest order derivative terms with nonlinear term appearing in Eq.(3).

It it to be noted that, the solutions of Eq.(5) for $(\frac{G'(\xi)}{G(\xi)})$ term can be written in the form of hyperbolic, trigonometric and rational functions as given below.

The first type: When $\lambda^2 - 4\mu > 0$,

$$(\frac{G'(\xi)}{G(\xi)}) = \frac{\sqrt{\lambda^2 - 4\mu}}{2} [\frac{C_1 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}] - \frac{\lambda}{2}. \tag{15}$$

The second type: When $\lambda^2 - 4\mu < 0$,

$$(\frac{G'(\xi)}{G(\xi)}) = \frac{\sqrt{4\mu - \lambda^2}}{2} [\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}] - \frac{\lambda}{2}. \tag{16}$$

The third type: When $\lambda^2 - 4\mu = 0$,

$$(\frac{G'(\xi)}{G(\xi)}) = \frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}, \tag{17}$$

where C_1 and C_2 are integration constants.

Substituting Eq.(13) into Eq.(12) and making use of Eq.(14), collecting all terms with the same order $((\frac{G'(\xi)}{G(\xi)}))$ together, and equating each coefficients of this polynomial to zero, yields a set of algebraic equations for a_i, λ, and μ. With the knowledge of the coefficients a_i and general solution of Eq.(5) we have more traveling wave solutions of the nonlinear evolution Eq.(1).

To find the exact traveling wave solutions of QZK equation. Making use the transformation with

$$u(\xi) = \phi_1(X, Y, T), \xi = L_x X + L_y Y - \nu T, \tag{18}$$

where ν is a constant speed, L_x and L_y are the directional cosine of the propagation wave vector along the X and Y axes,respectively. Then Eq.(8) reduces to

$$-\nu\frac{du(\xi)}{d\xi}+A_0u(\xi)\frac{du(\xi)}{d\xi}+B_0\frac{d^3u(\xi)}{d\xi^3}=0, \tag{19}$$

where

$$A_0=AL_x, B_0=BL_x^3+CL_xL_y^2. \tag{20}$$

Intergrading Eq.(20) with respect to ξ, we have

$$-\nu\mu(\xi)+\frac{A_0}{2}\mu^2(\xi)+B_0\frac{\ddot{\mu(\xi)}}{\ddot{\xi}}+C=0, \tag{21}$$

where C is an integration constant to be determined later. As mentioned above, by using Eq.(21) and considering homogeneous balance between $u^2(\xi)$ and $u^{''}(\xi)$, we obtain $n=2$. So we can write Eq.(13) as

$$u(\xi)=a_0+a_1(\frac{G^{'}(\xi)}{G(\xi)})+a_2(\frac{G^{'}(\xi)}{G(\xi)})^2+a_{-1}(\frac{G^{'}(\xi)}{G(\xi)})^{-1}+a_{-2}(\frac{G^{'}(\xi)}{G(\xi)})^{-2}. \tag{22}$$

Inserting Eq.(22) into Eq.(21) and equating each coefficients of this polynomial with the same powers of $(\frac{G^{'}(\xi)}{G})$ to zero yields a set of simultaneous algebraic equations for $a_0, a_1, a_2, a_{-1}, a_{-2}, \nu, C$. Solving set of algebraic equations with the aid of Maple, we have

Case(a):

$$a_2=-\frac{12B_0}{A_0}, a_0=a_0, a_{-1}=0, a_{-2}=0, \nu=\lambda^2B_0+A_0a_0+8\mu B_0, a_1=-\frac{12B_0\lambda}{A_0},$$
$$C=\frac{(24B_0^2\lambda^2\mu+48B_0^2\mu^2+A_0^2a_0^2+2A_0a_0B_0\lambda^2+16A_0a_0B_0\mu)}{2A_0}. \tag{23}$$

Case(b):

$$a_{-2}=-\frac{12B_0\mu^2}{A_0}, a_{-1}=-\frac{12B_0\lambda\mu}{A_0}, a_0=a_0, a_1=0, a_2=0, \nu=\lambda^2B_0+A_0a_0+8\mu B_0,$$
$$C=(1/2)\frac{(24B_0^2\lambda^2\mu+48B_0^2\mu^2+A_0^2a_0^2+2A_0a_0B_0\lambda^2+16A_0a_0B_0\mu)}{A_0}. \tag{24}$$

Now the general solution of quantum Zakharov-Kuznetsov equation

$$\mu(\xi)=-\frac{12B_0}{A_0(\frac{G'(\xi)}{G}(\xi))^2-\frac{12B_0\lambda}{A_0}(\frac{G'(\xi)}{G})+a_0}, \tag{25}$$

$$\xi=\xi=L_xX+L_yY-(\lambda^2B_0+A_0a_0+8\mu B_0)T. \tag{26}$$

With the aid of the general solutions of Eq.(14), we have three types of traveling wave solutions that propagate in quantum Zakharov-Kuznetsov equation as follows:

Case(1a): Hyperbolic function traveling wave solutions when $\lambda^2-4\mu>0$,

$$u_{1a}(\xi)=-\frac{12B_0}{A_0}[\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}-\frac{\lambda}{2}]^2$$
$$-\frac{12B_0\lambda}{A_0}[\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}-\frac{\lambda}{2}]+a_0. \tag{27}$$

Case(2a): Trigonometric function traveling wave solutions when $\lambda^2-4\mu<0$,

$$u_{2a}(\xi) = -\frac{12B_0}{A_0}[\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi} - \frac{\lambda}{2}]^2$$
$$-\frac{12B_0\lambda}{A_0}[\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi} - \frac{\lambda}{2}] + a_0. \tag{28}$$

Case(3a):

$$u_{3a}(\xi) = -\frac{12B_0}{A_0}[\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}]^2 - \frac{12B_0\lambda}{A_0}[\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}] + a_0. \tag{29}$$

According to case(b), with the general solutions of Eq.(24), we have three types of traveling wave solutions as

Case(1b): Hyperbolic function traveling wave solutions when $\lambda^2 - 4\mu > 0$,

$$u_{1b}(\xi) = a_0 - \frac{12B_0\mu^2}{A_0}[\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi} - \frac{\lambda}{2}]^{-2}$$
$$-\frac{12B_0\lambda\mu}{A_0}[\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi} - \frac{\lambda}{2}]^{-1}. \tag{30}$$

Case(2b): Trigonometric function traveling wave solutions when $\lambda^2 - 4\mu < 0$,

$$u_{2b}(\xi) = a_0 - \frac{12B_0\mu^2}{A_0}[\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi} - \frac{\lambda}{2}]^{-2}$$
$$-\frac{12B_0\lambda\mu}{A_0}[\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi} - \frac{\lambda}{2}]^{-1}. \tag{31}$$

Case(3b): Rational function solution when $\lambda^2 - 4\mu = 0$,

$$\mu_{3b}(\xi) = a_0 - \frac{12B_0\mu^2}{A_0}[\frac{C_2}{C_1 + C_2\xi}]^{-2} - \frac{12B_0\lambda\xi}{A_0}[\frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}]^{-1}. \tag{32}$$

3.2 Modified truncated expansion method (MTEM)

In what the follows the summarize of the Truncated Expansion (MTEM) method. For a given the nonlinear evolution equations in the form

$$\phi_1[u_t, u_x, \cdots, x, t] = 0. \tag{33}$$

We use the transformation $u(x,t) = y(z)$, $z = kx - ct$. Then Eq.(33) reduces to

$$\phi_2[-wy_z, ky_z, k^2y_{zz}, \cdots] = 0. \tag{34}$$

We assume the exact solution of Eq.(34) can be expressed as

$$y(z) = \sum_{i=0}^{N} a_i Q(z)^N, \tag{35}$$

where a_i are unknown constants, $Q(z)$ is the following equation

$$Q(z) = \frac{1}{1+e^z}, \tag{36}$$

which satisfies

$$Q_z = Q(z)^2 - Q(z). \tag{37}$$

Differentiating Eq.(35) with respect to z and using Eq.(37) admits

$$y_z = \sum_{i=1}^{N} a_i i(Q-1)Q^i, \tag{38}$$

$$y_{zz} = \sum_{i=1}^{N} a_i i((i+1)Q^2 - (2i+1)Q + i)Q^i, \tag{39}$$

$$y_{zzz} = \sum_{i=1}^{N} a_i i((Q-1)Q^i[(i^2+3i+2)Q^2 - (2i^2+3i+1)Q + i^2], \tag{40}$$

$$y_{zzzz} = \sum_{i=1}^{N} a_i i((Q-1)Q^i[(i^3+6i^2+11i+6)Q^3 - (3i^3+12i^2+15i+6]Q^2 + (3i^3+6i^2+4i+1)Q - i^3]. \tag{41}$$

Substituting Eq.(35) into Eq.(8). Then, we collect all terms the same power in the function $Q(z)$ and equate the expression to zero, we obtain an algbraic system of equations. Solving this system we get the values of the unknown parameters.

To find the exact solutions of QZK equation. Using the traveling waves

$$u(\xi) = y(z), z = L_x X + L_y Y - \nu T. \tag{42}$$

Eq.(8) admits to

$$-\nu y(z) + \frac{A_0}{2} y^2(z) + B_0 y_{zz} + C = 0, \tag{43}$$

where $A_0 = AL_x$,$B_0 = BL_x^3 + CL_x L_y^2$.

The pole of Eq.(43) is equal to $N = 2$, then we look for exact solution as

$$y(z) = a_0 + a_1 Q(z) + a_2 Q(z)^2, \tag{44}$$

where a_0, a_1 and a_2 are unknown constants to be determined later.

Substituting Eq.(44) into Eq.(43), we obtain the polynomial of function $Q(z)$ and equate the expression to zero we obtain an algebraic system of equations.Solving this system we get the values of the unknown parameters as

$$\nu = A_0 a_0 + B_0, a_0 = a_0, a_2 = -\frac{12B_0}{A_0}, a_1 = \frac{12B_0}{A_0}, C = \frac{1}{2} A_0 a_0 + a_0 B_0. \tag{45}$$

The solitary wave solution of Eq.(44) admits to

$$y(z) = a_0 + \frac{12B_0}{(A_0(1 + e^{(L_x x + L_y y - (A_0 a_0 + B_0 t))})} - \frac{12B_0}{(A_0(1 + e^{(L_x x + L_y y - (A_0 a_0 + B_0 * t))^2})}, \tag{46}$$

$$z = L_x X + L_y Y - [A_0 a_0 + B_0]T. \tag{47}$$

4 Conclusions

Here, two proposed method are employed to investigate the solutions of the QZK equation. The results obtained will enrich previous results and help us further understand the physical structures and analyze the nonlinear propagation of the quantum ion-acoustic waves in quantum magneto-plasma, namely, the extended $\frac{G'}{G}$-expansion method and truncated expansion method for constructing the new exact traveling wave solutions of the QZK equation. The results obtained will enrich previous results and help us further understand the physical structures and analyze the nonlinear propagation of the quantum ion-acoustic waves in quantum magneto-plasma. The obtained traveling wave solutions are expressed by the hyperbolic functions, the trigonometric functions and rational functions. Finally, it is worthwhile to mention that the two methods are straightforward, concise and can also be applied to other nonlinear problems in science and engineering. This is our task in future works.

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