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The Peakons and Periodic Cusp Waves Solutions of the Osmosis K(2,2) Equation with the Dispersion Term

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Abstract: In this paper, we investigate soliton wave solutions for the osmosis $K(2,2)$ equation with the dispersion term by the qualitative analysis methods of planar dynamical systems. With the phase portrait bifurcation of traveling wave system, periodic cusp wave solutions and peakon wave solutions are obtained and the graph of the numerical simulation solutions is showed.

Keywords: Planar dynamical system; The osmosis K(2,2) equation with dispersion term; Periodic cusp wave solution; Peakon wave solution

1 Introduction

Rosenau and Hyman [1] gave and studied the $K(m,n)$ equations:

$$u_t + (u^m)_x + (u^n)_{xxx} = 0, m > 0, 1 < n \leq 3. \tag{1.1}$$

Its compacton solutions were obtained and the stability of the compacton solutions were investigated by means of both linear stability analysis as well as Lyapunov stability criteria [2, 3]. The stability of compacton solutions of fifth-order nonlinear dispersive equations was studied in [4]. Then Rosenau and Hyman studied $K(2,2)$, $K(3,3)$ equation further. The solutions of the $K(2,2)$ equation are typical of the $K(m,n)$ equation. Tian and Yin [5] introduced a fifth-order $K(m,n)$ equation with nonlinear dispersion to obtain multi-compaction solution. Wazwaz [6, 7] studied $K(m,n)$ equation

$$u_t + a(u^m)_x + (u^n)_{xxx} = 0,$$

with Adomian decomposition method, and obtained compacton solution for $a = 1$ and peakon solution for $a = -1$. We investigated the osmosis $K(2,2)$ equation:

$$u_t + (u^2)_x - (u^2)_{xxx} = 0,$$

which is denoted as *OK(2,2)* in [8]. Yin, Tian and Fan studied the symmetric and non-symmetric waves of the osmosis $K(2,2)$ equation [9]. Chen and Li [10] studied single peak solitary wave solutions for the equation under inhomogeneous boundary condition. Chen, Ding and Huang [11] continued studying nonuniform continuity about the equation. Lu et al. [12, 13] studied the soliton solutions for the other equation. To study the effect of the dispersion term, we will continue studying the osmosis $K(2,2)$ equation with the dispersion term:

$$u_t + (u^2)_x - (u^2)_{xxx} + \varepsilon u_{xxx} = 0,$$

where ε is a coefficient.

Three sections are organized in the paper. In section 1, the object is investigated and the method is introduced. In section 2, by using the bifurcation method of planar dynamical system, we change the osmosis $K(2,2)$ equation with the dispersion term into the traveling wave system and draw the bifurcation of phase portraits. In section 3, solitary wave solutions and periodic wave solutions are constructed in two different ways. The general explicit expression of peaked solitary wave solutions is obtained, and the graph of the solution is given with the numerical simulation.

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2 The bifurcation of phase portraits of *OK(2,2)* with the dispersion term

Consider the osmosis $K(2,2)$ equation with the dispersion term:

$$u_t + (u^2)_x - (u^2)_{xxx} + \varepsilon u_{xxx} = 0, \tag{2.1}$$

Namely

$$u_t + 2uu_x - 6u_x u_{xx} - 2uu_{xxx} + \varepsilon u_{xxx} = 0.$$

Substituting $u = \varphi(\xi)$ with $\xi = x - ct$ into (2.1), we get the following ODE:

$$-c\varphi' + 2\varphi\varphi' - 6\varphi'\varphi'' - 2\varphi\varphi''' + \varepsilon\varphi''' = 0, \tag{2.2}$$

Integrating (2.2) once, (2.2) becomes

$$-c\varphi + \varphi^2 - 2(\varphi')^2 - 2\varphi\varphi'' + \varepsilon\varphi'' = g,$$

where g is integral constant. Let $\frac{d\varphi}{d\xi} = y$, and we get a planar autonomous system

$$\begin{cases} \frac{d\varphi}{d\xi} = y, \\ \frac{dy}{d\xi} = \frac{g + c\varphi - \varphi^2 + 2y^2}{\varepsilon - 2\varphi}. \end{cases} \tag{2.3}$$

Since the traveling wave solutions of Eq.(2.1) is determined by the phase portraits of system (2.3), the next step is to study it. But it is not convenient to investigate (2.3) directly because there is the singular line $\varphi = \frac{\varepsilon}{2}$. To avoid the line temporarily, the following transformation is introduced:

$$\mathrm{d}\xi = (\varepsilon - 2\varphi)\mathrm{d}\tau.$$

Under the transformation, system (2.3) becomes

$$\begin{cases} \frac{d\varphi}{d\tau} = (\varepsilon - 2\varphi)y, \\ \frac{dy}{d\tau} = g + c\varphi - \varphi^2 + 2y^2. \end{cases} \tag{2.4}$$

If let

$$H(\varphi, y) = (2\varphi - \varepsilon)^2[\frac{1}{2}(\mathrm{g} + \frac{c\varepsilon}{6} - \frac{\varepsilon^2}{24}) + (\frac{\mathrm{c}}{3} - \frac{\varepsilon}{12})\varphi - \frac{1}{4}\varphi^2 + y^2], \tag{2.5}$$

then both systems (2.3) and (2.4) have the same first integral

$$H(\varphi, y) = h.$$

Therefore system (2.3) should have the same topological phase portraits as system (2.4) except the straight line $\varphi = \frac{\varepsilon}{2}$. Thus we can obtain the topological phase portraits of system (2.3) from those of system (2.4).

Now the bifurcation behavior of system (2.4) is considered by using the theory of planar dynamical systems. For system (2.4), the distribution and property of singular points will be showed in the following proposition.

Let $A_\pm(\frac{\varepsilon}{2}, \pm\sqrt{-\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})$ and $B_\pm(\frac{c \pm \sqrt{c^2+4g}}{2}, 0)$, then $H(A_\pm) = 0$,

$$H(B_+) = \frac{1}{48}(c + \sqrt{c^2 + 4g} - \varepsilon)^2[2c^2 + 2c\varepsilon + 12g + 2\sqrt{c^2 + 4g}(c - \varepsilon) - \varepsilon^2],$$

$$H(B_-) = \frac{1}{48}(-c + \sqrt{c^2 + 4g} + \varepsilon)^2[2c^2 + 2c\varepsilon + 12g - 2\sqrt{c^2 + 4g}(c - \varepsilon) - \varepsilon^2].$$

Let $K_1 = -\frac{c^2}{4}$, $K_2 = \frac{\varepsilon^2 - 2c\varepsilon - 8c^2}{36}$, $K_3 = \frac{\varepsilon^2 - 2c\varepsilon}{4}$ and $\varepsilon \neq c$, then $K_1 < K_2 < K_3$.

When $g > K_3$, system (2.4) has two singular points $B_\pm$.

When $g = K_3$, system (2.4) has two singular points $(\frac{\varepsilon}{2}, 0), (\frac{2c-\varepsilon}{2}, 0)$.

When $K_1 < g < K_3$, system (2.4) has four singular points $A_\pm, B_\pm$.

When $g = K_1$, system (2.4) has three singular points $(\frac{\varepsilon}{2}, \pm\frac{\varepsilon-c}{2\sqrt{2}}), (\frac{c}{2}, 0)$.

When $g < K_1$, system (2.4) has two singular points $A_\pm$.

Let $M(\varphi, y)$be the coefficient matrix of linearized system of the system (2.4) at a singular point(φ, y). By the theory of planar dynamical system, we know that for a singular point $(\varphi$, y) of a planar system, if $J(\varphi, y) < 0$ then the singular point is a saddle point; if $J(\varphi, y) > 0$ and $Trace(M(\varphi, y)) = 0$ then it is a center point; if $J(\varphi, y) > 0$ and $(T(M(\varphi, y)))^2 - 4J(\varphi, y) > 0$ then it is a node; if $J = 0$ and the Poincare index of the equilibrium point is zero then it is a cusp.

According to the theory, one can see the following facts:

When $g > K_3$, $J(B_\pm) = -4g - c^2 \pm (\varepsilon - c)\sqrt{c^2 + 4g}$, then $J(B_\pm) < 0$, thus $B_\pm$ both are saddle points.

When $g = K_3$, $J(\frac{2c-\varepsilon}{2}, 0) = -2(\varepsilon - c)^2 < 0$, then $(\frac{2c-\varepsilon}{2}, 0)$ is a saddle point.

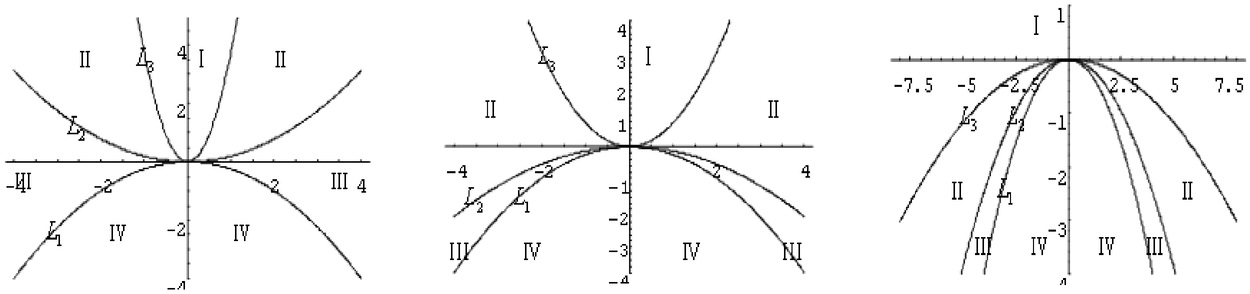
When $K_1 < g < K_3$, $J(A_\pm) = 4g < 0$, then $A_\pm$ both are saddle pionts.

$i)$ When $c > \varepsilon$, $J(B_+) < 0$,then B_+ is a saddle point; $J(B_-) = -c^2 - 4g + (c-\varepsilon)\sqrt{c^2 + 4g} > 0$, and $M(B_-) = 0$, then B_- is a center point.

$ii)$ When $c < \varepsilon$, $J(B_-) < 0$, then B_- is a saddle point; $J(B_+) = -c^2 - 4g + (\varepsilon - c)\sqrt{c^2 + 4g} > 0$, and $M(B_+) = 0$, then B_+ is a center point.

When $g = K_1$, $J(M(\frac{c}{2}, 0)) = 0$, then $(\frac{c}{2}, 0)$ is a cusp; $J(A_\pm) = -\frac{(\varepsilon-c)^2}{8} < 0$, $A_\pm$ both are saddle points.

When $g < K_1$, $J(A_\pm) < 0$, then $A_\pm$ both are saddle points.



(1)$c > 0$, $\varepsilon < -2c$ or $\varepsilon > 4c$; $c < 0, \varepsilon > -2c$ or $\varepsilon < 4c$.

(2)$c > 0$, or $2c < \varepsilon < 4c$; $c < 0, 0 < \varepsilon < -2c$ or $4c < \varepsilon < 2c$.

(3)$c > 0$, $0 < \varepsilon < c$ or $c < \varepsilon < 2c$; $c < 0, 2c < \varepsilon < c$ or $c < \varepsilon < 0$.

Figure 1: The curves and areas on the c-g plane, where $L_1 : g = K_1, L_2 : g = K_2, L_3 : g = K_3$.

According to the qualitative theory of dynamical systems and the results in proposition, we draw the bifurcation of phase portraits of system (2.4) as Fig. 1 and Fig. 2. Note that system (2.3) has the same topological phase portraits as system (2.4) except the line $\varphi = \frac{\varepsilon}{2}$.

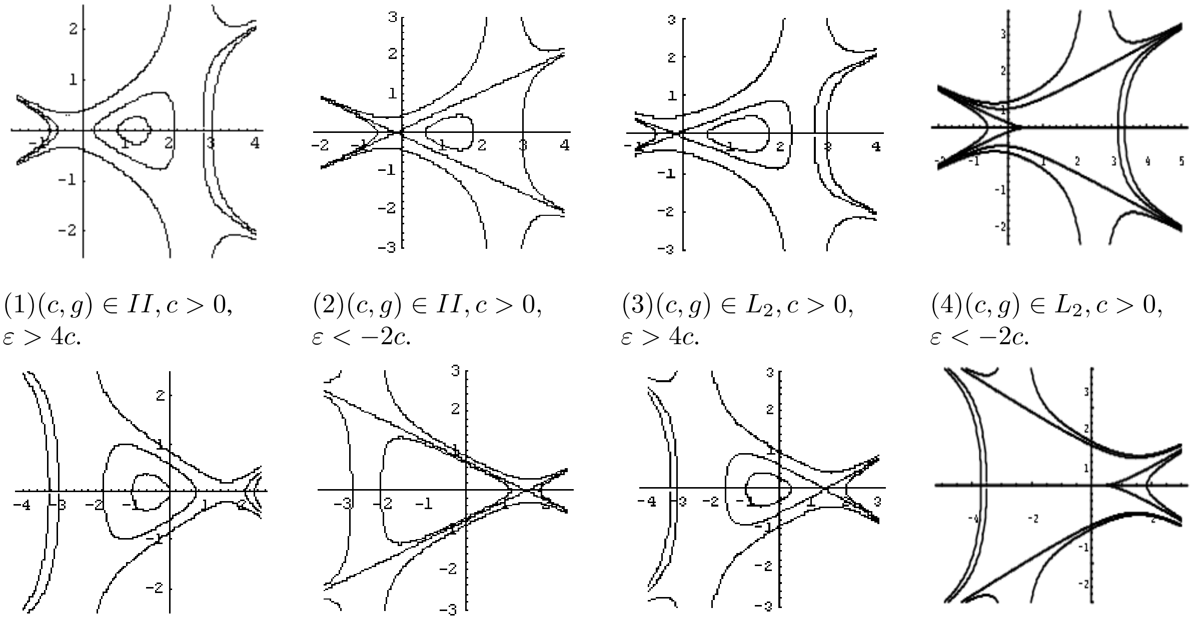
When $c < 0$, there are similar figures to Fig.2.

3 The peakons

3.1 Peakons from the limit of solitary waves

In this section, firstly we give a lemma to indict the relationship of solitary waves of Eq. (2.1) and homoclinic orbits of system (2.3). Secondly the information obtained from the topological phase portraits of system (2.3) is used to derive the peakons from the limit of solitary waves corresponding to homoclinic orbits.

Lemma 1 *Assume that Γ is a homoclinic orbit of system (2.3) and its parameter expression is $\varphi = \varphi(\xi)$ and $y = y(\xi)$, then $u = \varphi(\xi)$ with $\xi = x - ct$ is a solitary wave solution of Eq. (2.1).*



$(1)(c, g) \in II, c > 0,$
$\varepsilon > 4c.$

$(2)(c, g) \in II, c > 0,$
$\varepsilon < -2c.$

$(3)(c, g) \in L_2, c > 0,$
$\varepsilon > 4c.$

$(4)(c, g) \in L_2, c > 0,$
$\varepsilon < -2c.$

$(5)(c, g) \in III, c > 0,$
$\varepsilon > 4c.$

$(6)(c, g) \in III, c > 0,$
$\varepsilon < -2c.$

$(7)(c, g) \in L_1, c > 0,$
$\varepsilon > 4c.$

$(8)(c, g) \in L_1, c > 0,$
$\varepsilon < -2c.$

Figure 2: The phase portrait bifurcation of system (2.4).

Proof. A traveling wave solution of Eq. (2.1) is called a solitary wave if $\varphi(\xi)$ has a well-defined limit as $|\xi|$ approaches infinity. Usually, a solitary wave solution of Eq. (2.1) corresponds to a homoclinic orbit of system (2.3). Similarly, a periodically traveling wave solution of (2.1) corresponds to a periodic orbit of system (2.3). From the Fig. 2 (3), we see that Γ surrounds (φ_{1+}, 0) and connects with (φ_{1-}, 0), or surrounds (φ_{1-}, 0) and connects with (φ_{1+}, 0) (Fig. 3). Therefore, $\lim\limits_{|\xi|\to\infty} \varphi(\xi) = \varphi_{1+}$ or $\lim\limits_{|\xi|\to\infty} \varphi(\xi) = \varphi_{1-}$, where $\varphi_{1\pm} = \frac{c\pm\sqrt{c^2+4g}}{2}$.

On the other hand, $\varphi = \varphi(\xi)$ is the solution of system (2.4). This implies $\varphi = \varphi(\xi)$ is the solution of Eq.(2.2). Thus $u = \varphi(x - ct)$ is the solitary wave solution of Eq.(2.1). ■

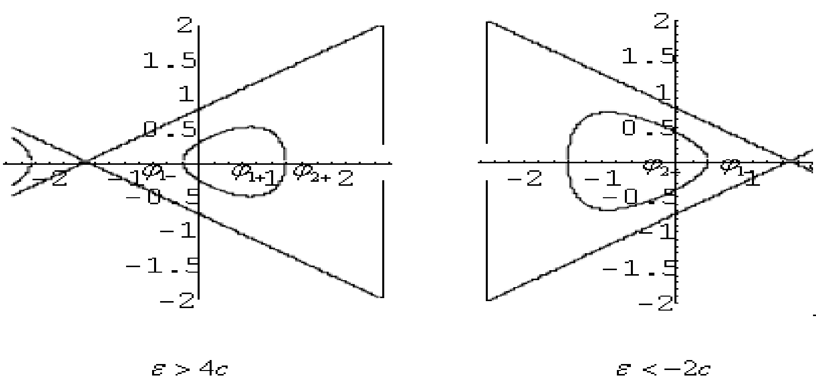


Figure 3: The homoclinic orbit of system (2.4).

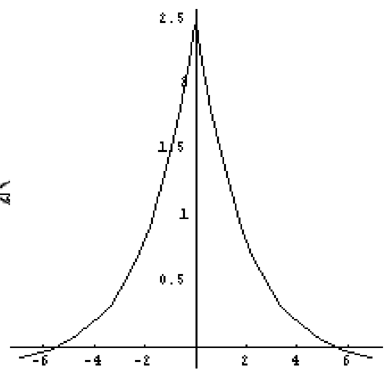


Figure 4: The peaked solitary wave solution for Eq. (2.1): $c = 1, \varepsilon = 5$.

In Fig. 3 the homoclinic orbit can be represented as $H(\varphi, y) = H(\varphi_{1-}, 0)$. Namely

$$(2\varphi-\varepsilon)^2[\tfrac{1}{2}(g+\tfrac{c\varepsilon}{6}-\tfrac{\varepsilon^2}{24})+(\tfrac{c}{3}-\tfrac{\varepsilon}{12})\varphi-\tfrac{1}{4}\varphi^2+y^2]=\tfrac{1}{48}(-c+\sqrt{c^2+4g}+\varepsilon)^2[2c^2+2c\varepsilon+12g-2\sqrt{c^2+4g}(c-\varepsilon)-\varepsilon^2]. \quad (3.1)$$

From (3.1), let $y = 0$, φ_{2+} can be obtained.

Have $H(\varphi, y) = H(\varphi_{1+}, 0)$, Namely

$$(2\varphi-\varepsilon)^2[\tfrac{1}{2}(g+\tfrac{c\varepsilon}{6}-\tfrac{\varepsilon^2}{24})+(\tfrac{c}{3}-\tfrac{\varepsilon}{12})\varphi-\tfrac{1}{4}\varphi^2+y^2]=\tfrac{1}{48}(c+\sqrt{c^2+4g}-\varepsilon)^2[2c^2+2c\varepsilon+12g+2\sqrt{c^2+4g}(c-\varepsilon)-\varepsilon^2]. \quad (3.2)$$

From (3.2), let $y = 0$, and φ_{2-} can be obtained.

Substituting φ_{2+} and φ_{2-} into the first equation of system (2.4) and integrating along the homoclinic orbits, we get

$$\int_{\varphi}^{\varphi_{2+}} \frac{\sqrt{6}(\varepsilon-2\varphi)d\varphi}{\sqrt{c^4+6c^2g+6g^2+(c^2+4g)^{\frac{3}{2}}(c-\varepsilon)-c^3\varepsilon-6cg\varepsilon+12g\varepsilon\varphi-12g\varphi^2+6c\varepsilon\varphi^2-8c\varphi^3-4\varepsilon\varphi^3+6\varphi^4}} = -|\xi|, \varepsilon > 4c \quad (3.3)$$

$$\int_{\varphi_{2-}}^{\varphi} \frac{\sqrt{6}(\varepsilon-2\varphi)d\varphi}{\sqrt{c^4+6c^2g+6g^2-(c^2+4g)^{\frac{3}{2}}(c-\varepsilon)-c^3\varepsilon-6cg\varepsilon+12g\varepsilon\varphi-12g\varphi^2+6c\varepsilon\varphi^2-8c\varphi^3-4\varepsilon\varphi^3+6\varphi^4}} = -|\xi|, \varepsilon < -2c \quad (3.4)$$

Note that the following facts: When $K_1 < g < K_2$ and $g \to K_2$, where $K_1 = -\frac{c^2}{4}$, $\quad K_2 = \frac{\varepsilon^2-2c\varepsilon-8c^2}{36}$, the limiting cures of such homoclinic orbits of system (2.4) is a triangle with the following three line segments(Fig. 3), and two diagonal lines are expressed by

$$y = \pm\frac{1}{2}(\varphi - \frac{4c-\varepsilon}{6}).$$

Letting $K_1 < g < K_2$ and $g \to K_2$, in (3.3) and (3.4), we get

$$\varphi(\xi) \to \frac{2}{3}(\varepsilon - c)e^{-\frac{1}{2}|\xi|} + \frac{4c-\varepsilon}{6},$$

which implies that the Eq.(2.1) has peakons

$$u_1(x,t) = \frac{2}{3}(\varepsilon - c)e^{-\frac{1}{2}|x-ct|} + \frac{4c-\varepsilon}{6}. \quad (3.5)$$

Obviously u has peaks at $x - ct = 0$. The peakons expressed by $u(x,t)$ are shown in Fig. 4 under some parameter conditions.

3.2 Peakons from the limit of periodic cusp waves

In this section, firstly the relationship of periodic waves of Eq. (2.1) and periodic orbits of system (2.3) is given, secondly the information obtained from the topological phase portraits is used to derive the peakons from the limit of periodic cusp waves corresponding to periodic orbits. Similar to Lemma 1, we have

Lemma 2 *Assume that Γ is a periodic orbit of system (2.3) and its parameter expression is $\varphi = \varphi(\xi)$ and $y = y(\xi)$, then $u = \varphi(\xi)$ with $\xi = x - ct$ is a periodic wave solution of (2.1).*

From Fig. 2 (1) it is seen that when $K_2 < g < K_3$, the system (2.3) has a periodic orbit which consist of an arc and a line segment (Fig. 5).

In Fig. 5 the periodic orbit can be expressed

$$y^2 = \frac{1}{4}\varphi^2 - (\frac{1}{3}c - \frac{1}{12}\varepsilon)\varphi - \frac{1}{2}(g + \frac{c\varepsilon}{6} - \frac{\varepsilon^2}{24}), \quad (3.6)$$

and $\varphi = \frac{\varepsilon}{2}$, where $\varphi_{3\pm} = \frac{4c-\varepsilon}{6} \pm 2\sqrt{\frac{-\varepsilon^2+2c\varepsilon+8c^2+36g}{72}}$.

Note that when $K_2 < g < K_3$ and $g \to K_2$, the periodic orbits lose their smoothness and become non-smooth periodic orbits, and when $g = K_2$, the periodic orbits become periodic cusp orbits. Substituting (3.6) into the first equation of system (2.3) and integrating along the periodic orbit, we get

$$u_2(\xi) = \begin{cases} v_1(\xi + 2nT), & c > \varepsilon, \\ v_2(\xi + 2nT), & c < \varepsilon. \end{cases}$$

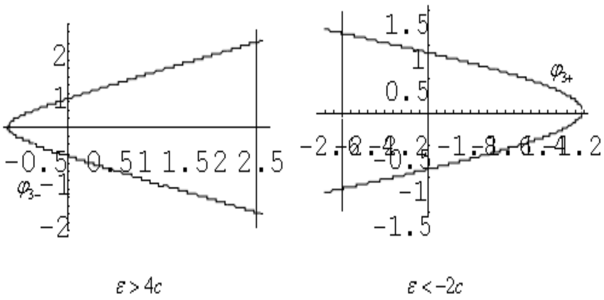


Figure 5: The periodic orbit of system (2.4) when $K_2 < g < K_3$.

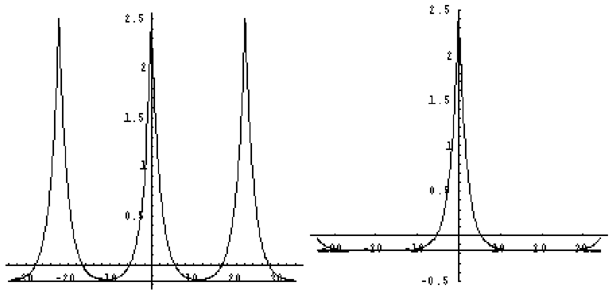


Figure 6: The periodic cusp wave solution for Eq. (2.1): $c = 1, \varepsilon = 5$.

where $n = 0, \pm1, \pm2, \cdots$, and $\xi \in [(2n-1)T, (2n+1)T]$.

$$v_1(\xi) = \frac{4c-\varepsilon}{6} + (\frac{\varepsilon - c}{3} + \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{\frac{1}{2}|\xi|} + (\frac{\varepsilon - c}{3} - \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{-\frac{1}{2}|\xi|},$$

$$v_2(\xi) = \frac{4c-\varepsilon}{6} + (\frac{\varepsilon - c}{3} - \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{\frac{1}{2}|\xi|} + (\frac{\varepsilon - c}{3} + \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{-\frac{1}{2}|\xi|},$$

$$-T < \xi < T, T = 2\left|\ln(\sqrt{\frac{-\varepsilon^2 + 2c\varepsilon + 8c^2 + 36g}{72}}) - \ln\left|\frac{\varepsilon - c}{3} + \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}}\right|\right|.$$

When $K_2 < g < K_3$, and $c > \varepsilon$, let $g \to K_2$, then $T \to \infty$, $\frac{\varepsilon-c}{3} + \sqrt{\frac{\varepsilon^2-2c\varepsilon-4g}{8}} \to 0$, and $\frac{\varepsilon-c}{3} - \sqrt{\frac{\varepsilon^2-2c\varepsilon-4g}{8}} \to \frac{2\varepsilon-2c}{3}$.
When $K_2 < g < K_3$, and $c < \varepsilon$, let $g \to K_2$, then $T \to \infty$, $\frac{\varepsilon-c}{3} - \sqrt{\frac{\varepsilon^2-2c\varepsilon-4g}{8}} \to 0$, and $\frac{\varepsilon-c}{3} + \sqrt{\frac{\varepsilon^2-2c\varepsilon-4g}{8}} \to \frac{2\varepsilon-2c}{3}$.

From above discussion one sees that when $K_2 < g < K_3$ and $g \to K_2$, the periodic wave solutions $v_1(\xi)$and $v_2(\xi)$ tend to the peakons

$$u_2(\xi) \to u_1(\xi) = \frac{2}{3}(\varepsilon - c)e^{-\frac{1}{2}|\xi|} + \frac{4c - \varepsilon}{6}, \tag{3.7}$$

The result (3.7) is identical to (3.5). The graph of some periodic wave for Eq. (2.1) is shown under some parameter condition (Fig. 6).

From Fig. 6 one can see that when $c = 1, \varepsilon = 5$, the period of periodic waves slowly become big. The period waves slowly lose their smoothness and become the periodic cusp wave. Finally the periodic cusp waves become the peakons and their periods become infinite.

4 Conclusion

In this paper the qualitative analysis methods of dynamical system are used to investigate the peaked wave solutions Eq. (2.1). By the phase portrait bifurcation of the traveling wave system, We obtain the peaked solitary wave solution: $u_1(\xi) = \frac{2}{3}(\varepsilon - c)e^{-\frac{1}{2}|\xi|} + \frac{4c-\varepsilon}{6}$ and the periodic cusp wave solution:

$$u_2(\xi) = \begin{cases} v_1(\xi + 2nT) & c > \varepsilon \\ v_2(\xi + 2nT) & c < \varepsilon \end{cases}, n = 0, \pm1, \pm2, \cdots, \xi \in [(2n-1)T, (2n+1)T]$$

$$v_1(\xi) = \frac{4c-\varepsilon}{6} + (\frac{\varepsilon - c}{3} + \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{\frac{1}{2}|\xi|} + (\frac{\varepsilon - c}{3} - \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{-\frac{1}{2}|\xi|},$$

$$v_2(\xi) = \frac{4c-\varepsilon}{6} + (\frac{\varepsilon - c}{3} - \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{\frac{1}{2}|\xi|} + (\frac{\varepsilon - c}{3} + \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}})e^{-\frac{1}{2}|\xi|},$$

$$-T < \xi < T, T = 2\left|\ln(\sqrt{\frac{-\varepsilon^2 + 2c\varepsilon + 8c^2 + 36g}{72}}) - \ln\left|\frac{\varepsilon - c}{3} + \sqrt{\frac{\varepsilon^2 - 2c\varepsilon - 4g}{8}}\right|\right|.$$

Finally when $K_2 < g < K_3$ and $g \to K_2$, the periodic wave solutions $u_2(\xi)$ tend to the peakons $u_1(\xi)$.

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