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A Double-Server Double-Class Queue with Constant Retrlal Policy in Reducing Delay in Healthcare Delivery

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Abstract: We consider in this paper a double-server double-class retrial queues. Servers, when free, help serve patients awaiting in the buffers of other servers. Patients arrive at both queues according to Poisson processes. Each station is fed by a renewal input with general independent identically distributed inter-arrival times and general identically independent distributed service times possibly different in the two stations. All service times are exponential, with rates depending on the queues. The cost to be minimized involve cost per unit time that a patient spends in the system. We find a sufficient conditions including all possible parameter choices such as arrival rates, retrial rates and service rates. We further present conditions for scheduling new arrivals, under which server assignment to either queue-1 or queue-2 is cost optimal.

Keywords: Markov process; Retrial queues; Control policy; Dynamic programming; Health care; Double server

1 Introduction

Assessing the quality of care is not new in healthcare delivery, the rapid growth of the managed-care industry in the world has lead to a variety of definitions and perceptions of quality. Today, several well-established agencies and organizations address improving health care quality and patient safety through a process known as continuous quality improvement [2, 3]. Organizations such as the Agency for Healthcare Research and Quality, the National Committee for Quality Assurance (NCQA), and the Joint Commission, to name a few, have emerged with the specific intent to support quality, safety, efficiency, and effectiveness of health care [1, 4].

Healthcare organizations worldwide are under increased pressure to deliver greater value and increased efficiency while guaranteeing the ever-higher quality of care [5]. At the same time, patients' needs are becoming more complex, advances in technology are enabling and forcing delivery model reform, expectations of service excellence are increasing, and payers are continuing to demand greater cost control [6].

With emphasis on "quality outcomes," it is becoming increasingly critical for healthcare organizations to develop and implement a sound strategy for providing effective care that is appealing to patients and focuses on controlling costs. Healthcare as a whole faces the challenges of attracting and retaining patients and talented employees while delivering consistently effective and efficient care. According to [7], developed triads with a structure-process-outcomes framework, which provide a solid foundation about quality-improvement efforts, but their opinion of structure needs to be updated to account for current tools and management capabilities that drive quality improvement. In an effort to develop a definition of structure for transforming quality-improvement initiatives, they discuss key elements of organizational attributes from a management perspective. Widespread use of health information technology(IT) could potentially increase patients access to their health information and facilitate future goals of advancing patient-centered care. Despite having increased access to their health data, patients do not always understand this information or its implications, and digital health data can be difficult to navigate when displayed in a small-format, complex interface [8].

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Single server queues with vacations and repeated attempts have been studied in [9]. According to [10, 20], the decomposition property of M/G/1 retrial queues with breakdowns and linear retrial policy depends linearly on the number of customers in orbit. A proposed single server linear retrial queue with multiple servers subject to breakdowns and repairs was developed by [11] to obtain the generating functions of the limiting distribution. Another closely related system is studied in the above mentioned paper [12], it turns out to be that if the objective is to minimize the queue length in a system with one retrial queue and several heterogeneous servers, the optimal control structure is of the threshold type and this control structure was evaluated numerically and dictates that a slow server has to be used only if the number of customers in the queue exceeds a certain threshold [13, 14]. Through an iterative process, [15] identified barriers and facilitators to the use of mHealth technology for HIV prevention for high-risk MSM, developed 'use cases' and identified relevant functional content and features for inclusion in a design document to guide future app development. Findings from [17] support the use of the ISR framework as a guide for designing future mHealth apps. Optimal scheduling of a single- server two - class retrial queue with level - dependent retrial rates under various control policies was considered by [16, 18].

Thus, the main motivation of this research was based on [19], their analysis was limited to a system with a single server with limited liability companies. Following [19], we establish a double-server with two-infinite capacity retrial queues for healthcare centers. In this work we place more emphasis on common retrial case where the idle time of the servers may be utilized for other secondary jobs, for instance to serve patients of another queue.
The rest of the article is organized as follows: In the next section, we describe the mathematical model. Section 3 presents the main results and proofs and the final section provides example on the network.

2 Methods

2.1 Model description

In this section, we consider the scheduling of a double queueing system which consist of four retrial rates (see Fig. 1). In such a system, each station has an infinite - capacity buffer for the awaiting type $x \in \{1, 2\}$ patients who arrive from outside to queue$-x$ according to Poisson process with intensity $\lambda_x \geq 0$.

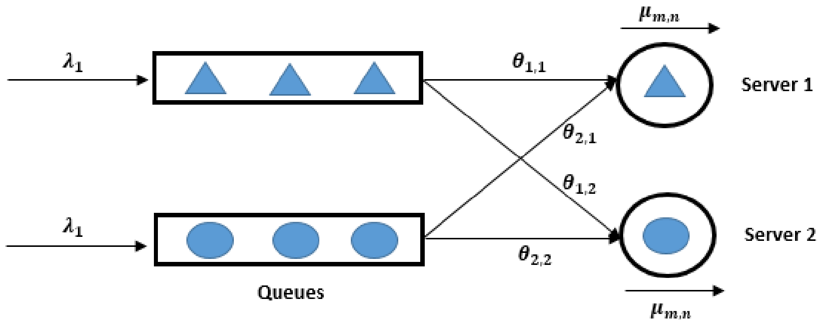

Figure 1: A double-server double-class queue with constant retrial policy

As soon as any of the server becomes free, the patient to be served next generates a Poisson stream of requests for service with rate $\theta_{m,n} > 0,\ (m,n) = (1,1),(1,2),(2,2),(2,1)$ for queue$-x$. The service times of the patients that are being served at time t are exponential with rate $\mu_{m,n} > 0,\ (m,n) = (1,1),(1,2),(2,2),(2,1)$. We assume non-preemptive and head-of-the-line (HoL) service discipline. We let $\xi(x) \geq 2$, be independent identically distributed (iid), inter-arrival times of the Class$-x$ patients entering the network after instant 0 $(x = 1, 2)$ and let $\eta(m,n) \geq 2$, be the iid service times of the Class$-(m,n)$ patient finishing service after instant 0. The residual service time $\eta(m,n)$ of class (m,n) patient initially being served, if any, is independent of $\{\eta(m,n),(m,n) \geq 2\}$ and $(m,n) = 1 =_{st} \eta(m,n)$ if class (m,n) is initially empty. We introduce linear holding costs at rate $c_x > 0$ for each unit time a patient of type x spends in a queue or on the server.

The starting point of our investigation of optimal control structure is, we first consider the scheduling problem without arrivals, that is $\lambda_1 = \lambda_2 = 0$. We assume that at the beginning, an arbitrary number of patients are present in the buffer of queue$-x$. The state space is denoted by $\mathbb{X} = \{m \times n \times s | m, n \in \mathbb{N}\}$ where m and n are the number of patients in the first and second buffers, respectively and s describes the state of the server $s \in \{(0,0),(1,1),(1,2),(2,2),(2,1)\}$.

$$\begin{cases} (0,0) & \text{the server is idle,} \\ (1,1) & \text{server} - 1 \text{ is processing a queue} - 1 \text{ patient,} \\ (1,2) & \text{server} - 2 \text{ is processing a queue} - 1 \text{ patient,} \\ (2,2) & \text{server} - 2 \text{ is processing a queue} - 2 \text{ patient,} \\ (2,1) & \text{server} - 1 \text{ is processing a queue} - 2 \text{ patient.} \end{cases}$$

Our control policy consists of assigning the idle server to one of the non-empty queues with control action $(m,n) = \{(1,1),(1,2),(2,2),(2,1)\}$ at every decision time. The decision times comprise the starting point and all service completion points. Our objective is to obtain an optional control policy that minimizes the expected total holding cost (see eg. [13]) until the system is cleared.
For ease, we describe this optimal control policy in a form of a control matrix $\Delta^*(m,n) \geq 0$. An empty buffers m and n gives the optimal control action for system state $(m,n,(0,0))$, where there are m patients in buffer-1 of queue-1 and n patients in buffer-2 of queue-2. This can be the initial or any other state

$$\begin{cases} \Delta^*(m,n) = (1,1),\ (2,1) & \text{server} - 1 \text{ is assigned to queue} - \text{x}, \quad \text{x} = 1,2 \\ \Delta^*(m,n) = (2,2),\ (1,2) & \text{server} - 2 \text{ is assigned to queue} - \text{x}, \quad \text{x} = 1,2. \end{cases}$$

For obvious reasons,

$$\Delta^*(m,0) = (1,1) \quad \text{and } \Delta^*(0,\text{n}) = (2,2), \quad \text{for m}, \text{n} \geq 1.$$

2.2 Stability condition

$$\lambda_1 \left[\left(\frac{1}{\theta_{1,1}} + \frac{1}{\mu_{1,1}} \right) + \left(\frac{1}{\theta_{2,1}} + \frac{1}{\mu_{2,1}} \right) \right] + \lambda_2 \left[\left(\frac{1}{\theta_{2,2}} + \frac{1}{\mu_{2,2}} \right) + \left(\frac{1}{\theta_{1,2}} + \frac{1}{\mu_{1,2}} \right) \right] < 1 \tag{1}$$

We consider the scheduling problem with new arrivals; that is $\lambda_1, \lambda_2 > 0$, if the stability condition in Eq. 1 hold, with control action of assigning the idle server to one of the nonempty queues (Control action $(1,1),(2,1),(2,2),(1,2)$), then the server cannot be idle if there are waiting patients in the buffers of queue-1 and queue-2 for service.
The aim is to identify a policy which minimizes the average cost per unit time g [13] over an infinite horizon. Uniformization according to [13] was applied with

$$\lambda_1 + \lambda_2 + \theta_{1,1} + \theta_{2,1} + \theta_{2,2} + \theta_{1,2} + \mu_{1,1} + \mu_{2,1} + \mu_{2,2} + \mu_{1,2} = 1. \tag{2}$$

In infinite horizon problems, the data is stationary. This means that the sets of states, the set of allowable actions in each state, the rewards, the transition or transfer functions and the decision sets are the same at every stage. The primary focus of this study is to consider problem in which decisions are made periodically at discrete time points (eg. minutes hours, days, weeks and months). The class of allowable policies is taken to be the set of non-preemptive stationary policies where the server decides the precise optimal policy to take under the optimal control actions, which depends only on the current state. It is independent of time and previous states.

3 Results

Addressing scheduling calls with and without new arrivals respectively.

3.1 Scheduling calls without new arrivals

We start with explicit conditions which guarantee that assigning the server to queue 1 (or 2) is the optimal control action in the absence of new arrivals. It is based on a simple comparison of costs arising from different control actions. We use

$V(m,n,s)$ to denote the minimum expected total cost from buffer m and buffer n respectively. We obtain the following

$$\begin{aligned} V(m,n,(0,0)) =& min\Big[\Big(\frac{1}{\theta_{1,1}}(mc_1+nc_2)+v\big((m-1),n,(1,1)\big),\\ &\frac{1}{\theta_{2,1}}(mc_1+nc_2)+v\big((m-1),n,(2,1)\big)\Big),\\ &\Big(\frac{1}{\theta_{2,2}}(mc_1+nc_2)+v\big(m,(n-1),(2,2)\big),\\ &\frac{1}{\theta_{1,2}}(mc_1+nc_2)+v\big(m,(n-1),(1,2)\big)\Big)\Big], \quad m,n\geq 1, \end{aligned} \tag{3}$$

$$V(m,n,(1,1)) = \Big(\frac{1}{\mu_{1,1}}\big((m+1)c_1+nc_2\big)+v\big(m,n,(0,0)\big)\Big), \quad m,n\geq 1, \tag{4}$$

$$V(m,n,(2,1)) = \Big(\frac{1}{\mu_{2,1}}\big((m+1)c_1+nc_2\big)+v\big(m,n,(0,0)\big)\Big), \quad m,n\geq 1, \tag{5}$$

$$V(m,n,(2,2)) = \Big(\frac{1}{\mu_{2,2}}\big(mc_1+(n+1)c_2\big)+v\big(m,n,(0,0)\big)\Big), \quad m,n\geq 1, \tag{6}$$

$$V(m,n,(1,2)) = \Big(\frac{1}{\mu_{1,2}}\big(mc_1+(n+1)c_2\big)+v\big(m,n,(0,0)\big)\Big), \quad m,n\geq 1, \tag{7}$$

The terms to be minimized in Eq. (3), correspond with control action $(1,1),\ (2,1)$ and control action $(2,2),\ (1,2)$ respectively. First we show that $\Delta^*(m,n)=(1,1),\ (2,1)$, with $m,n\geq 1$ on condition that

$$\frac{c_1}{\theta_{2,2}}+\frac{c_1}{\mu_{2,2}} > \frac{c_2}{\theta_{1,1}}+\frac{c_2}{\mu_{1,1}} \quad \text{or} \quad \frac{c_1}{\theta_{1,2}}+\frac{c_1}{\mu_{1,2}} > \frac{c_2}{\theta_{2,1}}+\frac{c_2}{\mu_{2,1}}$$

.

Assume that this conditions holds true. Recall that $\Delta^*(m,0)=1$ for $m\leq 1$; to be precise, the lowest row of the control matrix consist of ones. We use this row as a starting point and prove that every control matrix entry $\Delta^*(m,n-1)=(1,1)$ or $(2,1)$ leads to $\Delta^*(m,n)=(1,1)$ or $(2,1)$. Suppose that $\Delta^*(m,(n-1))=(1,1)$ or $(2,1)$, we evaluate the minimum in Eq. (3) for the system state $(m,n,0)$. Control action 2 and $\Delta^*(m,n-1)=(1,1)$ or $(2,1)$ leads to:

$$\begin{aligned} &\frac{1}{\theta_{2,2}}(mc_1+nc_2)+V\big(m,(n-1),(2,2)\big)=\frac{1}{\theta_{2,2}}(mc_1+nc_2)+\frac{1}{\mu_{2,2}}(mc_1+nc_2)+\\ &\frac{1}{\theta_{1,1}}\big(mc_1+(n-1)c_2\big)+\frac{1}{\mu_{1,1}}(mc_1+(n-1)c_2)+v\big((m-1),(n-1),(0,0)\big). \end{aligned} \tag{8}$$

The first two terms on the right-hand side of Eq. (8) are the costs which accumulate while processing a queue-2 patient, the next two terms represent the costs for one queue-1 patient.

$$\begin{aligned} &\frac{1}{\theta_{1,2}}(mc_1+nc_2)+V\big(m,(n-1),(1,2)\big)+=\frac{1}{\theta_{1,2}}(mc_1+nc_2)+\frac{1}{\mu_{1,2}}(mc_1+nc_2)\\ &\frac{1}{\theta_{2,1}}\big(mc_1+(n-1)c_2\big)+\frac{1}{\mu_{2,1}}(mc_1+(n-1)c_2)+v\big((m-1),(n-1),(0,0)\big). \end{aligned} \tag{9}$$

In this case server-1 is never idle if there are waiting class-2 patients in buffer-2. The first two terms on the right-hand side of Eq. (9) are the costs which accumulate while processing a class$-(2,1)$ patient. The next two terms represent the costs for one Class$-(1,2)$ patient.
For control action $(1,1),\ (2,1)$, we receive (using the fact that we do not know whether control action $(2,2),\ (1,2)$ is cost - optimal in state $((m-1),n,0)$.

We again evaluate the minimum in Eq. (3) for the system state $(m, n, 0)$. Control action $\Delta^*((m-1), n) = (2, 2)$ or $(1, 2)$ on condition that

$$\frac{c_1}{\theta_{1,1}} + \frac{c_1}{\mu_{1,1}} > \frac{c_2}{\theta_{2,2}} + \frac{c_2}{\mu_{2,2}} \quad \text{or} \quad \frac{c_1}{\theta_{2,1}} + \frac{c_1}{\mu_{2,1}} > \frac{c_2}{\theta_{1,2}} + \frac{c_2}{\mu_{1,2}}.$$

This leads to

$$\frac{1}{\theta_{1,1}}(mc_1 + nc_2) + V\big((m-1), n, (1,1)\big) = \frac{1}{\theta_{1,1}}(mc_1 + nc_2) + \frac{1}{\mu_{1,1}}(mc_1 + nc_2) + \frac{1}{\theta_{2,2}}\big((m-1)c_1 + nc_2\big) + \frac{1}{\mu_{2,2}}((m-1)c_1 + nc_2) + v\big((m-1), (n-1), (0,0)\big). \tag{10}$$

Yet again, the first two terms on the right-hand side of Eq. (10) are the costs which accumulate while processing a queue-1 patient, the next two terms represent the costs for one queue-2 patient.

$$\frac{1}{\theta_{2,1}}(mc_1 + nc_2) + V\big((m-1), n, (2,1)\big) + = \frac{1}{\theta_{2,1}}(mc_1 + nc_2) + \frac{1}{\mu_{2,1}}(mc_1 + nc_2) \; \frac{1}{\theta_{1,2}}\big((m-1)c_1 + nc_2\big) + \frac{1}{\mu_{1,2}}((m-1)c_1 + nc_2) + v\big((m-1), (n-1), (0,0)\big). \tag{11}$$

From Eq. (11), server-2 is never idle if there are waiting class-1 patients in buffer-1. The first two terms on the right-hand side are the costs which accumulate while processing a class$-(1,2)$ patient. The next two terms represent the costs for one Class$-(2,1)$ patient.
Proceeding, we analyze the equivalence of control actions for

$$\frac{c_1}{\theta_{2,2}} + \frac{c_1}{\mu_{2,2}} = \frac{c_2}{\theta_{1,1}} + \frac{c_2}{\mu_{1,1}}.$$

We introduce the notation $\Delta^*(m, n) = (1,1) \equiv (2,2)$ together with the entry $\Delta^*(m, (n-1)) = (1,1)$ or $\Delta^*(m, (n-1)) = (1,1) \equiv (2,2)$ leads to $\Delta^*(m, n) = (1,1) = (1,1) \equiv (2,2)$. We use $\Delta^*(0, n) = (2,2)$ and $\Delta^*(m, 0) = (1,1)$ for $m, n \leq 1$ and starting from $\Delta^*(1,1)$, we obtain each successively. We further evaluate the minimum in Eq. (3) for the system state $(m, n, (0,0))$.
Considering $\Delta^*(m, (n-1)) = (1,1)$ or $\Delta^*(m, (n-1)) = (1,1),\ (2,2)$. For control action $(2,2)$ we obtain

$$\begin{aligned}
&\frac{1}{\theta_{2,2}}(mc_1 + nc_2) + V\big(m, (n-1), (2,2)\big) = \\
&\frac{1}{\theta_{2,2}}(mc_1 + nc_2) + \frac{1}{\mu_{2,2}}(mc_1 + nc_2) + min\Big[\frac{1}{\theta_{1,1}}\big(mc_1 + (n-1)c_2\big) + v\big((m-1), (n-1), (1,1)\big), \\
&\frac{1}{\mu_{1,1}}(mc_1 + (n-1)c_2) + v\big(m, (n-1), (2,2)\big)\Big] = \\
&\frac{1}{\theta_{2,2}}(mc_1 + nc_2) + \frac{1}{\mu_{2,2}}(mc_1 + nc_2) + \frac{1}{\theta_{1,1}}\big(mc_1 + (n-1)c_2\big) + \frac{1}{\mu_{1,1}}(mc_1 + (n-1)c_2) + \\
&v\big((m-1), (n-1), (0,0)\big).
\end{aligned} \tag{12}$$

We again consider $\Delta^*((m-1), n) = (2,2)$ or $\Delta^*((m-1), n) = (1,1) \equiv (2,2)$, we analogously for control action $(1,1)$ obtain

$$\begin{aligned}
&\frac{1}{\theta_{1,1}}(mc_1 + nc_2) + \frac{1}{\mu_{1,1}}(mc_1 + nc_2) + \frac{1}{\theta_{2,2}}\big((m-1)c_1 + nc_2\big) + \frac{1}{\mu_{2,2}}((m-1)c_1 + nc_2) + \\
&v\big((m-1), (n-1), (0,0)\big).
\end{aligned} \tag{13}$$

Eventually the cost difference (12)-(13) yield

$$\frac{c_1}{\theta_{2,2}} + \frac{c_1}{\mu_{2,2}} - \frac{c_2}{\theta_{1,1}} - \frac{c_2}{\mu_{1,1}} = 0,$$

control action $(2,2)$ equals control action $(1,1)$.

Similarly

$$\frac{c_1}{\theta_{1,2}} + \frac{c_1}{\mu_{1,2}} = \frac{c_2}{\theta_{2,1}} + \frac{c_2}{\mu_{2,1}}.$$

Considering $\Delta^* = (1,1)$ or $\Delta^* = (m,(n-1)) = (1,2) \equiv (2,1)$ or $\Delta^*(m,(n-1)) = (1,2),\ (2,1)$, we obtain for control action $(1,2)$

$$\begin{aligned}
&\frac{1}{\theta_{1,2}}(mc_1+nc_2) + V\big(m,(n-1),(1,2)\big) = \\
&\frac{1}{\theta_{1,2}}(mc_1+nc_2) + \frac{1}{\mu_{1,2}}(mc_1+nc_2) + min\Big[\frac{1}{\theta_{2,1}}\big(mc_1+(n-1)c_2\big) + v\big((m-1),(n-1),(2,1)\big), \\
&\frac{1}{\mu_{2,1}}(mc_1+(n-1)c_2) + v\big(m,(n-1),(2,1)\big)\Big] = \\
&\frac{1}{\theta_{1,2}}(mc_1+nc_2) + \frac{1}{\mu_{1,2}}(mc_1+nc_2) + \frac{1}{\theta_{2,1}}\big(mc_1+(n-1)c_2\big) + \frac{1}{\mu_{2,1}}(mc_1+(n-1)c_2) + \\
&v\big((m-1),(n-1),(0,0)\big).
\end{aligned} \tag{14}$$

Setting $\Delta^*((m-1),n) = (2,1) \equiv (1,2)$ or $\Delta^*((m-1),n) = (2,1),(1,2)$ we obtain control action $(2,1)$

$$\begin{aligned}
&\frac{1}{\theta_{2,1}}(mc_1+nc_2) + \frac{1}{\mu_{2,1}}(mc_1+nc_2) + \frac{1}{\theta_{1,2}}\big((m-1)c_1+nc_2\big) + \frac{1}{\mu_{1,2}}((m-1)c_1+nc_2) + \\
&v\big((m-1),(n-1),(0,0)\big).
\end{aligned} \tag{15}$$

In the end, the cost difference (14)-(15) gives

$$\frac{c_1}{\theta_{1,2}} + \frac{c_1}{\mu_{1,2}} - \frac{c_2}{\theta_{2,1}} - \frac{c_2}{\mu_{2,1}} = 0,$$

Hence control action $(1,2)$ equals control action $(2,1)$ this implies

$$\frac{c_1}{\theta_{1,2}} + \frac{c_1}{\mu_{1,2}} = \frac{c_2}{\theta_{2,1}} + \frac{c_2}{\mu_{2,1}}.$$

3.2 Scheduling problem with new arrivals

Theorem 1 *Let $\lambda_1, \lambda_2 > 0$ and $\frac{c_2}{\theta_{1,1}} \leq [(\geq)]\frac{c_1}{\theta_{2,2}}$, $\frac{\theta_{1,1}}{\mu_{1,1}} \leq [(\geq)]\frac{\theta_{2,2}}{\mu_{2,2}}$. The optimal stationary policy with respect to the average cost per unit time for the infinite - capacity system assigns the server to $(1,1), [(2,2)]$ in states $(m,n,(0,0))$ of queue$-(i,j)$ with $i,j \geq 1$ in that, $\Delta^*(m,n) = (1,1)[(2,2)]$ for $i,j \geq 1$. If $\frac{c_2}{\theta_{1,1}} = \frac{c_1}{\theta_{2,2}}$ and $\frac{c_2}{\mu_{1,1}} = \frac{c_1}{\mu_{2,2}}$, the optimal control action is not uniquely determined.*

Proof. We find the average cost optimal stationary policy for the model using the average cost optimality equations **(ACOE)** [14].

We formed the equation using relative value functions $v(m,n,s)$ for states (m,n,s) and the average cost (reward) g. Describing the relationship between relative value functions due to possible state transitions gives

$$\begin{aligned}
V(m,n,(0,0)) &= mc_1 + nc_2 - g + \lambda_1 v\big((m+1),n,(0,0)\big) + \lambda_2 v\big(m,(n+1),(0,0)\big) + \\
&min\begin{cases} \theta_{1,1} v\left((m-1),n,(1,1)\right) + (1-\theta_{1,1}-\lambda_1-\lambda_2) v(m,n,(0,0)) \\ \theta_{2,2} v\left(m,(n-1),(2,2)\right) + (1-\theta_{2,2}-\lambda_1-\lambda_2) v(m,n,(0,0)). \quad m,n \geq 1 \end{cases}
\end{aligned} \tag{16}$$

The first three terms denote the costs of present patients in the system in state $(m,n,(0,0))$ and the average cost. The subsequent two terms result from allocating the server to queue-1. Here the system then proceed to state $((m-1),n,(1,1))$

or remain in state $(m, n, (0, 0))$. The second row designates the state changes when server is assigned to queue-2. If one of the queues is empty, the only possible control action is assigning the server to the nonempty queue. We obtain

$$V\big(m,(0,0),(0,0)\big) = mc_1 - g + \lambda_1 v\left((m+1),(0,0),(0,0)\right) + \lambda_2 v\left(n,(1,1),(0,0)\right) + \theta_{1,1} v\big((m-1),(0,0),(1,1)\big) + (1-\theta_{1,1}-\lambda_1-\lambda_2) v\big(m,(0,0),(0.0)\big) \; m \geq 1, \tag{17}$$

$$V\big((0,0),n,(0,0)\big) = nc_2 - g + \lambda_1 v\left((1,1),n,(0,0)\right) + \lambda_2 v\left((0,0),(n+1),(0,0)\right) + \theta_{2,2} v\big((0,0),(n-1),(2,2)\big) + (1-\theta_{2,2}-\lambda_1-\lambda_2) v\big((0,0),n,(0.0)\big) \; n \geq 1, \tag{18}$$

$$V\big((0,0),(0,0),(0,0)\big) = -g + \lambda_1 v\left((1,1),(0,0),(0,0)\right) + \lambda_2 v\left((0,0),(1,1),(0,0)\right) + (1-\lambda_1-\lambda_2)\, v\big((0,0),(0,0),(0.0)\big), \tag{19}$$

with $v\big((0,0),(0,0),(0.0)\big) = 0$, we finally have the equation for states $\big(m, n, (1,1)\big)$ which include transitions due to service completion.

$$V\big(m,n,(0,0)\big) = (m+1)c_1 + mc_2 - g + \mu_{1,1} v\left(m,n,(0,0)\right) + (1-\mu_{1,1}) v\big(m,n,(1,1)\big) \; m,n \geq 1, \tag{20}$$

$$V\big(m,n,(2,2)\big) = mc_1 + (n+1)c_2 - g + \mu_{2,2} v\left(m,n,(0,0)\right) + (1-\mu_{2,2}) v\big(m,n,(2,2)\big) \; m,n \geq 1, \tag{21}$$

We rewrite the above Eq. (16)-(21) for $m, n \geq 1$ as

$$V(m,n,(0,0)) = mc_1 + nc_2 - g + \lambda_1 v\big((m+1),n,(0,0)\big) + \lambda_2 v\big(m,(n+1),(0,0)\big) + min\begin{cases} \dfrac{\theta_{1,1}}{\mu_{1,1}}\big(mc_1+nc_2-g\big) + \theta_{1,1} v\big((m-1),n,(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,1}) v\big(m,n,(0,0)\big) \\ \dfrac{\theta_{2,2}}{\mu_{2,2}}\big(mc_1+nc_2-g\big) + \theta_{2,2} v\big(m,(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,1}) v\big(m,n,(0,0)\big) \end{cases} \tag{22}$$

$$V\big(m,(0,0),(0,0)\big) = mc_1 - g + \lambda_1 v\left((m+1),(0,0),(0,0)\right) + \lambda_2 v\left(m_1,(1,1),(0,0)\right) + \frac{\theta_{1,1}}{\mu_{1,1}}(mc_1 - g) + \theta_{1,1} v\big((m-1),(0,0),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,1}) v\big(m,(0,0),(0.0)\big), \tag{23}$$

$$V\big((0,0),n,(0,0)\big) = nc_2 - g + \lambda_1 v\left((1,1),n,(0,0)\right) + \lambda_2 v\left((0,0),(n+1),(0,0)\right) + \frac{\theta_{2,2}}{\mu_{2,2}}(mc_2 - g) + \theta_{2,2} v\big((0,0),(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,2}) v\big((0,0),n,(0.0)\big). \tag{24}$$

The solution of the above **ACOE** (and the corresponding optimal policy) can be found by solving the finite horizon optimality equations **(FHOE)**. These equations are formed by the $y-$stage cost function $v_y(m, n, s)$ for states m, n, s. From the above **ACOE** we obtain for $y > 0$ and $m, n \geq 1$. The following **FHOE**

$$V_{y+1}\big(m,n,(0,0)\big) = mc_1 + nc_2 + \lambda_1 v_y\big((m+1),n,(0,0)\big) + \lambda_2 v_y\big(m,(n+1),(0,0)\big) + min\begin{cases} \dfrac{\theta_{1,1}}{\mu_{1,1}}(mc_1+nc_2\big) + \theta_{1,1} v_y\big((m-1),n,(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,1}) v_y\big(m,n,(0,0)\big) \\ \dfrac{\theta_{2,2}}{\mu_{2,2}}(mc_1+nc_2) + \theta_{2,2} v_y\big(m,(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,2}) v_y\big(m,n,(0,0)\big), \end{cases} \tag{25}$$

$$V_{y+1}\big((m,(0,0),(0,0)\big) = mc_1 + \lambda_1 v_y\big((m+1),(0,0),(0,0)\big) + \lambda_2 v_y\big(m,(1,1),(0,0)\big) + \frac{\theta_{1,1}}{\mu_{1,1}}(mc_1 + \theta_{1,1} v_y\big((m-1),(0,0),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,1}) v_y\big(m,(0,0),(0,0)\big), \tag{26}$$

$$V_{y+1}\big(((0,0),n,(0,0)\big) = nc_2 + \lambda_1 v_y\big((1,1),n,(0,0)\big) + \lambda_2 v_y\big((0,0),(n+1),(0,0)\big) + \frac{\theta_{2,2}}{\mu_{2,2}}(nc_2 + \theta_{2,2} v_y\big((0,0),(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,2}) v_y\big((0,0),n,(0,0)\big). \quad (27)$$

For $y = 0$, we have $v_0(m,n,s) = 0$. We aim to prove that the main results holds for the **FHOE** for all y. We take limits as $y \to \infty$ the average cost per unit time result follows [13] and [14]. Assume that

$$\frac{c_2}{\theta_{1,1}} \leq \frac{c_1}{\theta_{2,2}} \quad \text{and} \quad \frac{\theta_{1,1}}{\mu_{1,1}} \leq \frac{\theta_{2,2}}{\mu_{2,2}}.$$

Hence the minimum term in (25) yield

$$\frac{\theta_{1,1}}{\mu_{1,1}}(mc_1 + nc_2) + \theta_{1,1} v_y\big((m-1),n,(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,1}) v_y\big(m,n,(0,0)\big) \leq \frac{\theta_{2,2}}{\mu_{2,2}}(mc_1 + nc_2) + \theta_{2,2} v_y\big(m,(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,2}) v_y\big(m,n,(0,0)\big), \quad (28)$$

that is $\Delta^*(m,n) = (1,1)$ for all y. ■

Theorem 2 *Let $\lambda_1, \lambda_2 > 0$ and $\frac{c_2}{\theta_{2,1}} \leq [(\geq)] \frac{c_1}{\theta_{1,2}}$, $\frac{\theta_{2,1}}{\mu_{2,1}} \leq [(\geq)] \frac{\theta_{1,2}}{\mu_{1,2}}$. The optimal stationary policy with respect to the average cost per unit time for the infinite - capacity system assigns the server to $(2,1),[(1,2)]$ in states $(m,n,(0,0))$ of queue$-x$ with $m,n \geq 1$ in that, $\Delta^*(m,n) = (2,1)[(1,2)]$. If $\frac{c_2}{\theta_{2,1}} = \frac{c_1}{\theta_{1,2}}$ and $\frac{c_2}{\mu_{2,1}} = \frac{c_1}{\mu_{1,2}}$, the optimal control action is not uniquely determined.*

Proof. We again find the average cost optimal stationary policy for servers who served patients from Class$-(2,1)$ or Class$(1,2)$ using the average cost optimality equations **(ACOE)** [14].
We formed the equation using relative value functions $v(m,n,s)$ for states (m,n,s) and the average cost (reward) g. Describing the relationship between relative value functions due to possible state transitions gives

$$V(m,n,(0,0)) = mc_1 + nc_2 - g + \lambda_1 v\big((m+1),n,(0,0)\big) + \lambda_2 v\big(m,(n+1),(0,0)\big) + min \begin{cases} \theta_{2,1} v\left((m-1),n,(2,1)\right) + (1-\theta_{2,1}-\lambda_1-\lambda_2) v(m,n,(0,0)) \\ \theta_{1,2} v\left(m,(n-1),(1,2)\right) + (1-\theta_{1,2}-\lambda_1-\lambda_2) v(m,n,(0,0)). \quad m,n \geq 1 \end{cases} \quad (29)$$

The first three terms denote the costs of patients in a change buffer in state $(m,n,(0,0))$ and the average cost. The next two terms result from transitions due to new arrivals in the queues. The term to be minimized depends on the control action in the state $(m,n,(0,0))$. The fist row consist of transitions which result from assigning the server to queue-1. Here the system can proceed to state $((m-1),n,(2,1))$ or remain in state $(m,n,(0,0))$. The second row describes the state changes when server is assigned to queue-2. If one of the queues is empty, the only possible control action is assigning the server to the nonempty queue. We obtain

$$V\big(m,(0,0),(0,0)\big) = mc_1 - g + \lambda_1 v\left((m+1),(0,0),(0,0)\right) + \lambda_2 v\left(n,(2,1),(0,0)\right) + \theta_{2,1} v\big((m-1),(0,0),(2,1)\big) + (1-\theta_{2,1}-\lambda_1-\lambda_2) v\big(m,(0,0),(0.0)\big) \; m \geq 1, \quad (30)$$

$$V\big((0,0),n,(0,0)\big) = nc_2 - g + \lambda_1 v\left((2,1),n,(0,0)\right) + \lambda_2 v\left((0,0),(n+1),(0,0)\right) + \theta_{1,2} v\big((0,0),(n-1),(1,2)\big) + (1-\theta_{1,2}-\lambda_1-\lambda_2) v\big((0,0),n,(0.0)\big) \; n \geq 1, \quad (31)$$

$$V\big((0,0),(0,0),(0,0)\big) = -g + \lambda_1 v\left((2,1),(0,0),(0,0)\right) + \lambda_2 v\left((0,0),(2,1),(0,0)\right) + (1-\lambda_1-\lambda_2)\, v\big((0,0),(0,0),(0.0)\big), \quad (32)$$

with $v\big((0,0),(0,0),(0.0)\big) = 0$, we finally have the equation for states $\big(m,n,(2,1)\big)$ which include transitions due to service completion.

$$V\big(m,n,(0,0)\big) = (m+1)c_1 + mc_2 - g + \mu_{2,1} v\left(m,n,(0,0)\right) + (1-\mu_{2,1}) v\big(m,n,(2,1)\big) \; m,n \geq 1, \quad (33)$$

$$V\big(m,n,(1,2)\big) = mc_1 + (n+1)c_2 - g + \mu_{1,2}v\left(m,n,(0,0)\right) + (1-\mu_{1,2})v\big(m,n,(1,2)\big)\ m,n \geq 1, \tag{34}$$

We rewrite the above Eq. (29)-(34) for $m,n \geq 1$ as

$$V(m,n,(0,0)) = mc_1 + nc_2 - g + \lambda_1 v\big((m+1),n,(0,0)\big) + \lambda_2 v\big(m,(n+1),(0,0)\big) + \\ min\begin{cases} \dfrac{\theta_{2,1}}{\mu_{2,1}}\big(mc_1+nc_2-g\big) + \theta_{2,1}v\big((m-1),n,(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,1})v\big(m,n,(0,0)\big) \\ \dfrac{\theta_{1,2}}{\mu_{1,2}}\big(mc_1+nc_2-g\big) + \theta_{1,2}v\big(m,(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,1})v\big(m,n,(0,0)\big) \end{cases} \tag{35}$$

$$\begin{aligned} V\big(m,(0,0),(0,0)\big) =& mc_1 - g + \lambda_1 v\left((m+1),(0,0),(0,0)\right) + \lambda_2 v\left(m_1,(2,1),(0,0)\right) \\ &+\frac{\theta_{2,1}}{\mu_{2,1}}(mc_1-g) + \theta_{2,1}v\big((m-1),(0,0),(0,0)\big) \\ &+(1-\lambda_1-\lambda_2-\theta_{2,1})v\big(m,(0,0),(0.0)\big), \end{aligned} \tag{36}$$

$$\begin{aligned} V\big((0,0),n,(0,0)\big) =& nc_2 - g + \lambda_1 v\left((2,1),n,(0,0)\right) + \lambda_2 v\left((0,0),(n+1),(0,0)\right) \\ &+\frac{\theta_{1,2}}{\mu_{1,2}}(mc_2-g) + \theta_{1,2}v\big((0,0),(n-1),(0,0)\big) \\ &+(1-\lambda_1-\lambda_2-\theta_{1,2})v\big((0,0),n,(0.0)\big). \end{aligned} \tag{37}$$

Again the solution of the above **ACOE** (and the corresponding optimal policy) can be found by solving the finite horizon optimality equations **(FHOE)** are formed by $y-$stage cost function $v_y(m,n,s)$ for states m,n,s. From the above **ACOE** we deduce for $y>0$ and $m,n \geq 1$. The following **FHOE**

$$V_{y+1}\big(m,n,(0,0)\big) = mc_1 + nc_2 + \lambda_1 v_y\big((m+1),n,(0,0)\big) + \lambda_2 v_y\big(m,(n+1),(0,0)\big) + \\ min\begin{cases} \dfrac{\theta_{2,1}}{\mu_{2,1}}\big(mc_1+nc_2\big) + \theta_{2,1}v_y\big((m-1),n,(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,1})v_y\big(m,n,(0,0)\big) \\ \dfrac{\theta_{1,2}}{\mu_{1,2}}(mc_1+nc_2) + \theta_{1,2}v_y\big(m,(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,2})v_y\big(m,n,(0,0)\big), \end{cases} \tag{38}$$

$$\begin{aligned} V_{y+1}\big((m,(0,0),(0,0)\big) =& mc_1 + \lambda_1 v_y\big((m+1),(0,0),(0,0)\big) + \lambda_2 v_y\big(m,(2,1),(0,0)\big) \\ &+\frac{\theta_{2,1}}{\mu_{2,1}}(mc_1 + \theta_{2,1}v_y\big((m-1),(0,0),(0,0)\big) \\ &+(1-\lambda_1-\lambda_2-\theta_{2,1})v_y\big(m,(0,0),(0,0)\big), \end{aligned} \tag{39}$$

$$\begin{aligned} V_{y+1}\big(((0,0),n,(0,0)\big) =& nc_2 + \lambda_1 v_y\big((2,1),n,(0,0)\big) + \lambda_2 v_y\big((0,0),(n+1),(0,0)\big) \\ &+\frac{\theta_{1,2}}{\mu_{1,2}}(nc_2 + \theta_{1,2}v_y\big((0,0),(n-1),(0,0)\big) \\ &+(1-\lambda_1-\lambda_2-\theta_{1,2})v_y\big((0,0),n,(0,0)\big). \end{aligned} \tag{40}$$

For $y=0$, we have $v_0(m,n,s)=0$, our objective is to prove that the main results holds for the **FHOE** for all y. We take limits as $y \to \infty$ the average cost per unit time result follows [13] and [14]. Assume that

$$\frac{c_2}{\theta_{2,1}} \leq \frac{c_1}{\theta_{1,2}} \quad \text{and} \ \frac{\theta_{2,1}}{\mu_{2,1}} \leq \frac{\theta_{1,2}}{\mu_{1,2}}.$$

Hence the minimum term in (38) yield

$$\frac{\theta_{2,1}}{\mu_{2,1}}\big(mc_1+nc_2\big) + \theta_{2,1}v_y\big((m-1),n,(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{2,1})v_y\big(m,n,(0,0)\big) \leq \\ \frac{\theta_{1,2}}{\mu_{1,2}}(mc_1+nc_2) + \theta_{1,2}v_y\big(m,(n-1),(0,0)\big) + (1-\lambda_1-\lambda_2-\theta_{1,2})v_y\big(m,n,(0,0)\big), \tag{41}$$

that is $\Delta^*(m,n) = (2,1)$ for all y. ■

4 An example

We consider a flexible healthcare center with routing flexibility. In other words, two different queues, which have differing primary functions and some overlapping secondary ones. Servers can perform the same operations. For assigning projects to servers, the most flexible alternative is to use control action $(1,1),\ (2,1),\ (2,2)$ and $(1,2)$, at every decision epoch. It can be used to route patients with different types of questions or languages to servers with various sets of expertise.

Assume that

$$\begin{cases} \lambda_1 = 0.075, \quad \lambda_2 = 0.015, \\ \theta_{1,1} = 0.13, \quad \theta_{2,1} = 0.15, \quad \theta_{2,2} = 0.176, \quad \theta_{1,2} = 0.169, \\ \mu_{1,1} = 0.4, \quad \mu_{2,1} = 0.4, \quad \mu_{2,2} = 0.4, \quad \mu_{1,2} = 0.4, \\ c_1 = 4.0, \quad c_2 = 2.0. \end{cases} \tag{42}$$

In the infinite case, for example, we have queues limited to 18, a boundary effects occurs. See Figure 2 for illustration. Here the cost of holding a full queue-1 and operating on queue-2 is lower than the cost incurred by a new arrival in queue-1 which is avoided if queue-1 is full. Queue-1 patients are more expensive, but it is better to hold the expensive patients than encourage the arrival of new expensive ones.

n																				
18	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
17	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
16	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
15	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
14	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
13	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
12	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
11	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
10	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
9	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
8	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
7	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
6	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
5	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
4	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
3	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
2	(2,2)	(1,1)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,2)	(1,2)	(1,2)	(2,2)	(2,2)	(2,1)	(2,2)	(2,2)	
1	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	
0		(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	m

Figure 2: Graphical representation of double-server double-queue with constant retrial policy at healthcare centers.

5 Conclusion

In this paper we have analyzed the non-preemptive scheduling of a double-server double-queue with constant retrial policy at healthcare centers with the scheduling algorithm of First Come First Serve (FCFS) service discipline. To be precise we have provided conditions for optimal from the first and the second queue with their rendition attributes being different from traditional routers and processing functions across multiple resources. The optimality criteria considered include finite and infinite horizon expected total reward with control policy of assigning the idle server to one of the non-empty queues with control action $(m,n) = \{(1,1),(1,2),(2,2),(2,1)\}$ at every decision time. The main objectives in analyzing sequential decision processes in general and Markov Decision Processor's (MDP's) in particular include: providing an optimality equation which characterizes the supremal value of the objective function and characterizing the form of an optimal policy if it exists. It was evident that a system without new arrivals can be acknowledge as a standard system without retrials but with a general service time. Nevertheless, for a system with new arrivals we utilize dynamic programming in order to find sufficient conditions for the network policies.

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